EM FIELDS + MICRO ENGINEERING NOTES

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EM Fields and Micro Engineering Notes, First Edition

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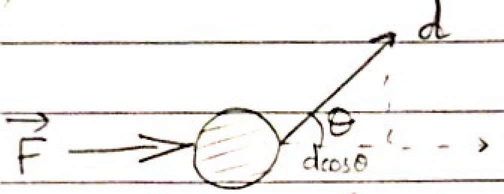
Vectors

BASICS

* DOT PRODUCT

Consider 2 vectors, force ($\vec{F}$) & displacement ($\vec{d}$)
Force applied on the ball displaces it. In this process, some work is done.

$$WD = |\vec{F}||\vec{d}|$$



Now, work done along direction of force is

$$WD = |\vec{F}||\vec{d}\cos\theta|$$
$$= |\vec{F}||\vec{d}|\cos\theta$$
$$\Rightarrow WD = \vec{F}\cdot\vec{d}$$

Basically, product of vectors in the dirⁿ of one vector is dot product

* Vector space operator : $\hat{\nabla}$
  └ takes only +ve incremental change

$$\hat{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

* $\vec{E} = -\frac{dV}{dx}$ or $-\nabla V$; $E = \frac{F}{q}$

* DOT product : Understood as DIVERGENCE

* $\hat{D}$: electric flux density
* $\hat{E}$: electric field intensity

$$\left.\right\} \vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P}$$

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* Angular displacement can be termed as AZIMUTHAL ANGLE (like, ϕ, from x axis)

* $\bar{\nabla} \cdot \bar{D} = \rho_v$: DIVERGENCE

~~$\bar{\nabla} \times \bar{E}$~~ $\hat{\nabla} \cdot \hat{E} = \rho_v / \epsilon$

$\bar{\nabla} \times \bar{F}$ = Curl.

Imp. * Note : ① $\vec{E}$ (static) always travels in straight line

$\Downarrow$

Its curl = 0 $\Rightarrow \vec{\nabla} \times \vec{E} = 0$

② $\vec{B}$ or $\vec{H}$ (magnetic field or magnetic field inten cannot diverge from one pt.

$\Downarrow$

Its divergence = 0 $\Rightarrow \vec{\nabla} \cdot \vec{B} = 0$

* Note : The direction of a plane is ⊥ to its surface. Its motion occurs in that direction.

Rectangular coordinate sys :-

* line, surface & volume vectors :-

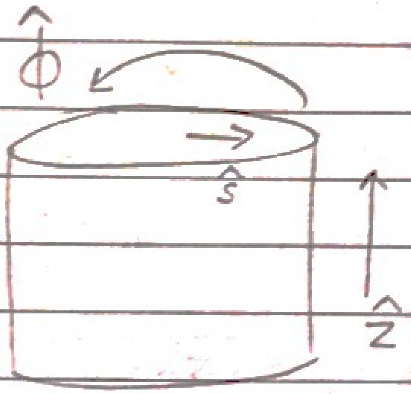
dl	ds	dv
differential line in	differential surface in	differential Volume in space
① → space	① → space	
$dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$	$ds = dxdy\,\hat{a_z} + dydz\,\hat{a_x} + dxdz\,\hat{a_y}$	$dv = dx\,dy\,dz$
② → line	② → plane	
x axis = $dx\,\hat{a_x}$	x plane = $dydz\,\hat{a_x}$	
y axis = $dy\,\hat{a_y}$	y plane = $dxdz\,\hat{a_y}$	
z axis = $dz\,\hat{a_z}$	z plane = $dxdy\,\hat{a_z}$	

*  $l = r\,d\theta$

* Cylindrical coordinate sys:-

Parameters: $r,\ \phi,\ z$ (Also represented as : $\rho,\ \phi,\ z$ or $r,\ \phi,\ z$)

r → radius, ϕ → angle (direction is tangent to circle of cylinder), z → height



$$\vec{ds} = (ds)_r,\ (ds)_z,\ (ds)_\phi$$

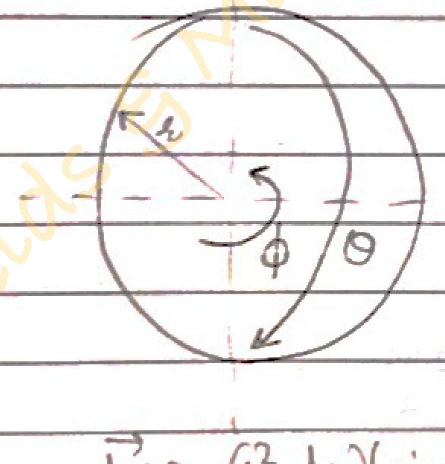
$$= (\rho\, d\phi)\,dz\,\hat{\rho},\ d\rho(\rho\, d\phi)\hat{z},\ d\rho\, dz\,\hat{\phi}$$

$$\vec{dv} = (d\rho)(dz)(\rho\, d\phi)$$

* Spherical Coordinate Sys:-

Parameters: $r,\ \theta,\ \phi$

$$\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$$



$$\vec{ds} = dr(r\,d\theta)\hat{\phi}^*$$

$$(r\,d\theta)(r\sin\theta\, d\phi)\hat{r}^*$$

$$(dr)(r\sin\theta\, d\phi)\hat{\theta}^*$$

$$\vec{dv} = (dr)(r\,d\theta)(r\sin\theta\, d\phi)$$

Extra:- $\vec{dv} = (r^2 dr)(\sin\theta\, d\theta)(d\phi)$

$$\Rightarrow V = \iiint (r^2 dr)(\sin\theta\, d\theta)(d\phi) = \int_0^r r^2 dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\Rightarrow V = \left(\frac{r^3}{3}\right)(2)(2\pi) = \frac{4}{3}\pi r^3 \text{ (result known before)}$$

H.P

✯ STATIC field

1. COULOMB'S LAW

→ Imp: Direction of force directed along the line joining the charges.

→ Notation :- F_{ij} , r_{ij} → distance from i^{th} charge to j^{th} charge $(j-i)$*

Force on i^{th} charge due to j^{th} charge

→ Prev. knowledge : Distance formula : Distance b/w any 2 pts (x_1, y_1, z_1) & (x_2, y_2, z_2) in space

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

→ Law :-

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

ϵ → Absolute permittivity $= \epsilon_0 \epsilon_r$

ϵ_0 → permittivity in free space or vacuum

ϵ_r → Relative permittivity

→ $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$

2. GAUSS LAW

$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{enclosed}}{\epsilon}.$$

$\oint$ → integral over closed surface

$\vec{dA}$ → or $\vec{ds}$

or

$$\oint \vec{D} \cdot \vec{dA} = Q_{enclosed}$$

Doesn't give direction

$\vec{D}$ → Electric flux density

→ Prev. knowledge :- $D = \epsilon E = \epsilon_0 \epsilon_r E$

$= \epsilon_0 E + P$ → Polariz

$= \epsilon_0 (1 + \chi) E$

χ → Electric susceptibility

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* Note, for a symmetrical surface, $\vec{E}$ (or $\vec{D}$) is constt. So, $\oint \vec{E}\cdot d\vec{s} = E\oint d\vec{s} = (E)(S)$ ← surface area
* $\vec{E} = \frac{\vec{F}}{q}$ (already known)

* **Potential** : It's the potential diff. with reference pt. at infinity

  - $V = \int \vec{E}\cdot d\vec{l}$
  - $\vec{E} = -\vec{\nabla}\cdot V$

* **Gradient of Potential field**

  - Cartesian Coordinate (x, y, z) ; $\nabla(x,y,z) = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
  - Cylindrical coordinate (s, ϕ, z), $\nabla(s,\phi,z) = \frac{\partial}{\partial s}\hat{s} + \frac{\partial}{s\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}$
  - Spherical coordinate (r, θ, ϕ), $\nabla(r,\theta,\phi) = \frac{\partial}{\partial r}\hat{r} + \frac{\partial}{r\partial\theta}\hat{\theta} + \frac{\partial}{r\sin\theta\partial\phi}\hat{\phi}$

* **Divergence of $\hat{D}$ (electric flux density) ($\vec{D}$)**

$$\vec{\nabla}\cdot\vec{D} = \lim_{\Delta V\to 0}\frac{\oint\vec{D}\cdot\vec{dA}}{\Delta V} = \frac{Q_{encl}}{\Delta V} = \rho_v$$

Gauss law; ρ_v → volume charge density

$\Rightarrow \vec{\nabla}\cdot\vec{D} = \rho_v$ → Gauss law in Point form

* **GAUSS DIVERGENCE THM.**

from Gauss law, $\oint\vec{D}\cdot\vec{dA} = Q_{encl} = \int\rho_v\,dV$

$= \int(\vec{\nabla}\cdot\vec{D})\,dV$

$$\Rightarrow \oint\vec{D}\cdot\vec{dA} = \iiint(\vec{\nabla}\cdot\vec{D})\,dV$$

* Avoid mistake :- whenever you take dl in ϕ dir, $\vec{dl}\big|_{\hat{\phi}} = \rho \, d\phi$ (or $s\,d\phi$)
||ly for other coordinate sys.

Q. Find volume charge density which establishes electric flux density,

$$\vec{D} = 4\rho^2 \phi^2 \, \hat{a}_\rho + 8\rho\phi^2 \, \hat{a}_\phi \quad C/m^2$$

We know $\vec{\nabla} \cdot \vec{D} = \rho_v$

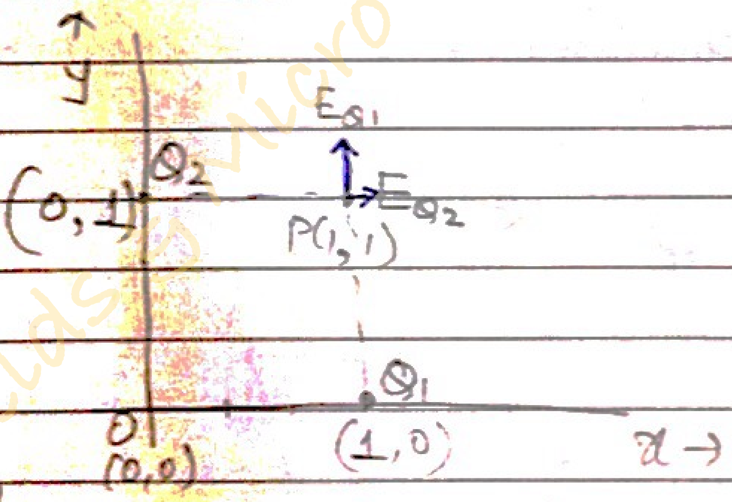
$$\Rightarrow \left(\hat{a}_\rho \frac{\partial}{\partial \rho}\right)\left(\vec{D}_\rho\right) + \left(\hat{a}_\phi \frac{\partial}{\rho\,\partial \phi}\right)\left(\vec{D}_\phi\right) + \left(\hat{a}_z \frac{\partial}{\partial z}\right)\left(\vec{D}_z\right) = \rho_v$$

$$\Rightarrow \frac{\partial}{\partial \rho}\left(4\rho^2\phi^2\right) + \frac{1}{\rho}\frac{\partial}{\partial \phi}\left(8\rho\phi^2\right) + \frac{\partial}{\partial z}(0) = \rho_v$$

$$\Rightarrow 8\rho\phi^2 + 16\phi = \rho_v$$

$$\Rightarrow \rho_v = 8\rho\phi^2 + 16\phi$$

Q. Find electric field intensity at pt $P(1,1,0)$ due to point charge $Q_1 = 9\times10^{-9}\,C$ at $(1,0,0)$ & $Q_2 = 9\times10^{-9}\,C$ at $(0,1,0)$



2 D diagram
∵ its only in X-Y plane
(Z axis = 0)

We know

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$$

So, $$\vec{E}_{Q_1} = \frac{Q_1}{4\pi\epsilon_0(1)^2}\,\hat{y} \quad V/m$$

$$\vec{E}_{Q_2} = \frac{Q_2}{4\pi\epsilon_0(1)^2}\,\hat{x} \quad V/m$$

So, net

$$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$$

$$= \frac{1}{4\pi\epsilon_0}\left[Q_1\right]\left[\hat{x} + \hat{y}\right]$$

$$= \frac{Q_1}{4\pi\epsilon_0}(\hat{x} + \hat{y})$$

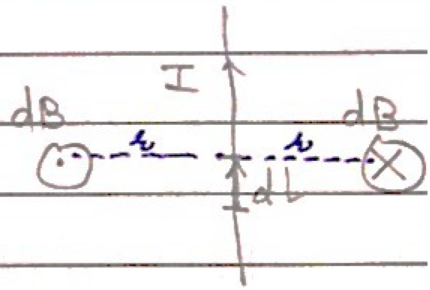
$$\Rightarrow \vec{E} = (9\times10^{9})(9\times10^{-9})(\hat{x} + \hat{y})$$

$$\Rightarrow \vec{E} = 81(\hat{x} + \hat{y}) \ V/m$$

Static Magnetic field.

1. BIOT-SAVART LAW :

$$d\vec{B} = \frac{\mu}{4\pi}\frac{I(\vec{dl} \times \vec{r})}{r^2}$$

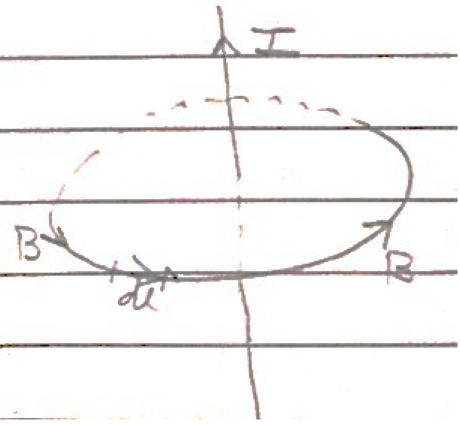


2. AMPERE'S ~~to~~ LAW

↳ Concept of Amperial loop.

$$\oint \vec{B} \cdot \vec{dl} = I_{enclosed}$$

line integral → ~~or~~ $\vec{H}$.



* **CURL** :- (Ampere's law in differential form)

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$\vec{H}$ or $\vec{B}$.

$\vec{J}$ → current density $= \frac{I}{A}$, A → or S

mathematically

$$\lim_{\Delta s \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta S} = \frac{I}{\Delta s} = \vec{J}$$

* ~~STOKE'S Theorem :~~

In cylindrical coordinate sys,

$$\vec{H} = H_s \hat{s} + H_\phi \hat{\phi} + H_z \hat{z}$$

So, $$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{s\,\partial \phi} & \frac{\partial}{\partial z} \\ H_s & H_\phi & H_z \end{vmatrix}$$

or, $$\frac{1}{s} \begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_s & sH_\phi & H_z \end{vmatrix}$$

s or ρ

In spherical coordinate sys,

$$\vec{H} = H_r \hat{r} + H_\theta \hat{\theta} + H_\phi \hat{\phi}$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{r\,\partial \phi} & \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \\ H_r & H_\theta & H_\phi \end{vmatrix}$$

or $\dfrac{1}{(r)(r\sin\theta)}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{vmatrix}$

eg find current density ($\vec{J} = \vec{\nabla}\times\vec{H}$) which establishes magnetic field intensity,

$$\vec{H} = 2\phi\,\hat{r} + 2\hat{\theta} + \frac{1}{r}\hat{\phi}.$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ 2\phi & 2r & \sin\theta \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{r}(\cos\theta - 0) - r\hat{\theta}(0-2) + r\sin\theta\,\hat{\phi}(2)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\cos\theta\,\hat{r} + 2r\,\hat{\phi} + 2r\sin\theta\,\hat{\phi}\right]$$

☆ STOKE'S Theorem :-

We know, $\oint \vec{H}\cdot\vec{dl} = I_{en}$ (Ampere's law) & $I = \iint \vec{J}\cdot\vec{ds}$

$\Rightarrow \oint \vec{H}\cdot\vec{dl} = \iint \vec{J}\cdot\vec{ds}$

$\Rightarrow \oint \vec{H}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{H})\,ds$

So, $\oint \vec{H}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{H})\,ds$

* In static field, for a closed loop,

$$\oint \vec{E} \cdot \vec{dl} = 0$$

↳ By Stoke's law,

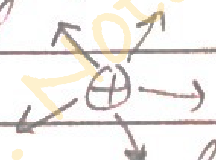
$$\oint \vec{H} \cdot \vec{dl} = \iint (\vec{\nabla} \times \vec{H}) ds$$

$$\Rightarrow \oint \vec{E} \cdot \vec{dl} = \iint (\vec{\nabla} \times \vec{E}) \cdot ds = 0$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = 0$$

↳ $\Rightarrow$ $\vec{E}$ always travels in st line

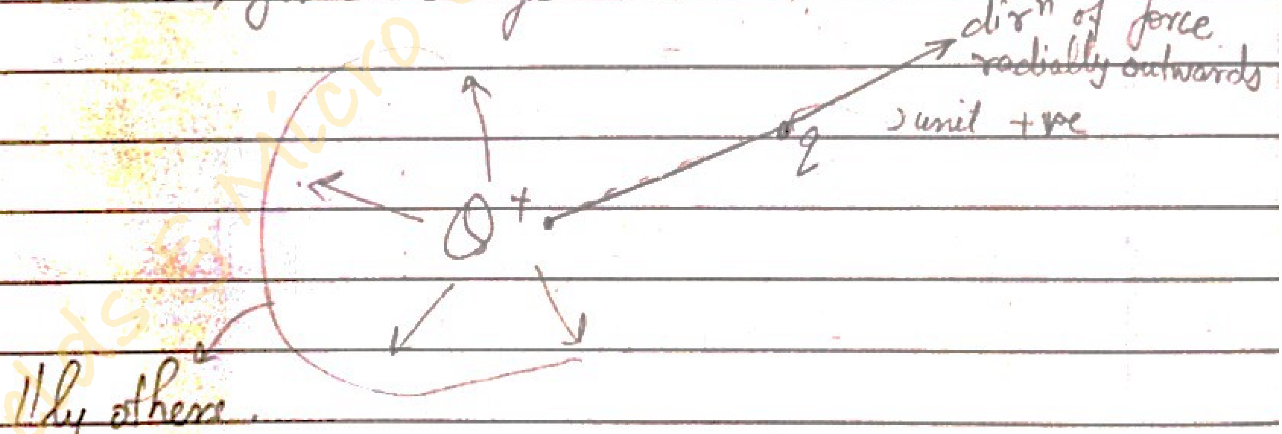
Q Why, for a +ve charge, $\vec{E}$ is radially outwards? i.e, why



?

Ans :- $\vec{E}$ is defined with respect to a unit +ve test charge & force due to that

So, for a charge Q (+ve)



* For a closed surface, $\vec{B}$ inside it starts from one surface & comes back from other. So, its not diverged. $\Rightarrow \vec{\nabla} \cdot \vec{B} = 0$.

Now, $\iiint (\vec{\nabla} \cdot \vec{B}) = \oint \vec{B} \cdot \vec{ds}$

$$\Rightarrow \oint \vec{B} \cdot \vec{ds} = 0$$

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* Inconsistency in Ampere's Law:

~~Sat~~ Said : $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0$.

Actually, $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H})$

$= \vec{\nabla}\cdot\vec{J} \quad \left(= -\frac{\partial \rho_v}{\partial t}\right)$

$\neq 0 \quad$ (so, inconsistent)

* $\vec{J} = \sigma\vec{E}$

↳ σ : conductivity $= nq\mu$ → mobility.

→ Let $\vec{\nabla}\times\vec{H} = \vec{J} + \vec{G}$

↳ some $\vec{G}$ s.t. $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0$.

$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot(\vec{J}+\vec{G})$

$= \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{G}$

$= \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{G}$

0 (as per defin^n)

$\Rightarrow \vec{\nabla}\cdot\vec{G} = -\vec{\nabla}\cdot\vec{J}$

$= -\left(-\frac{\partial \rho_v}{\partial t}\right)$

Also, $\vec{\nabla}\cdot\vec{D} = \rho_v$

$\Rightarrow \vec{\nabla}\cdot\vec{G} = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$

$= \nabla\cdot\left(\frac{\partial \vec{D}}{\partial t}\right)$

$\Rightarrow \vec{G} = \frac{\partial \vec{D}}{\partial t}$.

So, modific^n for Time varying field:

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$ (modified form of Ampere's law)

↳ In static field :- $\vec{\nabla}\times\vec{H} = \vec{J}$.

★ FARADAY'S LAW

$$\text{emf}\Big|_{\text{induced}} = \boxed{-\frac{d\phi}{dt} \left(= -\oint \vec{E}\cdot\vec{dl} \right)}$$

(Current flowing across a closed path)

Solving further:-

By Stoke's theorem:-

$$\oint \vec{E}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{E})$$

$$\& \oint \vec{E}\cdot\vec{dl} = -\frac{d\phi}{dt}$$

$$\Rightarrow \iint \vec{\nabla}\times\vec{E} = -\frac{d\phi}{dt}$$

Now, $\phi = \iint \vec{B}\cdot\vec{ds}$

$$\Rightarrow \iint \vec{\nabla}\times\vec{E} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{ds}$$

or $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}}$

Time varying magnetic field produces varying electric field.

★ MAXWELL'S EQN^{s}

A for Static field

(i) Integral form

(a) Gauss law:- $\iint \vec{D}\cdot\vec{ds} = \iiint (\vec{\nabla}\cdot\vec{B})\,dV$

Proof :- $\iint \vec{D}\cdot\vec{ds} = Q_{encl}$

$= \iiint \rho_v \, dV$

$\iint \vec{D}\cdot ds = \iiint (\vec{\nabla}\cdot\vec{D})\,dV.$

(b) Ampere's law :- $\iint \vec{B}\cdot\vec{ds} = 0, \oint \vec{H}\cdot\vec{dl} = I = \iint \vec{J}\cdot\vec{ds}$

(c) Faraday's law: $\oint \vec{E}\cdot\vec{dl} = 0$

(ii) Differential form:-

$$\vec{\nabla}\cdot\vec{D} = \rho_v$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{H} = \vec{J}$$
$$\vec{\nabla}\times\vec{E} = 0$$

(B) for time varying field

(ii) Differential form:

$$\vec{\nabla}\cdot\vec{D} = \rho_v$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{dt}$$
$$= \sigma\vec{E} + \frac{\partial}{\partial t}(\epsilon\vec{E})$$
$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$

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(i) Integral form

$$\oint \vec{D}\cdot\vec{ds} = \iiint \rho_v \, dV$$

$$\oiint \vec{B}\cdot\vec{ds} = 0$$

remain same

$$\oint \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot\vec{ds}$$

$$= \text{emf} = -\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial t}\left(\iint \vec{B}\cdot\vec{ds}\right)$$

$$\oint \vec{H}\cdot\vec{dl} = I = \iiint\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\vec{ds}$$

★ MAXWELL'S EQNS IN PHASOR FORM.

Shortcut: Consider a time varying field

$$A = A_0 e^{j\omega t}$$

$$\frac{dA}{dt} = (j\omega)\underbrace{\left(A_0 e^{j\omega t}\right)}_{A}$$

$$\Rightarrow \frac{dA}{dt} = j\omega A \quad \text{or} \quad \frac{d}{dt}(A) = j\omega(A)$$

So, whenever we do $\frac{d}{dt}$ of a time varying field,

replace $\frac{d}{dt} \rightarrow j\omega$

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In differential form:

$$\vec{\nabla}\cdot\vec{D} = \rho_v$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \longrightarrow \text{time varying field}$$

$$\left.\begin{aligned} &= \vec{J} + (j\omega)\vec{D} \\ &= \sigma\vec{E} + (j\omega)(\epsilon\vec{E}) \\ \Rightarrow \vec{\nabla}\times\vec{H} &= (\sigma + j\omega\epsilon)E \end{aligned}\right\} \text{Phasor form}$$

$$\& \quad \vec{\nabla}\times\vec{E} = (-j\omega)\vec{B}$$

$$\left.\Rightarrow \nabla\times\vec{E} = (-j\omega)\mu\vec{H}\right\} \text{Phasor form.}$$

In integral form:

$$\oiint \vec{D}\cdot\vec{ds} = \iiint (\nabla\cdot\vec{D})\,dv$$

$$\oiint \vec{B}\cdot\vec{ds} = 0$$

$$\oint \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot\vec{ds}$$

$$\left.\begin{aligned} &= -j\omega\iint \vec{B}\cdot ds \\ &= -j\omega\mu\iint \vec{H}\cdot\vec{ds} \end{aligned}\right\} \text{Phasor form}$$

$$\oint \vec{H}\cdot\vec{dl} = \iint\left(\vec{J} + \frac{\partial}{\partial t}\vec{D}\right)ds$$

$$\left.\begin{aligned} &= \iint(\vec{J} + j\omega\vec{D})\,ds \\ \Rightarrow \oint \vec{H}\cdot\vec{dl} &= \iint(\vec{J} + j\omega\epsilon\vec{E})\,ds \end{aligned}\right\} \text{Phasor form}$$

$\star$ $I \times (II \times III) = (I \cdot III)II - (I \cdot II)III$

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$\star$ WAVE EQ^n

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times (\sigma + j\omega\epsilon)\vec{E}$$

$$= (\sigma + j\omega\epsilon)(\vec{\nabla} \times \vec{E})$$

$$= (\sigma + j\omega\epsilon)(-j\omega\mu H)$$

$$= -(j\omega\mu)(\sigma + j\omega\epsilon)\vec{H}$$

Now RHS

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = (\vec{\nabla} \cdot \vec{H})\nabla - \nabla^2 H$$

Now,

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\Rightarrow \vec{\nabla} \cdot (\mu\vec{H}) = 0$$

$$\Rightarrow \mu(\vec{\nabla} \cdot \vec{H}) = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{H} = 0$$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = -\nabla^2 H$$

$$\Rightarrow \cancel{-} \nabla^2 \vec{H} = \cancel{-}(j\omega\mu)(\sigma + j\omega\epsilon)\vec{H}$$

$$\Rightarrow \nabla^2 \vec{H} = (j\omega\mu)(\sigma + j\omega\epsilon)\vec{H}$$

$$\equiv \nabla^2 \vec{H} = \gamma^2 (\vec{H}) \longrightarrow (1)$$

$\rightarrow \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$\Rightarrow \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\rightarrow \gamma$: propagation constt

$\rightarrow \gamma = (\alpha + j\beta)$

how much red^n in amplitude takes place : Attenuation constt

$\rightarrow$ how much change in phase happens : Phase constt.

Now, $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}\times(-j\omega\mu\vec{H})$

$= (-j\omega\mu)(\vec{\nabla}\times\vec{H})$

$= (-j\omega\mu)(\sigma + j\omega\epsilon)E$

$\vec{\nabla}\cdot\vec{E}=0$ as field exists, but source doesn't

LHS

$\vec{\nabla}\times(\vec{\nabla}\times\vec{F}) = (\vec{\nabla}\cdot\vec{E})\vec{\nabla} - \vec{\nabla}^2\cdot E$

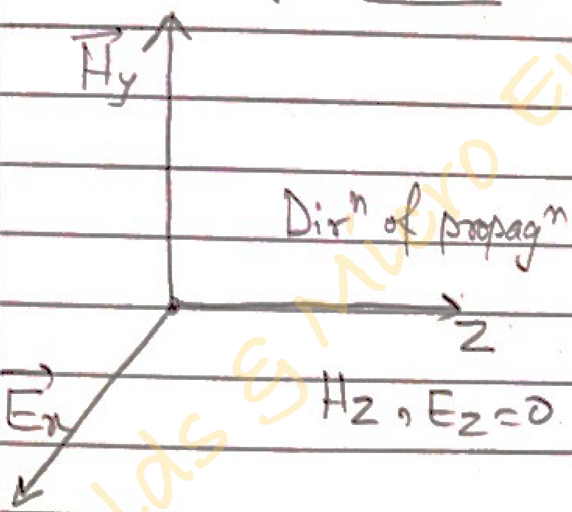
$= 0 - \vec{\nabla}^2\vec{E}$

$-\vec{\nabla}^2\vec{E} = -(j\omega\mu)(\sigma + j\omega\epsilon)\vec{E}$

$\Rightarrow \vec{\nabla}^2\vec{E} = \gamma^2\vec{E} \longrightarrow (2)$

$\hookrightarrow \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

Solving (1) & (2)



$H_z, E_z = 0$

From (1)

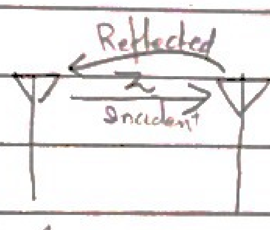
$\nabla^2 H_y = \gamma^2 H_y$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

- $\rightarrow H_x = 0$
- $\rightarrow H_y = \text{const.} \Rightarrow \frac{\partial^2}{\partial y^2}H = 0$
- $\rightarrow H_z$: variable w.r.t z.

$\Rightarrow \nabla^2 H_y = \frac{\partial^2}{\partial z^2}H_y = \gamma^2 H_y$

$\Rightarrow \frac{d^2 H_y}{dz^2} - \gamma^2 H_y = 0$



$\Rightarrow H_y = \underbrace{Ae^{-\gamma z}}_{\text{Incident component}} + \underbrace{Be^{\gamma z}}_{\text{Reflected component}} \rightarrow$ Ignore

* $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$: Propag^n constt

↳ μ, σ, ϵ : properties of medium

$\Rightarrow H_y = A e^{-\gamma z}$.

$\Rightarrow$ Magnetic intensity exponentially decays w.r.t distance

Finding A :-

At $z = 0$

$H_y = H_{y_0}$; $A = H_{y_0}$

$$\Rightarrow H_y = H_{y_0} e^{-\gamma z}$$

||ly,

$$E_x = E_{x_0} e^{-\gamma z}$$

for time varying field, we should have terms like $\sin\omega t$, $\cos\omega t$, $e^{j\omega t}$ etc. So, making $\vec{E}$ & $\vec{H}$ time varying,

$$E_x(z,t) = E_0 z e^{-\gamma z} \cdot e^{j\omega t}$$
$$= E_0 z e^{j\omega t - \gamma z}$$

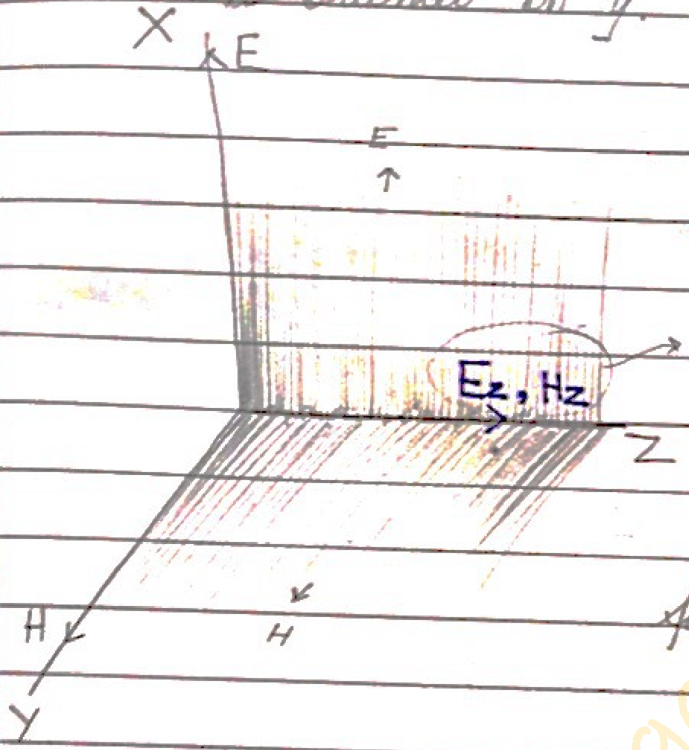
||ly, $H_y(z,t) = H_0 z e^{j\omega t - \gamma z}$

Intrinsic Impedance or wave impedance $= \dfrac{E}{H}$

* EM Wave in any medium

E is oriented in x, assuming $E_z = 0$

H is oriented in y. $H_z = 0$



$= 0$, here (assumed: no component of E or H along z)

now, let

$\vec{E} = E_x \hat{x}$ V/m

finding H_y

$$\vec{\nabla} \times \vec{E} = -j\omega\mu H$$

$$\Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -j\omega\mu H_y \hat{y}$$

$$\Rightarrow -\hat{y}\left(-\frac{\partial E_x}{\partial z}\right) + \underbrace{\hat{z}\left(-\frac{\partial E_x}{\partial y}\right)}_{0} = -j\omega\mu H_y \hat{y}$$

∂z → dir^n of propag^n

E_x → ≃ constt w.r.t y dir^n

$$\Rightarrow \hat{y}\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \hat{y}$$

or, $$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y$$

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Assume, $E_x(z) = E_{x0} e^{-\gamma z}$

$$\Rightarrow \frac{\partial}{\partial z} E_x(z) = -\gamma E_x(z)$$

So, we get

$$-\gamma E_x(z) = -j\omega\mu H_y(z)$$

$$\Rightarrow \frac{E_x(z)}{H_y(z)} = \frac{j\omega\mu}{\gamma}$$

$$\Rightarrow \left|\frac{E}{H}\right| = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

taking magnitude
(to get +ve value
of resistance)

$$\Rightarrow \eta = \left|\frac{E}{H}\right| = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

SEM Wave in Free Space :

for free space, $\sigma = 0$
$\epsilon = \epsilon_0$
$\mu = \mu_0$

$$\Rightarrow \gamma = \sqrt{j^2\omega^2 \epsilon_0 \mu_0} = j\omega\sqrt{\mu_0\epsilon_0}$$

now,

$$\gamma = \alpha + j\beta$$

$\beta \rightarrow$ phase constant

$\alpha \rightarrow$ attenuation constant comparing, $\alpha = 0$; $\gamma = j\beta = \omega\sqrt{\mu_0 \epsilon_0}$

So, $H(z,t) = H_0 z e^{(j\omega t - \gamma z)}$

$= H_0 z e^{(j\omega t - j\beta z)}$

$= H_0 z e^{j(\omega t - \beta z)}$

similarly, $E(z,t) = E_0 z e^{j(\omega t - \beta z)}$

now, Wave Impedance :

$$\left|\frac{E}{H}\right| = \sqrt{\frac{j\omega\mu_0}{\underbrace{\sigma}_{\to 0} + j\omega\epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$$

↳ for free space

for any medium, we had

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

rearranging, $= \sqrt{j^2\omega^2\mu\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$

$$\gamma = j\omega\sqrt{\mu\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$$

↳ $\frac{\sigma}{\omega\epsilon}$: determines whether a medium is conductor, dielectric or semi conductor

- $\frac{\sigma}{\omega\epsilon} > 100$: conductor
- $\frac{\sigma}{\omega\epsilon} < \frac{1}{100}$: dielectric
- $100 < \frac{\sigma}{\omega\epsilon} < \frac{1}{100}$: semi conductor

S EM ~~field in~~ wave in Conductor.

here,

$$\frac{\sigma}{\omega\epsilon} >> 100$$

approximating $\Rightarrow \gamma = j\omega\sqrt{\mu\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)}$

$$= j\omega\sqrt{\frac{\sigma\mu}{j\omega}}$$

$$\Rightarrow \gamma = \sqrt{\sigma\mu j\omega}$$

$$\Rightarrow \gamma = \sqrt{\omega\sigma\mu}\ \angle\theta/2$$

$\hookrightarrow |\sqrt{j}| = \theta/2$

$j \equiv \frac{\pi}{2}$

$$= \sqrt{\omega\sigma\mu}\ \angle 45^\circ$$

$$= \sqrt{\omega\sigma\mu}\ (\cos 45^\circ + j\sin 45^\circ)$$

$$= \sqrt{\frac{\omega\sigma\mu}{2}}\ (1+j)$$

now,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

&,

$$E(z,t) = E_0 z\, e^{j\omega t - \gamma z}$$

$$H(z,t) = H_0 z\, e^{j\omega t - \gamma z}$$

$\hookrightarrow$ γ changes for different mediums.

So, E, H also change.

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Types of EM Waves

Transverse Electric wave (TE wave)

Transverse Magnetic wave (TM wave)

Transverse Electromagnetic wave. (TEM wave)

for z : dir^n of propagation -

If $E_z = 0$: TE wave

$H_z = 0$: TM wave

$E_z = 0$ & $H_z = 0$: TEM wave

Poynting vector & theorem

We know,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \rightarrow (1)$$

Now, we know

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C} - \vec{B} \cdot (\vec{A} \times \vec{C})$$

So,

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = (\vec{\nabla} \times \vec{E}) \cdot \vec{H} - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \rightarrow (2)$$

Using (2) in (1),

$$\Rightarrow \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H})$$

we know, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\Rightarrow \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial}{\partial t}\vec{D} = \vec{H}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) \rightarrow (A)$$

Now, we know

$$\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \vec{E}\cdot\frac{\partial(\epsilon\vec{E})}{\partial t}$$

$$= \frac{\partial}{\partial t}\left(\vec{E}\cdot(\epsilon\vec{E})\right)$$

$$= \frac{\partial(\epsilon E^2)}{\partial t}$$

we see for rms value. So, just like power, we have $\frac{P}{\sqrt{2}}$ as rms. Similarly, Electric field, E changes as $\frac{E}{\sqrt{2}}$. So, $E^2 = \left(\frac{E}{\sqrt{2}}\right)^2 = \frac{E^2}{2}$

$$\text{So, } \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right) \rightarrow (3)$$

$$\text{Illy, } -\vec{H}\cdot\left(\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) \rightarrow (4)$$

$\vec{B} \rightarrow \mu\vec{H}$

Substitute (3) & (4) in (A)

$$\Rightarrow \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) - \vec{\nabla}\cdot(\vec{E}\times\vec{H})$$

Or,

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$$

$$\Rightarrow \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{E}\cdot\vec{J} - \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right]$$

$$= -\vec{J}\cdot\vec{E} - \frac{\partial}{\partial t}\left[\underbrace{\frac{1}{2}\epsilon E^2}_{\text{Energy stored in Electric field}} + \underbrace{\frac{1}{2}\mu H^2}_{\text{Energy stored in magnetic field}}\right]$$

Imp*

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \underbrace{-(\sigma E^2)}_{\text{represents Ohmic loss}} \underbrace{- \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)}_{\text{Energy change at negative rate}}$$

→ Poynting vector : gives dir^n of power.

→ Poynting theorem : what happens in expres^n of poynting vector : Ohmic loss and energy loss

* In transmission line, ∃ TEM wave ∴ E & H along dir^n of propag^n = 0.

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Waves at boundary

Sys. parameters

medium ①	medium ②
γ_1, η_1, Z_1	γ_2, η_2, Z_2
$\sigma_1, \mu_1, \epsilon_1$	$\sigma_2, \mu_2, \epsilon_2$

Z_2 → Characteristic impedance or wave impedance

E_i, H_i → EM wave → E_t, H_t

E_r, H_r ←

E_i, H_i: Incident component

E_r, H_r: reflected component

E_t, H_t: transmitted component

Type of wave	E_i, H_i	E_r, H_r	E_t, H_t
Standing wave	✓	✓	✗
Travelling wave	✓	✗	✓

* Transmission coefficient : $\dfrac{E_t}{E_i}$ or $\dfrac{H_t}{H_i}$

* Reflection coefficient : $\dfrac{E_r}{E_i}$ or $\dfrac{H_r}{H_i}$

Balancing vectors : $E_i + E_r = E_t$

& $H_i + H_r = H_t$

Using system components :

$Z_1 = \dfrac{E_i}{H_i}$, $Z_2 = \dfrac{E_t}{H_t}$.

& $Z_1 = \dfrac{E_r}{-H_r}$ → ∵ ∃ phase change, only in Magnetic field

∵ in same medium

Now, $H_t = \frac{E_t}{Z_2} = H_i + H_r$

$$\Rightarrow \frac{E_t}{Z_2} = \frac{E_i}{Z_i} - \frac{E_r}{Z_1} \rightarrow (1)$$

And, $E_t = (E_i + E_r)$

$\times \frac{Z_2}{Z_1}$, both sides

$$\Rightarrow \frac{Z_2}{Z_1} E_t = \frac{Z_2}{Z_1} E_i + \frac{Z_2}{Z_1} E_r \rightarrow (2)$$

From (1) & substituting

$$E_t = \frac{Z_2}{Z_1} (E_i - E_r)$$

$$\& \quad E_t = E_i + E_r$$

$$\Rightarrow E_i + E_r = \frac{Z_2}{Z_1} (E_i - E_r)$$

$$\Rightarrow (Z_1 + Z_2) E_r = (Z_2 - Z_1) E_i$$

$$\Rightarrow E_i = \left(\frac{Z_1 + Z_2}{Z_2 - Z_1}\right) E_r \quad \text{(form (1))}$$

Adding (1) & (2) & solving

$$E_t \left(1 + \frac{Z_2}{Z_1}\right) = \frac{2 Z_2}{Z_1} E_i$$

$$\Rightarrow \frac{E_t}{E_i} = \frac{2 Z_2}{Z_1 + Z_2} \quad \text{(form (2))}$$

only for normal incidence of any wave

★ $\tau = \frac{E_t}{E_i} \Rightarrow \tau = \frac{2 Z_2}{Z_1 + Z_2}$

$\rho = \frac{E_r}{E_i} \Rightarrow \rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

Subtracting (2) − (1)

defines property of wave going from one medium to another

& $\tau = 1 + \rho$

oblique incidence

normal incidence — $0°$

★ ANALYSING CONDUCTING MEDIUM

Free space	Conducting medium
$\eta_1 = Z_1 = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$	$Z_2 = Z_c$

Now, $Z_0 >> Z_c$
(for conducting medium, resistance is negligible)

now,
$$\tau = \frac{2Z_2}{Z_1+Z_2} = \frac{2Z_c}{Z_1+Z_c} \simeq \frac{2Z_c}{Z_0}.$$

$$\Rightarrow \tau_{conducting} = \frac{2Z_c}{Z_0}$$

&
$$\rho = \frac{Z_2-Z_1}{Z_2+Z_1} = \frac{Z_c-Z_1}{Z_c+Z_1} \simeq \frac{-Z_0}{Z_0} = -1$$

$$So,\ \rho = \frac{E_r}{E_i} = -1$$

$$\Rightarrow E_r = -E_i$$

↳ so, for conducting medium, for $\vec{E}$, the reflected wave has opp. dir^n as of incident wave.

★ ANALYSING PERFECT CONDUCTOR

↳ $Z_c = 0$

$$\Rightarrow \tau = \frac{2Z_c}{Z_1+Z_c} = 0$$

$$\rho = \frac{Z_2-Z_1}{Z_2+Z_1} = \frac{0-Z_0}{0+Z_0} = -1 = \frac{E_r}{E_i}$$

$$\Rightarrow E_r = -E_i$$

So, when EM wave passes/touches/crosses a conducting medium, ∃ no transmission (for perfect conductor)
almost 0 transmission (for conductor)

* for an antenna located horizontally, $\vec{E}$ is ∥ to earth surface
* Behaviour of earth : Low freq. : Dielectric
  High freq. : Conductor

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Polarization of EM Waves

$\Downarrow$

Direction of $\vec{E}$ field

Polarized waves

linear
- **horizontal**: $\vec{E}$ is ∥ to plane of polarizⁿ
- **vertical**: $\vec{E}$ is ⊥ to plane of incidence

circular: used to prevent distortion even in presence of Faraday rotation

elliptical: hyperbolic curve of antenna is there in it.

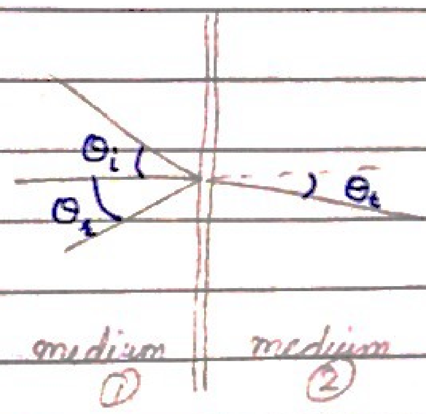
* FARADAY ROTATION

When an EM wave goes to ionosphere, → or any charged medium due to charges present in it, the polarizⁿ gets disturbed. So, a distortion occurs. This change of polarizⁿ is called Faraday rotⁿ.

* LINEAR polarization

for vertical

$$\rho_{\parallel} = \frac{E_r}{E_i} = \frac{Z_2 \cos\theta_t - Z_1 \cos\theta_i}{Z_1 \cos\theta_i - Z_2 \cos\theta_t}$$



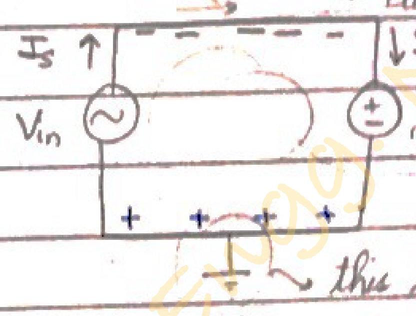
$$\tau_{\parallel} = 1 + \rho_{\parallel}$$

$$\rho_{\perp} = \frac{E_r}{E_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

$$\tau_{\perp} = 1 + \rho_{\perp}$$

✲ Transmission Lines

- using parameters (V, I)
  - in EM waves, we use $(E \& H)$
- has 4 primary constants
  1. has resistance, measured in Ω/km
     ✲ Transm^n occurs over a large distance, so, $V_{out} \neq V_{in}$
  2. Inductance (L), measured in mH/km
  3. Capacitance (C), measured in $\mu F/km$
     Seen as :-



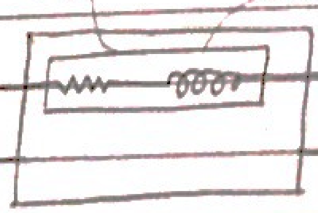
current ⇒ e^- flow -> -ve charge
; $I_R \neq I_s$
air gap
this line is PROPERLY Grounded!

Faraday's Induction Law: If one plate of C is charge & other plate is grounded; equal & opp. charge is induced on that plate

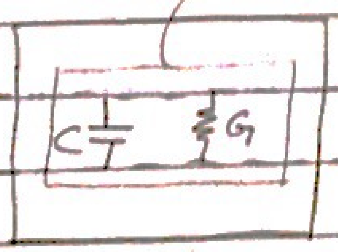
  4. Conductance, measured in $\mu ℧/km$

✲ Equivalent circuit of transmission lines :-

Z_1 → Series impedance



Z_2, shunt admittance



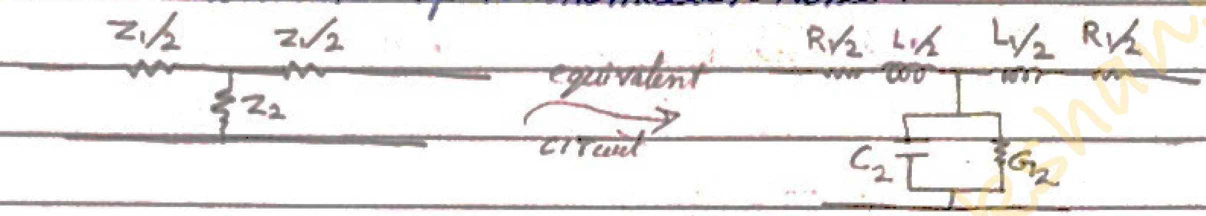
* SYMMETRICAL NETWORK

Network/Sys. whose electrical char. are unchanged when source & load are interchanged.

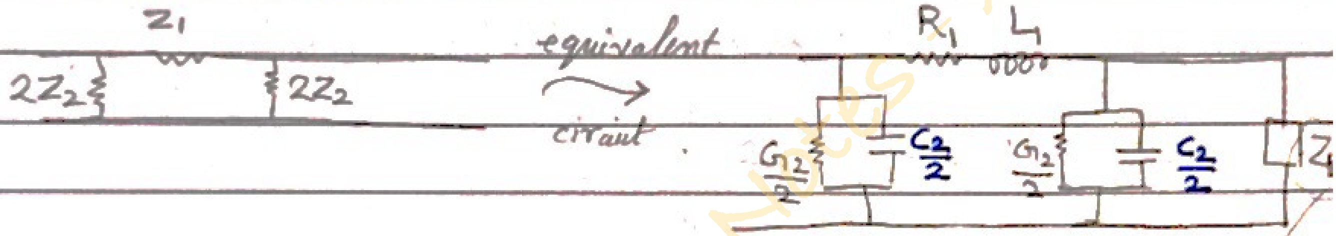
↳ eg: for a CE BJT, it is an unsymmetrical network

eg②: transmission lines are symmetrical networks.

T section network of transmission lines:



π section network



(using the relations:

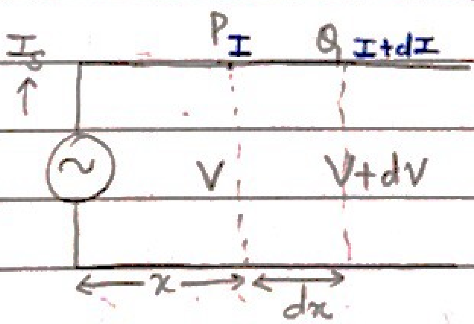
In parallel, $C = C_1 + C_2 = \frac{C}{2} + \frac{C}{2}$

$G = G_1 + G_2 = \frac{G}{2} + \frac{G}{2}$)

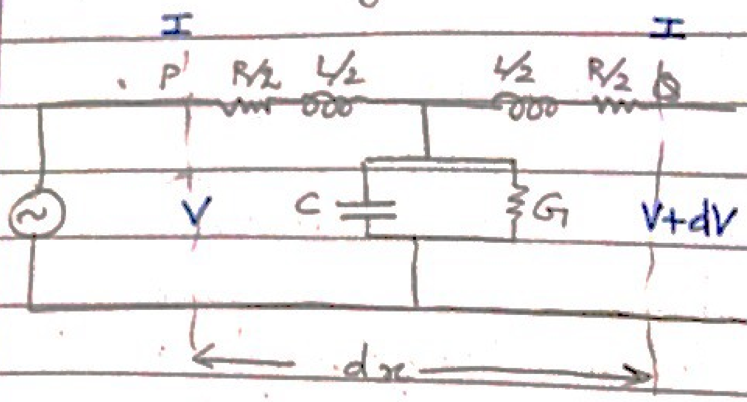
* Transmission Line eq^{ns}

↳ V & I at any pt on TL.

Consider an infinite TL & points P & Q at distance x & $x+dx$ from start



V & I are changing as we go through the line expanded analysis of dx length



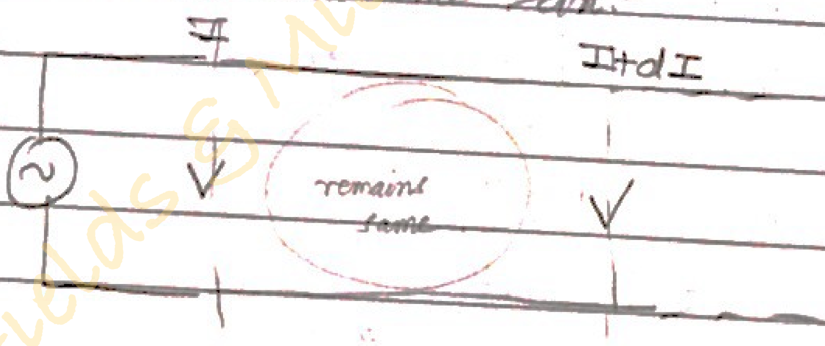
Finding change in voltage:
(assuming change in current is zero) across the distance dx,

$$V - (V + dV) = \left[(R + j\omega L)dx\right] I$$

Imp

$$\Rightarrow \boxed{\frac{dV}{dx} = -(R + j\omega L) I} \rightarrow (A)$$

Finding change in current:
(assuming change in voltage is zero) across the distance dx:



$$\Rightarrow I - (I + dI) = \left(\underbrace{G}_{\text{conductance}} + \underbrace{j\omega C}_{\text{susceptance}}\right) dx\, V$$

Imp

$$\Rightarrow \boxed{-\frac{dI}{dx} = (G + j\omega C) V} \rightarrow (B)$$

now, solving eq^n only in terms of V & only in terms of I.

* $e^{\pm px} = \cosh(px) \pm \sinh(px)$

Taking eqⁿ (A) & differentiating w.r.t x :

$$\Rightarrow -\frac{d^2V}{dx^2} = (R+j\omega L)\frac{dI}{dx}$$

$$\Rightarrow -\frac{d^2V}{dx^2} = (R+j\omega L)(-(G+j\omega C)V)$$

$$\Rightarrow \frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)\,V = p^2 V \rightarrow (C)$$

$\hookrightarrow p^2 = (R+j\omega L)(G+j\omega C)$

||ly, from eqⁿ (B) :

$$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega L)I$$

$$\Rightarrow \frac{d^2I}{dx^2} = p^2 I \rightarrow (D)$$

$\hookrightarrow p^2 = (R+j\omega L)(G+j\omega C)$

$p \rightarrow$ propagⁿ constt. $= \alpha + j\beta$

or

$$p = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Solⁿ of eqⁿ (C) & (D) :-

✓ TL eqⁿ with 4 unknown coeffs (a, b, c, d) in expo. form

$$V = ae^{px} + be^{-px}$$

$$I = ce^{px} + de^{-px}$$

✓ TL eqⁿ with 4 unknown coeffs, in hyperbolic form

$$V = a[\cosh px + \sinh px] + b[\cosh px - \sinh px]$$

$$= (a+b)\cosh px + (a-b)\sinh px$$

$$\Rightarrow V = A\cosh px + B\sinh px \rightarrow (E)$$

||ly,

$$I = C\cosh px + D\sinh px \rightarrow (F)$$

TL eqⁿ with 2 unknown coeffs in hyperbolic form

From (A)

$$-\frac{dV}{dx} = (R+j\omega L)I$$

Substituting (E) in (A)

$$\Rightarrow -\frac{d}{dx}\left[A\cosh px + B\sinh px\right] = (R+j\omega L)I$$

$$\Rightarrow -\left[Ap\sinh px + Bp\cosh px\right] = (R+j\omega L)I$$

$$\Rightarrow -p(A\sinh px + B\cosh px) = (R+j\omega L)I$$

$$\Rightarrow I = \frac{-p(A\sinh px + B\cosh px)}{(R+j\omega L)}$$

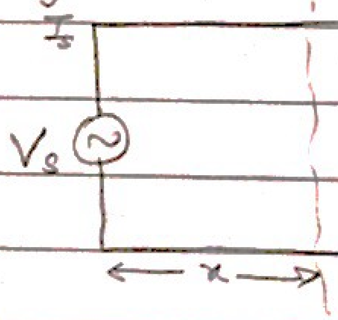
$$= \frac{-\sqrt{(R+j\omega L)(G+j\omega C)}\,(A\sinh px + B\cosh px)}{(R+j\omega L)}$$

$$\Rightarrow I = -\underbrace{\left[\sqrt{\frac{G+j\omega C}{R+j\omega L}}\right]}_{}\times(A\sinh px + B\cosh px)$$

Characteristic impedance of the line, $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$\Rightarrow I = -\frac{1}{Z_0}(A\sinh px + B\cosh px) \longrightarrow (G)$$

finding A & B



when $x=0$, $V=V_s$, $I=I_s$

* $\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\sinh x = \cosh x$

$\because x = 0, \Rightarrow$ in Eqⁿ (F)

$$V = V_s = A$$

& eqⁿ (G) $I = I_s = -\frac{1}{Z_0} B$

$$\text{or } B = -I_s Z_0$$

$$\Rightarrow \quad A = V_s$$
$$B = -I_s Z_0$$

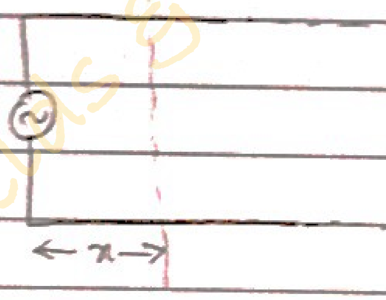
// TL eqⁿ with KNOWN coeff.

$$\Rightarrow \boxed{\begin{aligned} V &= V_s \cosh px - I_s Z_0 \sinh px \\ I &= I_s \cosh px - \frac{V_s}{Z_0} \sinh px \end{aligned}}$$

} finding V, I at any pt. on TL

~

* *Infinite length line*



finding V, I for infinite line

$$V = ae^{px} + be^{-px}$$
$$I = ce^{px} + de^{-px}$$

If $x \to \infty$, $V = V_R = 0$ → Receiving

& $I = I_R = 0$

* V & I have incident & reflected component

In a TL, considering power; some of power goes to load (Z_L) & remaining is reflected from load.

When $x \to \infty$
$e^{-Px} = 0$.

$\Rightarrow$ When line length $\to \infty$, we won't have any incident component at the load.

So, $(\ )e^{-Px}$: incident component
$(\ )e^{Px}$: reflected component.

* for infinite length line,
① $V_R = I_R = 0$
② No reflected component
③ Z_{in} (input impedance) $= Z_0$.

for $x \to \infty$, we have $V = V_R = 0$

$$\Rightarrow a e^{Px} + b e^{-Px} = 0$$
$$\Rightarrow a e^{P(\infty)} + b e^{-P(\infty)} = 0$$
$$\Rightarrow a(\infty) = 0$$

for this to be true, $a = 0$.

So, $V = b e^{-Px}$: the decreasing component; goes to load.

IIly, $I = d e^{-Px}$

Seeing initial condn (at $x=0$)

$\Rightarrow V = V_s = b$

source $\ I = I_s = d$.

So, my $V = V_s e^{-Px}$
$I = I_s e^{-Px}$

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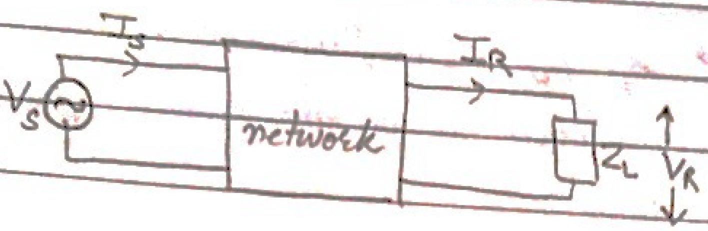
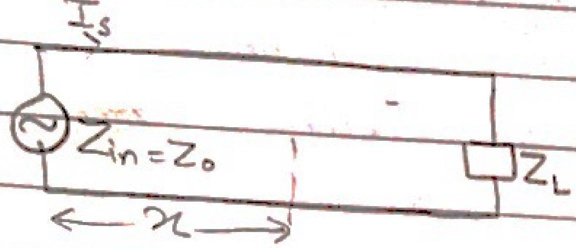
* Propagation Constant (P) = $\ln\left(\frac{I_s}{I_R}\right)$ or $\ln\left(\frac{V_s}{V_R}\right)$

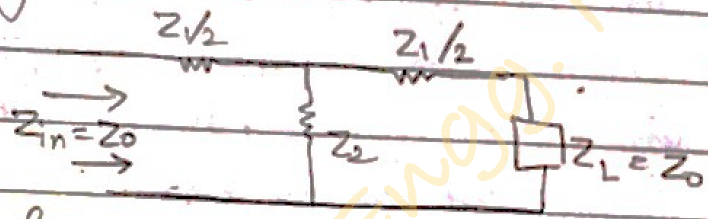
consider a std. network.





CASE — If $Z_L = Z_0$ (char. impedance), then $Z_{in} = Z_0$

eg:- for circuit



for this,
$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

we know,

TL eq^n in hyperbolic form is

$$V = V_s \cosh px - I_s Z_0 \sinh px$$
$$I = I_s \cosh px - \frac{V_s}{Z_0} \sinh px$$

When $x = l$

$$V_R = V_s \cosh pl - I_s Z_0 \sinh pl \rightarrow (1)$$
$$I_R = I_s \cosh pl - \frac{V_s}{Z_0} \sinh pl \rightarrow (2)$$

$$\frac{V_R}{I_R} = Z_0 = \frac{V_s \cosh pl - I_s Z_0 \sinh pl}{I_s \cosh pl - V_s/Z_0 \sinh pl}.$$

finding $Z_{in} = \frac{V_s}{I_s}$

$$\Rightarrow Z_0 = \frac{I_s}{I_s}\left(\frac{(V_s/I_s)\cosh pl - Z_0 \sinh pl}{\cosh pl - (V_s/I_sZ_0)\sinh pl}\right)$$

$\frac{V_s}{I_s}$

$\Rightarrow Z_0 \cosh pl - Z_{in} \sinh pl = Z_{in} \cosh pl - Z_0 \sinh pl$

$\Rightarrow Z_0 (\cosh pl + \sinh pl) = Z_{in} (\cosh pl + \sinh pl)$

$\Rightarrow Z_{in} = Z_0$

↳ If $Z_{in} = Z_0$, we have connected matched load impedance for transmission line.

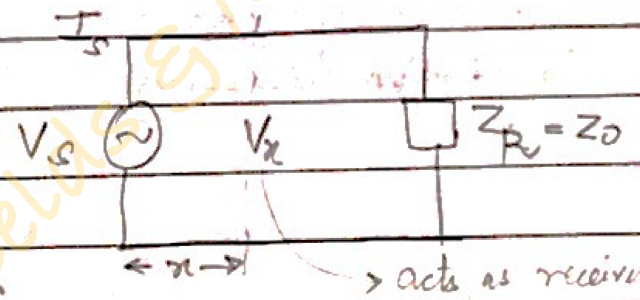
Basically, impedance is same everywhere.
So, change in power is nearly zero.
Hence, max power is transferred to load

⇓

- TL terminated by its char. impedance has NO REFLECTED Component.
- Infinite line becomes finite length line (& vice versa), when line is terminated by its char. impedance.

$\Rightarrow V = V_s e^{-px}, \; I_s = V_s e^{-px}$

For a TL terminated at $Z_R = Z_0$



> acts as receiving end at distance x.

$V_x = V_s e^{-px}$ & $I_x = I_s e^{-px}$

$\Rightarrow e^{-px} = \frac{V_x}{V_s} = \frac{I_x}{I_s}$

$\Rightarrow e^{px} = \frac{V_s}{V_x} = \frac{I_s}{I_x}$

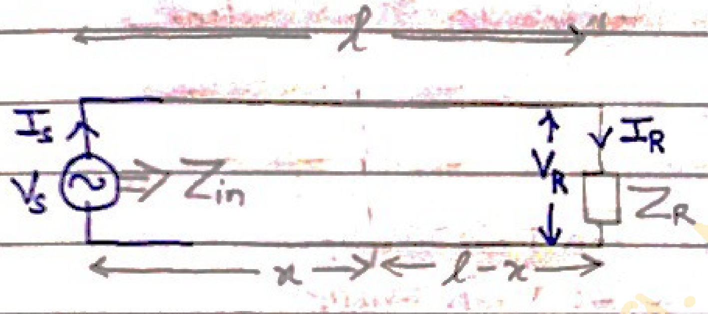
$\Rightarrow px = \ln\left(\frac{V_s}{V_x}\right) = \ln\left(\frac{I_s}{I_x}\right)$

Propagⁿ const. p.u. length $\left(= \frac{px}{x} \right)$

$$p = \ln\left(\frac{V_s}{V_x}\right) = \ln\left(\frac{I_s}{I_x}\right)$$

$$\Rightarrow \sqrt{(R+j\omega L)(G+j\omega C)} = p = \ln\left(\frac{V_s}{V_x}\right) = \ln\left(\frac{I_s}{I_x}\right)$$

* Input impedance of a TL.



from sending end,

$$V = V_s \cosh px - I_s Z_0 \sinh px$$
$$I = I_s \cosh px - \frac{V_s}{Z_0} \sinh px$$

from receiving end,

$$V = V_R \cosh p(l-x) + I_R Z_0 \sinh p(l-x)$$
$$I = I_R \cosh p(l-x) + \frac{V_R}{Z_0} \sinh p(l-x)$$

to find Z_{in}, reach sending pt. from receiving end.
Substituting $x=0$ in above eqⁿˢ

$$V = V_s \;\&\; I = V_s$$

$$\Rightarrow V_s = V_R \cosh p(l) + I_R Z_0 \sinh p(l)$$
$$I_s = I_R \cosh p(l) + \frac{V_R}{Z_0} \sinh p(l)$$

Now,

$$Z_{in} = \frac{V_s}{I_s} = \frac{V_R \cosh pl + I_R Z_0 \sinh pl}{I_R \cosh pl + \frac{V_R}{Z_0} \sinh pl}$$

$$\Rightarrow Z_{in} = \frac{1}{1/Z_0} \frac{(V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l)}{(I_R Z_0 \cosh \gamma l + V_R \sinh \gamma l)}$$

$$= (Z_0) \frac{(\cancel{I_R})}{(\cancel{I_R})} \frac{(V_R/I_R \cosh \gamma l + Z_0 \sinh \gamma l)}{(V_R/I_R \sinh \gamma l + Z_0 \cosh \gamma l)}$$

Now,

$$\frac{V_R}{I_R} = Z_R$$

$$\Rightarrow Z_{in} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_R \sinh \gamma l + Z_0 \cosh \gamma l} \right]$$

$$= Z_0 \frac{(\cancel{\cosh \gamma l})}{(\cancel{\cosh \gamma l})} \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

$$\Rightarrow Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

→ i/p impedance (Z_{in}) of a TL given any Z_R.

→ when $Z_R \to \infty$ (i.e, **Open circuit** at receiving end)

$$Z_{oc} = Z_{in} = Z_0 \left(\frac{Z_R}{Z_R} \right) \left(\frac{1 + \frac{Z_0}{Z_R} \tanh \gamma l}{\frac{Z_0}{Z_R} + \tanh \gamma l} \right)$$

$\frac{Z_0}{Z_R} \to 0$

$$\Rightarrow \boxed{Z_{oc} = Z_0 \coth \gamma l}$$

→ when $Z_R \to 0$ (i.e, **Short circuit** at receiving end)

$$Z_{sc} = Z_{in} = Z_0 \left(\frac{0 + Z_0 \tanh \gamma l}{Z_0 + 0} \right)$$

$$\Rightarrow \boxed{Z_{sc} = Z_0 \tanh \gamma l}$$

→ When a line is terminated by its char. impedance, $Z_{in} = Z_0$. Verifying it :

$$Z_{in} = Z_0 \left[\frac{Z_0 + Z_0 \tanh \gamma l}{Z_0 + Z_0 \tanh \gamma l} \right] = Z_0$$

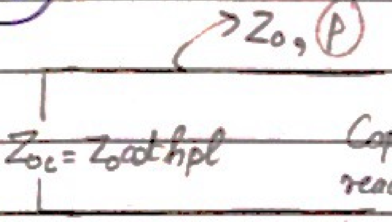
Verified

- for any TL,

$$Z_{sc} = Z_0 \tanh \gamma l$$
$$Z_{oc} = Z_0 \coth \gamma l$$
$$\Rightarrow Z_{sc} \times Z_{oc} = Z_0^2$$
$$\Rightarrow \boxed{Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}}$$

* Open circuited line and short circuited line

OC



Capacitive reactance

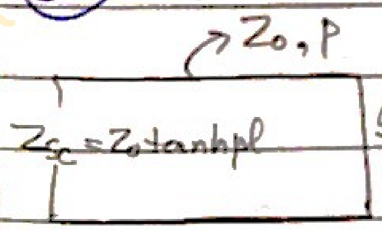
$\begin{pmatrix} V_R = V_{max} \\ I_R = 0 \end{pmatrix}$

* Assume length → very small
So, loss is very min.
$\Rightarrow \alpha \to 0. \Rightarrow \gamma = j\beta$

$$\& \; Z_{oc} = Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$
$$= Z_0 \frac{\cosh (j\beta l)}{\sinh (j\beta l)}$$
$$= Z_0 \frac{\cos(\beta l)}{(+j) \sin(\beta l)}$$
$$\Rightarrow Z_{oc} = -jZ_0 \cot(\beta l)$$

for capacitor, $X_C = \dfrac{-j}{\omega C}$

SC



Inductive reactance

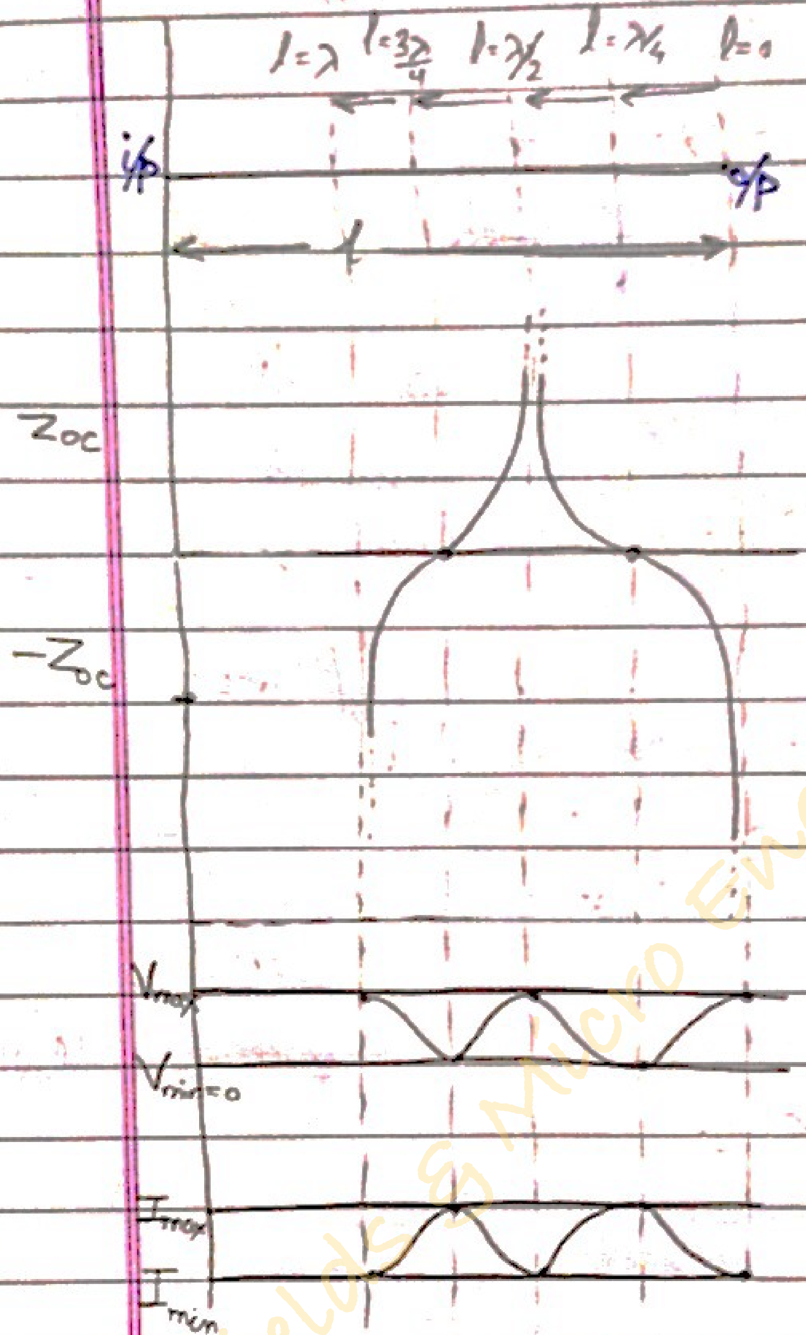
$\begin{pmatrix} V_R = 0 \\ I_R = max \end{pmatrix}$

$|||^{ly}$, $Z_{sc} = j \tan(\beta l)$

$\& \; X_L = j\omega L$

β: phase constt $= \dfrac{\text{Total phase}}{\text{Wavelength}} = \dfrac{2\pi}{\lambda}$

Now, finding values of Z_{oc} & Z_{sc}
So, travelling from receiving to sending end:



OPEN CIRCUIT ANALYSIS

For OC
$V_R = max$
$I_R = 0$.

Now, $Z_{oc} = -jZ_0 \cot\left(\frac{2\pi}{\lambda} l\right)$

→ when $l = 0$
$Z_{oc} = -j Z_0 \cot 0$
$\rightarrow -j(\infty)$

→ when $l = \frac{\lambda}{4}$
$Z_{oc} = -jZ_0 \cot\left(\frac{\pi}{2}\right)$
$= 0$

→ when $l = \frac{\pi}{2}$
$Z_{oc} = -jZ_0 \cot(\pi)$
$\rightarrow +j(\infty)$

Voltage :-
when $Z \rightarrow \pm\infty$, $V \rightarrow V_{max}$
$Z \rightarrow 0$, $V \rightarrow V_{min}$.

Current,
when $Z \rightarrow \pm\infty$, $I \rightarrow I_{min}$
$Z \rightarrow 0$, $I \rightarrow I_{max}$

Illy, for Short Circuit Analysis (self)

Line distortion
- Freq. distortion : Particular freq. is attenuated.
- Delay distortion : Changes in vel. of propag^n.

* LOSSLESS LINE (v_p remains constt)
  - $\alpha = 0$ (attenuation $= 0$)
  - velocity of propag^n, v_p = constt $\forall$ freq.
    - $v_p = \frac{\omega}{\beta}$ → β → phase constt of line

Idea: make v_p independent of ω.

how? ↓

we know; $p = \sqrt{(R+j\omega L)(G+j\omega C)}$

↳ If we take $R=0$ & $G=0$

$\Rightarrow p = \alpha + j\beta = j\omega\sqrt{LC}$

comparing

↳ $\Rightarrow \alpha = 0$ & $\beta = \omega\sqrt{LC}$

$\Rightarrow v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}}$

* Whenever $R=0$ & $G=0$, we get a lossless line (not practical)

$\Rightarrow \boxed{v_p = \frac{1}{\sqrt{LC}}}$
- velocity of propag^n
- $\exists$ no delay $\Rightarrow$ no loss

* DISTORTIONLESS LINE ($\Rightarrow \exists$ some constt distortion $\forall$ freq & vel. of propag^n of signal = constant)
  ↳ cond^n: $\frac{R}{L} = \frac{G}{C}$

Now, we know, $p = \sqrt{(R+j\omega L)(G+j\omega C)}$

$= \sqrt{(LC)\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)}$

$= \sqrt{LC\left(\frac{R}{L}+j\omega\right)^2}$ (or) $\sqrt{LC\left(\frac{G}{C}+j\omega\right)^2}$

$\Rightarrow p = \left(\frac{R}{L} + j\omega\right)\sqrt{LC}$ or $p = \left(\frac{G}{C} + j\omega\right)\sqrt{LC}$

Now, $p = \alpha + j\beta$

$\Rightarrow \alpha = R\sqrt{\frac{C}{L}}$ or $G\sqrt{\frac{L}{C}}$ & $\beta = \omega\sqrt{LC}$

attenuation, Independent of freq.

Now,

$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

velocity of propag^n. Independent of freq.

So, we got what was desired

Now,

how to get $\frac{R}{L} = \frac{G}{C}$?

Thought process : way : increase L (i.e, place inductor throughout the TL)

usually, $R \uparrow$ throughout TL, making $\frac{R}{L} \gg \frac{G}{C}$.

So, by $\uparrow L$, $\frac{R}{L} = \frac{G}{C}$. becomes possible

Method :

* Cond^n req^d for line to have min attenuation

We know

$$p = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta \longrightarrow (A)$$

Squaring both sides

$$\Rightarrow (\alpha^2 - \beta^2) + j(2\alpha\beta) = (R+j\omega L)(G+j\omega C)$$

$$\Rightarrow (\alpha^2 - \beta^2) + j(2\alpha\beta) = (RG - \omega^2 LC) + j\omega(RC + LG).$$

$$\Rightarrow \alpha^2 - \beta^2 = RG - \omega^2 LC \longrightarrow (B)$$

From (A), finding its magnitude

$$= \sqrt{\alpha^2 + \beta^2} = \sqrt{\sqrt{R^2+\omega^2L^2} \times \sqrt{G^2+\omega^2C^2}}$$

Squaring both sides

$$\Rightarrow \alpha^2 + \beta^2 = \sqrt{R^2+\omega^2L^2} \times \sqrt{G^2+\omega^2C^2} \longrightarrow (C)$$

Adding (B) & (C)

$$\Rightarrow 2\alpha^2 = (RG - \omega^2LC) + \left(\sqrt{(R^2+\omega^2L^2)}\sqrt{(G^2+\omega^2C^2)}\right)$$

$$\Rightarrow \alpha = \underbrace{\sqrt{\frac{1}{2}\left(\left[RG - \omega^2LC\right] + \sqrt{\underbrace{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}_{y}}\right)}}_{= x,\ \text{say}}$$

Subtracting (B) from (C).

$$\Rightarrow \beta = \sqrt{\frac{1}{2}\left(\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right] - (RG - \omega^2LC)\right)}$$

Now,

finding extreme cond^{ns} for min. attenuation,

Now, lets do $\frac{d\alpha}{dL} = 0$.

$$\Rightarrow \frac{d\alpha}{dL} = \frac{1}{2\sqrt{x}}\left\{\frac{1}{2}\left([-\omega^2C] + \frac{1}{2\sqrt{y}}(G^2+\omega^2C^2)(2L\omega^2)\right)\right\} = 0.$$

$$\Rightarrow -\omega^2 C + \frac{(G^2\omega^2C)(2L\omega^2)}{2\sqrt{y}} = 0.$$

$$\Rightarrow \omega^2 C = \frac{(G^2+\omega^2C^2)(\omega^2 2L)}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

$$\Rightarrow \omega^2 C = (\omega^2 L)\sqrt{\frac{G^2+\omega^2C^2}{R^2+\omega^2L^2}}$$

$\Rightarrow$ Squaring both sides & solving.

$$\Rightarrow C^2(R^2+\omega^2L^2) = L^2(G^2+\omega^2C^2)$$

$$\Rightarrow \omega^2(C^2L^2 - C^2L^2) = L^2G^2 - C^2R^2$$

$$\Rightarrow L^2G^2 = C^2R^2$$

$$\Rightarrow \frac{R^2}{L^2} = \frac{G^2}{C^2}$$

Taking square root, both sides

$$\Rightarrow \boxed{\frac{R}{L} = \frac{G}{C}}$$

$\hookrightarrow$ same result for $\frac{d\alpha}{dC}$ or $\frac{d\beta}{dL}$ or $\frac{d\beta}{dL} = 0$.

Q. A π has $Z_{oc} = 900\angle-30^\circ\ \Omega$.

$Z_{sc} = 400\angle-10^\circ\ \Omega$

find Z_0

We know, $Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$

$$= \sqrt{(900\angle-30^\circ)(400\angle-10^\circ)}$$

$$= 60\sqrt{\angle(-30-10)}$$

$$= 30\angle\left(\frac{-30-10}{2}\right)$$

$$\Rightarrow Z_0 = 30\angle-20^\circ\ \Omega.$$

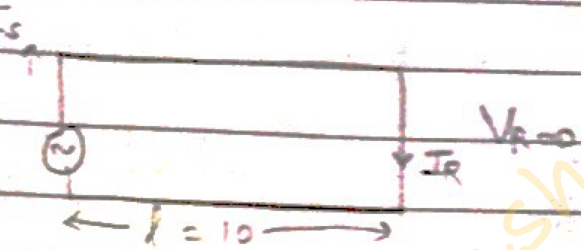
Q. A TL has following constants :

$Z_0 = 600\angle 0^\circ\ \Omega$

$\alpha = 0.1$ neper/km

$\beta = 0.05$ rad/km

$l = 10$ km.



The line has sending current, $I_s = 20$ mA. If the line is short circuited, find receiving end current.

We know $P = \alpha + j\beta$.

$\Rightarrow P = 0.1$ (neper/km) $+ j$ (0.05 rad/km)

Now, we know,

$V = V_s \cosh px - I_s Z_0 \sinh px$.

$I = I_s \cosh px - \frac{V_s}{Z_0} \sinh px$

Now, we have to find I_R i.e, $I\big|_{x=l}$.

$\Rightarrow I_R = I_s \cosh pl - \frac{V_s}{Z_0} \sinh pl$.

$= I_s \cosh pl - \frac{I_s Z_{in}}{Z_0} \sinh pl$. $\rightarrow Z_{sc} = Z_0 \tanh pl$

$= I_s \cosh pl - I_s \tanh pl \sinh pl$

$= I_s \left(\cosh pl - \frac{\sinh^2 pl}{\cosh pl}\right)$

$= I_s \left(\frac{\cosh^2 pl - \sinh^2 pl}{\cosh pl}\right)$

$= I_s \left(\frac{(\cos pl)^2 - (j \sin pl)^2}{\cosh pl}\right)$

$= I_s \left(\frac{\cos^2 pl + \sin^2 pl}{\cosh pl}\right)$

$\Rightarrow I_R = \frac{I_s (1)}{\cosh pl}$.

☆ $\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$

☆ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

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$$\Rightarrow I_R = \frac{I_S}{\cosh\left((0.1+j0.05)(10)\right)}$$

$$= \frac{I_S}{\cosh(1+0.5j)}$$

$\cosh(A+B)$

$$= \frac{20\,mA}{\cosh 1 \cosh(j0.5) + \sinh 1 \sinh(j0.5)}$$

$$= \frac{20\,mA}{\cosh 1 \cos(0.5) + \sinh 1\,(j\sin(0.5))}$$

(using calculator, find all values keeping it in RADIANS (change mode))

$$= \frac{20\,mA}{(1.5)(0.87) + j(1.17)(0.47)}$$

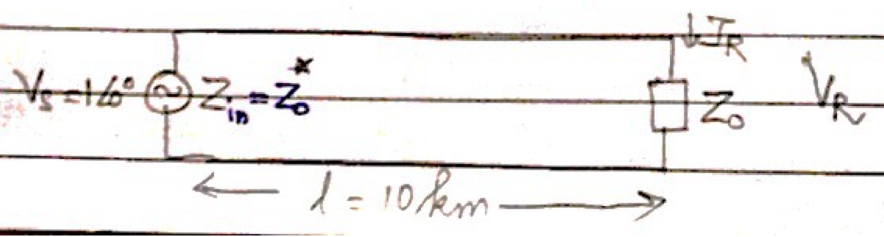
$$= \frac{20\,mA}{1.305 + j(0.5499)}$$

$$= \frac{20\,mA}{1.41612\angle 0.39879^c}$$ $\rightarrow$ radians

$$\Rightarrow I_R = 0.01412 \angle -0.398^c \quad A$$

Q. A TL is terminated by its char. impedance $Z_0 = 600\angle 0°\ \Omega$. Device is energized by voltage source, $V_S = 1\angle 0°\ V$. The length of the line is $l = 10$ km & attenuation constt of the line $\alpha = 0.1$ neper/km & phase constt of line $\beta = 0.05$ rad/km.

Find receiving end voltage, V_R



Now, when a line is terminated by its char impedance, $V = V_s e^{-Px}$

$I = I_s e^{-Px}$

when $x = l$, $V = V_R = V_s \, e^{-P \cdot l}$

$= 1\angle 0^\circ e^{-(0.1 + j0.05)(10)}$

$= 1\angle 0^\circ e^{-1} \cdot e^{-j(0.5)}$

" e^{j0}

$= 1 \, e^{-1} [\cos(0.5) - j\sin(0.5)]$

(put calculator in rad while evaluating)

$\Rightarrow V_R = 0.3228 - j0.1763^c$

or $V_R = 0.3678 \angle -0.4998^c$

Q Find primary constants R, G, L, C of the line which gives 1) $Z_{oc} = 1920\angle -68.9$, given $l = 1$ km, $\omega = 6000$ rad/s.

2) $Z_{sc} = 1308 \angle -76.2$

1) $\rightarrow R, G, L, C$

$Z_{in} = Z_{oc} = 1920\angle -68.9$

We know, $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

& $P = \sqrt{(R + j\omega L)(G + j\omega C)}$

$Z_0 P = R + j\omega L$

$\dfrac{P}{Z_0} = G + j\omega C$

2)

$Z_{in} = Z_{sc} = 1308\angle -76.2$

Now, $Z_0 = \sqrt{Z_{oc} \times Z_{sc}}$

$= 1584.727 \angle \left(\dfrac{-68.9 - 76.2}{2}\right)$

$\Rightarrow Z_0 = 1584.727 \angle -72.55$ $\rightarrow$ (A)

$\Rightarrow Z_0 = -1517 + j(458.26)$

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Now, $Z_{sc} = Z_0 \tanh \gamma l$

$Z_{oc} = Z_0 \coth \gamma l$

$$\Rightarrow \tanh^2 \gamma l = \frac{Z_{sc}}{Z_{oc}}$$

$$\Rightarrow \tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\Rightarrow \gamma = \frac{1}{l} \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right)$$

$$= \frac{1}{1} \tanh^{-1}\left(\underbrace{0.825\angle -3.65^\circ}_{\text{keep it in degrees}}\right)$$

$$= \tanh^{-1}\left(0.82 - j(0.052)\right)$$

or $\tanh \gamma = (0.82 - j(0.052))$

$$\Rightarrow \frac{e^{\gamma} - e^{-\gamma}}{e^{\gamma} + e^{-\gamma}} = \frac{0.82 - j(0.05)}{1}$$

Applying componendo & dividendo operation

$\Rightarrow$ $\dfrac{\text{denominator} + \text{numerator}}{\text{den.} - \text{num}}$ on both sides

$$\Rightarrow \frac{(e^{\gamma} + e^{-\gamma}) + (e^{\gamma} - e^{-\gamma})}{(e^{\gamma} + e^{-\gamma}) - (e^{\gamma} - e^{-\gamma})} = \frac{(1+0.82) - j(0.05)}{(1-0.82) + j(0.05)}$$

$$\Rightarrow \frac{2e^{\gamma}}{2e^{-\gamma}} = \frac{1.82 - j(0.05)}{0.18 + j(0.05)}$$

$$\Rightarrow e^{2\gamma} = \frac{1.82 - j(0.05)}{0.18 + j(0.05)}$$

↳ M1 : Solve by rationalizing

M2 : Solve by converting to polar (using calculator)

$$\Rightarrow e^{2\gamma} = \frac{1.82\angle -1.573}{0.186\angle 15.52} = 9.784\angle -17.093^\circ$$

$\Rightarrow e^{2P} = 9.784\, e^{j(-17.093^\circ)}$

taking $\ln$ on both sides

$\Rightarrow 2p = \ln\left(9.784\, e^{-j(17.093^\circ)}\right)$

$= \ln(9.784) + (-j17.093^\circ + j2n\pi^c)$; $n \in Z^+$

added, to ensure β is always +ve

$\Rightarrow 2p = 2.2807 + (-j17.093^\circ + j2n\pi^c)$; $n = 0, 1, \ldots$

$\Rightarrow p = 1.1403 + (-j\underbrace{8.546^\circ}_{\text{degree}} + j\underbrace{n\pi^c}_{\text{rad}})$; $n = 0, 1, \ldots$

Convert deg. to rad

$180^\circ \longrightarrow \pi^c$

$1^\circ \longrightarrow \frac{\pi^c}{180}$

$8.546^\circ \longrightarrow \frac{\pi}{180} \times 8.546^c$

$\Rightarrow p = 1.1403 + (-j\,0.149^c + j\underbrace{n}_{n}\pi^c)$

or $\alpha + j\beta = 1.1403 + (-j0.149 + jn\pi)$; $n = 0, 1, 2, \ldots$

$\rightarrow$ for $n = 0$, $\beta = -ve$. So, reject $n = 0$

$\rightarrow$ for $n = 1$, $\beta = 2.99$ (+ve) So, ✓ $n = 1$

$\rightarrow$ Ⓑ (taking only $n = 1$, i.e., min value of n)

Using Ⓐ & Ⓑ,

$R + j\omega L = P \times Z_0$

$= \underbrace{(1.1403 + j(2.99))}_{P} \times \underbrace{(1584.727\angle{-72.55})}_{Z_0}$

$= (3.2\angle 69.12)(1584.72\angle{-72.55})$

$= 5071.104\angle{-3.43}$

$\Rightarrow R + j\omega L = 5062 - j(303.39)$

Comparing, we get

$R = 5062 \ \Omega/km$

$\omega L = -303.39$

$\Rightarrow L = \frac{-303.39}{6000} = -0.0505 \ H/km$

Now,

$(G + j\omega C) = \frac{P}{Z_0}$

$= \frac{3.2 \angle 69.12}{1584.72 \angle -72.55}$

$= (2.019 \times 10^{-3}) \angle 141.67$

$(G + j\omega C) = (-1.583 \times 10^{-3}) + j(1.252 \times 10^{-3})$

$\Rightarrow G = -1.583 \times 10^{-3} \ \mho/km$

$\& \ \omega C = 1.25 \times 10^{-3}$

$\Rightarrow C = 2.083 \times 10^{-4} \ F/km$

Comparing, we get

$R = 5062\ \Omega/km$

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& $\omega C = 1.25 \times 10^{-3}$

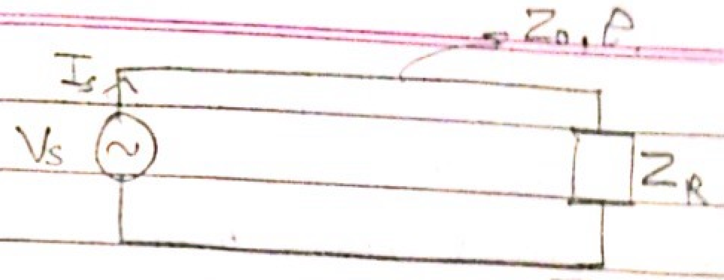
$\Rightarrow C = 2.083 \times 10^{-4}\ F/km$

✩ Reflection Coefficient

$k = \frac{\text{Reflected component (RC)}}{\text{Incident component (IC)}}$

$V = \underbrace{V_i}_{IC} + \underbrace{V_r}_{RC}$

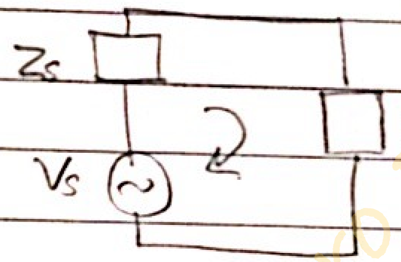
$\Rightarrow k = \left|\frac{V_r}{V_i}\right| = \left|\frac{I_r}{I_i}\right|$



$Z_R = Z_0 \Rightarrow$ Matched line

- Max power will be transferred when $\frac{Z_R}{Z_0} = 1$.

- <u>Max. Power Transfer Theorem</u>
  It states that if you have any source with internal impedance Z_L, max. power will be delivered from load to source iff load impedance is complex conjugate of source impedance



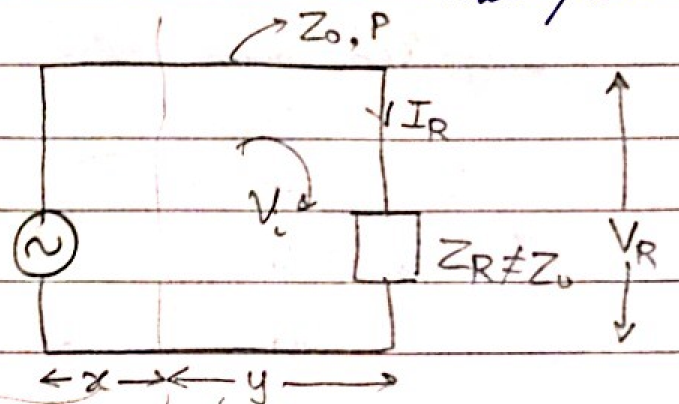
$$Z_L = Z_s^*$$
$$Z_L = R_L + jX_L$$
$$Z_s = R_s - jX_s$$
$$\Rightarrow R_L = R_s$$
$$\Rightarrow jX_L = -jX_s$$
$$\Rightarrow Z_L + Z_s = R_L + R_s$$

$\Rightarrow$ We should have only resistance.
Imaginary part : corresponds to inductor ~~of~~ or capacitor (no energy transfer)
In resistance, energy dissipates $\Rightarrow$ energy transfers.

In order to have power transfer, we shouldn't have any reactance :
So, $Z_L = Z_s^*$; for max power transfer, $R_s = R_L$.

If $Z_R = Z_0$: no imaginary part & real part is equal

$Z_R \neq Z_0$: part of power will be transferred. Other part reflected.



some distance from i/p side

from o/p side

$$V = \underbrace{ae^{px}}_{RC} + \underbrace{be^{-px}}_{IC} \quad \left(\because I = \frac{1}{Z_0}\left(ae^{px} + be^{-px}\right) \right)$$

In direction of y, $x = -y$.

So, $V = ae^{-py} + be^{py}$.

& $I = \frac{-1}{Z_0}\left(ae^{-py} - be^{py}\right)$

Now, $-\frac{dV}{dx} = (R + j\omega L) I$.

$$\Rightarrow -\frac{d}{dx}\left(ae^{px} + be^{-px}\right) = (R + j\omega L) I$$

$$\Rightarrow -p\left(ae^{px} - be^{-px}\right) = (R + j\omega L) I$$

We know, $k = \frac{ae^{-py}}{be^{py}}$

When $y = 0$, $V_R = a + b$, $I_R Z_0 = -a + b$

Adding, $2b = V_R + I_R Z_0$.

$$\Rightarrow b = \frac{V_R + I_R Z_0}{2}.$$

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Ill^{y}, $a = \dfrac{V_R - I_R Z_0}{2}$.

Now, $k = \dfrac{a e^{-py}}{b e^{py}} = \dfrac{\left(\dfrac{V_R - I_R Z_0}{2}\right) e^{-py}}{\left(\dfrac{V_R + I_R Z_0}{2}\right) e^{py}}$

$= \dfrac{V_R - I_R Z_0}{V_R + I_R Z_0} e^{-2py}$

$\Rightarrow k = \left(\dfrac{\dfrac{V_R}{I_R} - Z_0}{\dfrac{V_R}{I_R} + Z_0}\right) e^{-2py}$

$\Rightarrow k = \dfrac{Z_R - Z_0}{Z_R + Z_0} e^{-2py}$

At $y = 0$, $k = \dfrac{Z_R - Z_0}{Z_R + Z_0}$

↳ when $Z_R = Z_0$: matched cond^n.
$\Rightarrow \exists$ no reflection
$\Rightarrow k = 0$.

- In open circuit,
when $Z_R \to \infty \Rightarrow$ Max. reflection
(no load to draw power
$\Rightarrow$ complete reflection)

hence, $k = \dfrac{1 - Z_0/Z_R}{1 + Z_0/Z_R} = 1 = \dfrac{V_R}{V_i}$

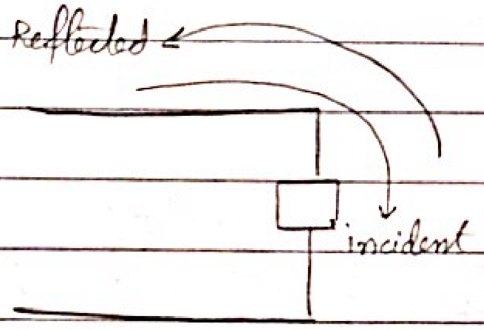
- In short circuit,
$Z_R \to 0$
$\Rightarrow k = \dfrac{-Z_0}{Z_0} = -1$ → Phase difference (180°)

In both cases, magnitude remains the same. Hence, magnitude of reflection coefficient lies between 0 & 1

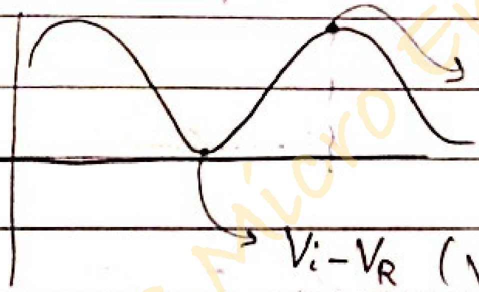
$$\Rightarrow \underline{\underline{0 \leq |k| \leq 1}}.$$

☆ Standing wave ratio (S)

VSWR : Voltage Standing Wave Ratio
ISWR : Current " " "



Sometimes,
in-phase : Addition
out of phase : Subtraction



$\underbrace{V_i + V_R \; (V_{max})}$ → In phase

$\underbrace{V_i - V_R \; (V_{min})}$ → Out of phase

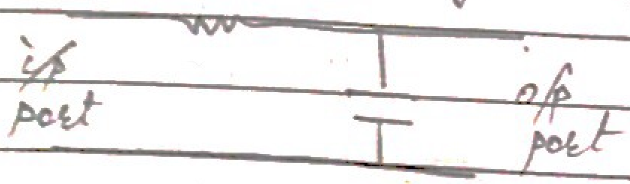
$$S = \frac{V_{max}}{V_{min}} = \frac{|V_i| + |V_R|}{|V_i| - |V_R|}$$

So, $\boxed{S = \frac{1+k}{1-k}}$ $\left(\because \frac{V_i}{V_R} = k \right)$

↳ If $k = 0$, $S = 1$

$k = 1$, $S \to \infty$.

In RF circuit design, appears as S parameter.



(2 port network)

★ When freq. of transmission is increased, we have,

$$\omega L >> R$$
$$\& \ \omega C >> G.$$

Seeing distortionless line,

$$p = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}.$$

$\hookrightarrow$ $\omega L >> R$. So, $\left(1+\frac{R}{j\omega L}\right)^{1/2}$, when expanded, higher order terms of series are neglected.

Illy for $\left(1+\frac{G}{j\omega C}\right)^{1/2}$.

$$\Rightarrow p = j\omega\sqrt{LC}\Big(1+\frac{1}{2}\frac{R}{j\omega L}\underbrace{(+\ldots)}_{\text{neglect}}\Big)\Big(1+\frac{1}{2}\frac{G}{j\omega C}\underbrace{+(\ldots)}_{\text{neglect}}\Big)$$

$$\Rightarrow p = j\omega\sqrt{LC}\Big(1+\frac{1}{2}\frac{R}{j\omega L}+\frac{1}{2}\frac{G}{j\omega C}+\frac{1}{4}\frac{RG}{(-1)\underbrace{\omega^2}_{\text{very large so, neglect}}LC}\Big)$$

$$\Rightarrow p = j\omega\sqrt{LC}\Big(1+\frac{1}{2}\frac{R}{j\omega L}+\frac{1}{2}\frac{G}{j\omega C}\Big)$$

Now, we know,

$$p = \alpha + j\beta = \frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{1}{2}G\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}$$

↳ Attenuation constt is independent of freq.

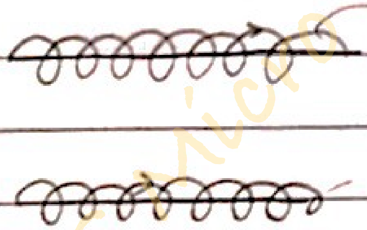
$$\Rightarrow \alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\beta = \omega\sqrt{LC}$$

Now, velocity of propagⁿ, $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

↳ Independent of freq.

★ In practical scenario, increasing freq. of TL, SKIN effect comes into picture.

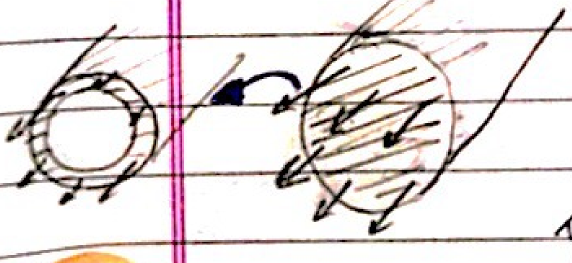


→ for a current carrying conductor, ∃ magnetic field around it (Faraday's law)

For lower freq, the induced magnetic field wont have effect.

For higher freq, time varying current ∃ time varying magnetic field. So, induced emf is generated.

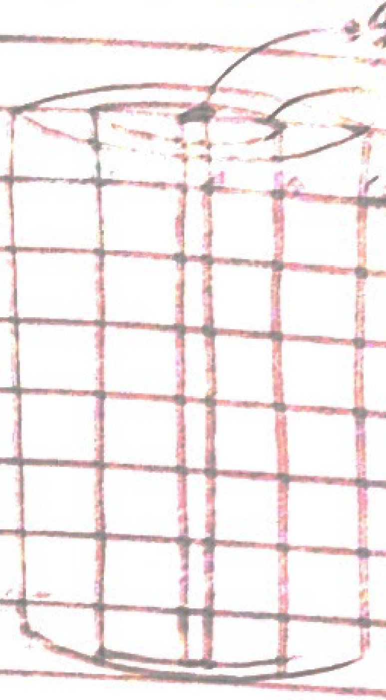
Consider a crossection of current wire. Now,



the emf that was induced, will oppose the current flowing through the complete crossection (Lenz law). So, now, current flows only near the border (= skin)

This is skin effect.

COAXIAL CABLE



When current flows through it, emf is generated on outer conductor. Hence, no effect on inner conductor. i.e. Skin effect is prevented

SMITH CHART

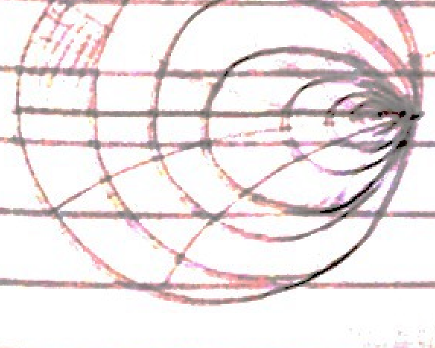
- ✓ RF Circuit Design
- ✓ TL Calculⁿ.
- ✓ Consists of 2 circles :- R circles → Resistance; X circles → Reactance

↳ corresponding to normalised terminating impedance

$$z_L = \frac{Z_R}{Z_0}$$

Black Magic Design

Something like:



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★ How to draw Smith Chart

The chart is made b/w k_r & k_x axis
k : reflection coeff

We know, $k = \dfrac{Z_R - Z_0}{Z_R + Z_0} \; (= k_r + jk_x)$

$$\Rightarrow k = \left(\frac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1}\right)$$

$$\Rightarrow k = \left(\frac{Z_r - 1}{Z_r + 1}\right) \qquad \left(\because Z_r = \frac{Z_R}{Z_0}\right)$$

Using Componendo & Dividendo

$$\Rightarrow \frac{k+1}{k-1} = \frac{(Z_r - 1) + (Z_r + 1)}{(Z_r - 1) - (Z_r + 1)}$$

$$\Rightarrow \frac{k+1}{k-1} = \frac{2Z_r}{-2}$$

$$\Rightarrow Z_r = \frac{1+k}{1-k}$$

$R + jX$

gives Resistance circle of Smith Chart

Gives reactance circle of smith chart

$$= \frac{1 + (k_r + jk_x)}{1 - (k_r + jk_x)}$$

$$= \frac{(1 + k_r) + jk_x}{(1 - k_r) - jk_x}$$

Now, finding real part & imaginary part

$$R + jX = \frac{(1+k_r) + jk_x}{(1-k_r) - jk_x} \times \frac{(1-k_r) + jk_x}{(1-k_r) + jk_x}$$

$$= \frac{\left[(1-k_r^2) - k_x^2\right] + j\left[k_x(1+k_r) + k_x(1-k_r)\right]}{(1-k_r)^2 + k_x^2}$$

$$= \frac{(1-k_r^2-k_x^2) + j(2k_x)}{(1-k_r)^2 + k_x^2}$$

$$\Rightarrow R + jX = \frac{1-k_r^2-k_x^2}{(1-k_r)^2+k_x^2} + j\frac{2k_x}{(1-k_r)^2+k_x^2}.$$

* drawing Smith Circles

(I) For R circle,

$$R = \frac{1-k_r^2-k_x^2}{(1-k_r)^2+k_x^2}$$

Now, we know,

eq^n of circle :- $(x-a)^2 + (y-b)^2 = r^2$.

↳ radius → r

↳ Center → (a, b)

Now, trying to bring this eq^n

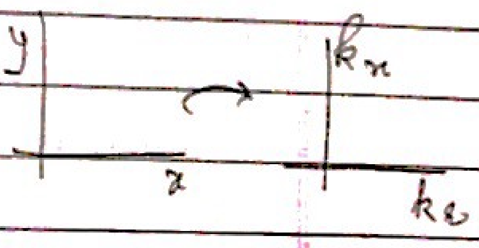
Idea :- Our variables are k_x & k_r.

So, we should have something like

$$k_r^2 + k_x^2 = r^2$$

↳ Center (0, 0)

↳ radius → r.



Cross multiplying

$$\Rightarrow R(1-k_r)^2 + R k_x^2 = 1 - k_r^2 - k_x^2$$

$$\Rightarrow R + R k_r^2 - 2R k_r + R k_x^2 = 1 - k_r^2 - k_x^2$$

$$\Rightarrow k_r^2 (1+R) + k_x^2 (1+R) - 2R k_r = 1 - R$$

$$\Rightarrow k_r^2 + k_x^2 - \frac{2R}{1+R} k_r = \frac{1-R}{1+R}$$

$$\Rightarrow \left[k_r^2 - 2\left(\frac{R}{1+R}\right) k_r\right] + k_x^2 = \frac{1-R}{1+R}$$

$$\Rightarrow \left[k_r^2 - 2\left(\frac{R}{1+R}\right) k_r + \left(\frac{R}{1+R}\right)^2 - \left(\frac{R}{1+R}\right)^2\right] + k_x^2 = \frac{1-R}{1+R}$$

$$= \left(k_r - \frac{R}{1+R}\right)^2 - \left(\frac{R}{1+R}\right)^2 + k_x^2 = \frac{1-R}{1+R}$$

$$\Rightarrow \left(k_r - \frac{R}{1+R}\right)^2 + k_x^2 = \frac{1-R}{1+R} + \left(\frac{R}{1+R}\right)^2$$

$$= \frac{1 - R^2 + R^2}{(1+R)^2}$$

$$\Rightarrow \left(k_r - \frac{R}{1+R}\right)^2 + k_x^2 = \left[\frac{1}{(1+R)}\right]^2$$

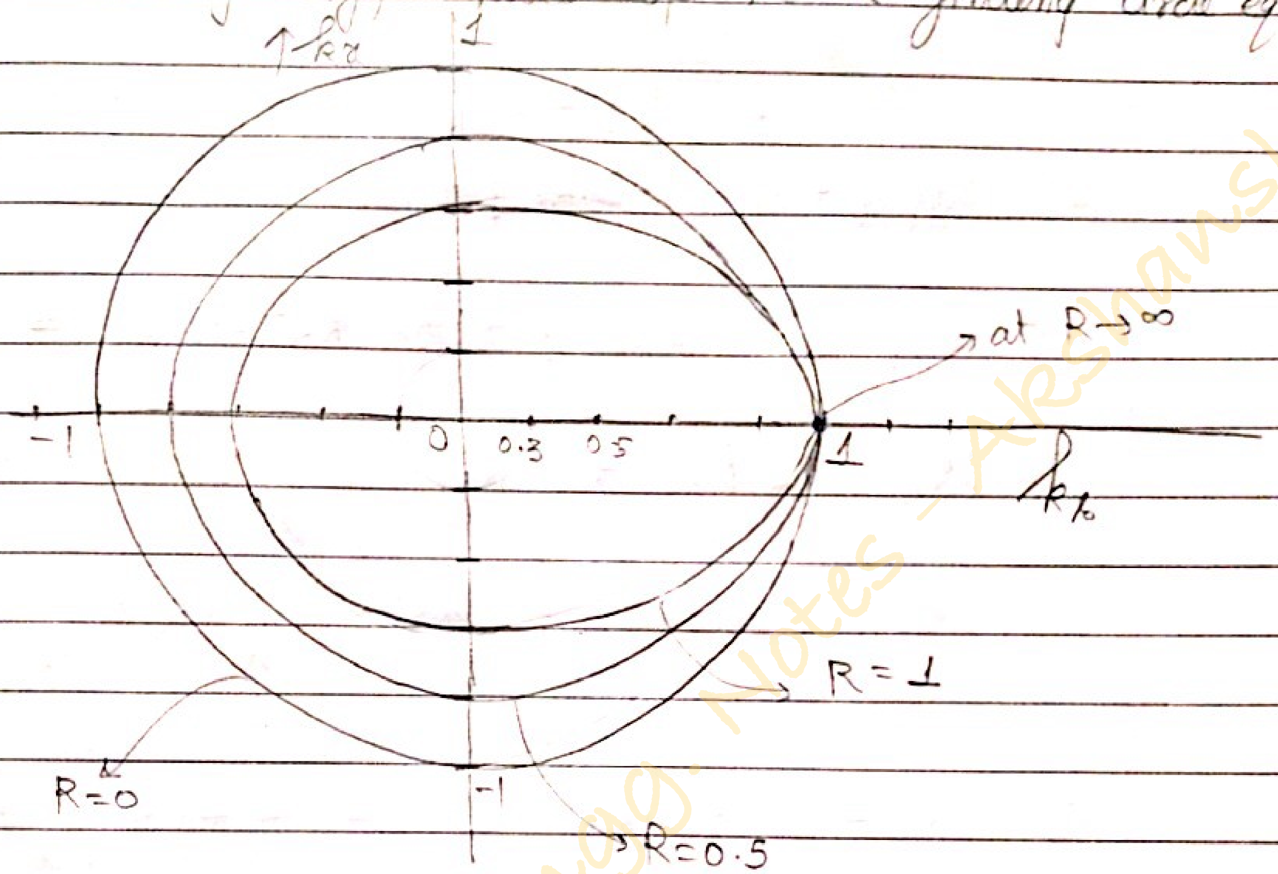
$$(x-a)^2 + (y-0)^2 = a^2$$

So, its a circle

→ radius $= \dfrac{1}{1+R}$

→ center :- $\left(\dfrac{R}{1+R}, 0\right)$

Drawing circle :
Putting diff^t values of R & finding circle eq^ns



for $R=0$

$\hookrightarrow k_r^2 + k_x^2 = 1$

$\hookrightarrow$ center $(0,0)$, radius $= 1$

for $R = 0.5$

$\hookrightarrow (k_r - 0.3)^2 + k_x^2 = (0.66)^2$.

$\hookrightarrow$ center : $(0.3, 0)$, radius $= 0.66$

for $R = 1$

$\hookrightarrow$ radius $= 0.5$, center $(0.5, 0)$

for $R \to \infty$

$$\text{radius} = \lim_{R\to\infty}\left(\frac{1}{1+R}\right) = 0$$

$$\text{center} = \lim_{R\to\infty}\left(\frac{R}{1+R}, 0\right)$$

$$= \lim_{R\to\infty}\left(\frac{1}{1+1/R}, 0\right) = (1, 0)$$

so, it becomes a pt. at $(1, 0)$.

(II) drawing X circle :-

$$X = \frac{2k_x}{(1-k_r)^2 + k_x^2}$$

$$\Rightarrow X\left[\left[1 + k_r^2 - 2k_r\right] + k_x^2\right] = 2k_x$$

$$\Rightarrow X\left[k_r^2 - 2k_r\right] + X k_x^2 - 2k_x = -X$$

$$\Rightarrow (k_r^2 - 2k_r) + k_x^2 - 2\left(\frac{1}{X}\right)k_x = -1$$

$$\Rightarrow (k_r^2 - 2k_r + 1 - 1) + k_x^2 - 2\left(\frac{1}{X}\right)k_x + \left(\frac{1}{X}\right)^2 - \left(\frac{1}{X}\right)^2 = -1$$

$$\Rightarrow (k_r - 1)^2 + \left(k_x - \frac{1}{X}\right)^2 = -1 + 1 + \left(\frac{1}{X}\right)^2$$

$$\Rightarrow (k_r - 1)^2 + \left(k_x - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

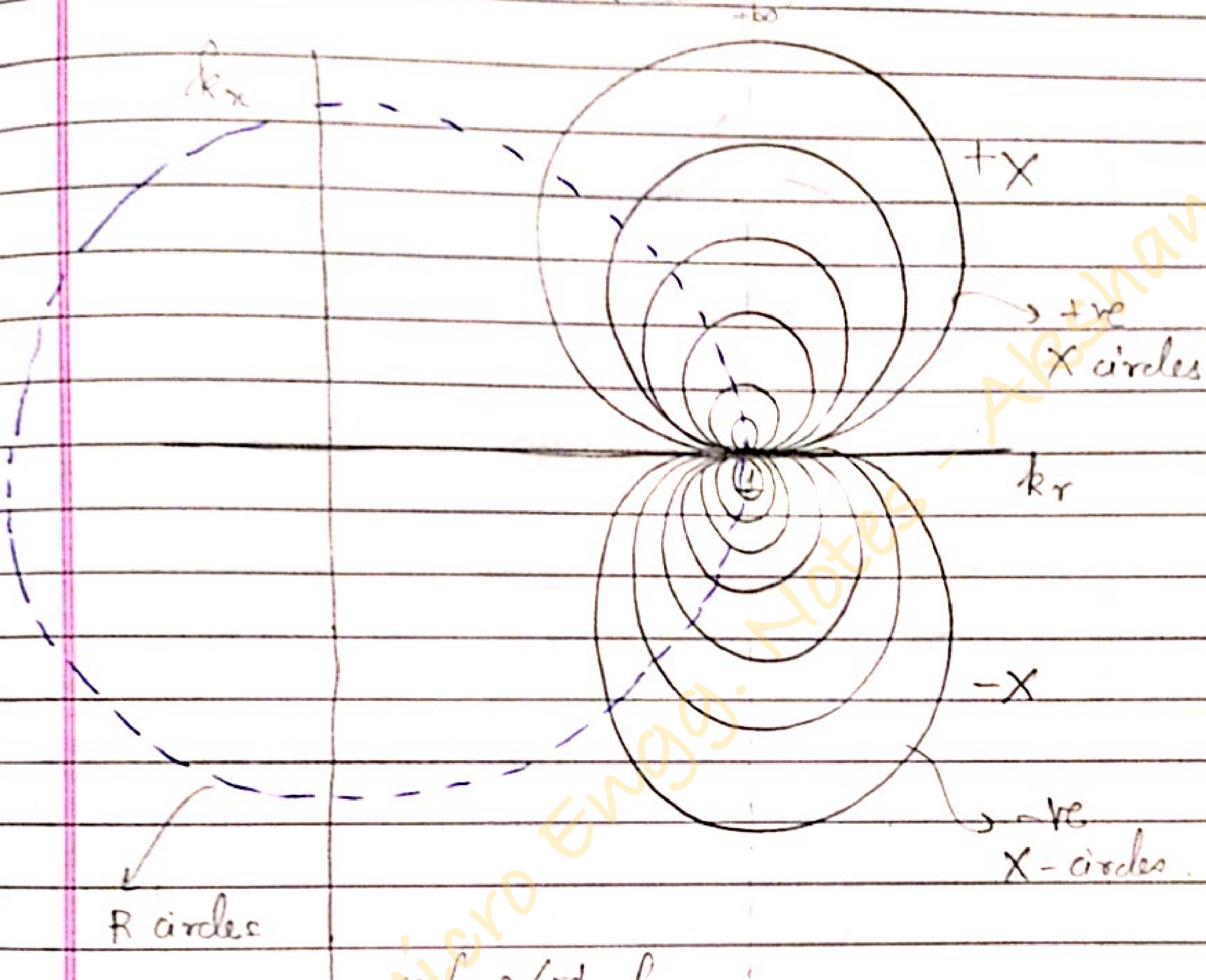
→ Circle with
radius $= 1/X$
center $= \left(1, \frac{1}{X}\right)$

for $X = 0$,
center $(1, \infty^+)$, radius (∞^+)
when center is at $(1, \infty)$ & infinite radius, it'll be effectively the k_r axis.

for $X = 0.1$
center $(1, 10)$, radius (10).

for $X = 0.5$
center $(1, 2)$, radius (2).

when $x \to \infty$
center $(1, 0)$ & radius (0)



only $\frac{2}{3}$rd of
X circles is covered in R-circle.
So, in the Smith chart, X circles are not complete circles

Q(A) Mark following pts. on Smith Chart.

$Z_R = 180 + j60$, $Z_0 = 100\ \Omega$

$Z_R = 300 - j200$, $Z_0 = 100\ \Omega$.

Z_R : SC Impedance

Z_R : OC Impedance

Matched Impedance $Z_R = Z_0$

Idea:

(1) Normalise Z_R,
$$Z_r = \frac{Z_R}{Z_0}.$$

(2) $Z_r = R + jX$.

(3) Trace R circle & trace X circle.

(4) Find pt. of intersection of R circle & X-circle

(5) This pt. of intersection is loc^n of normalised Z_r.

Sol^n: (a) $Z_{R_1} = 180 + j60$, $Z_0 = 100\ \Omega$, given

$$\Rightarrow Z_{r_1} = \frac{180 + j60}{100}$$

$$Z_{r_1} = 1.8 + j(0.6)$$
$$(\equiv R + jX)$$

Next,

(b) $Z_{R_2} = 300 - j200$, $Z_0 = 100\ \Omega$

$$\Rightarrow Z_{r_2} = \frac{300 - j(200)}{100}.$$

$$Z_{r_2} = 3 - j(2)$$

So, take R-circle $-(3)$
X circle $-(-2)$

(c) SC point, $Z_R = (0, 0)$.

So, $Z_r = \left(\frac{0}{Z_0}, \frac{0}{Z_0}\right) = (0, 0)$

R=0 is corresponding to biggest R circle.
X=0 is corresponding to k_r axis.
The pt. of intersection is (0,0), as shown.

(d) When OC pt. is plotted,

$$Z_R = (\infty, \infty)$$

$$\text{So, } Z_L = \frac{\infty}{Z_0}, \frac{\infty}{Z_0} = (\infty, \infty)$$

$R \to \infty$ is pt (1,0) & $X \to \infty$ is also (1,0) pt. So, that is point of intersection.

(e) Matched pt ; $Z_R = Z_0$

$$\Rightarrow Z_L = \frac{Z_R}{Z_0} = 1$$

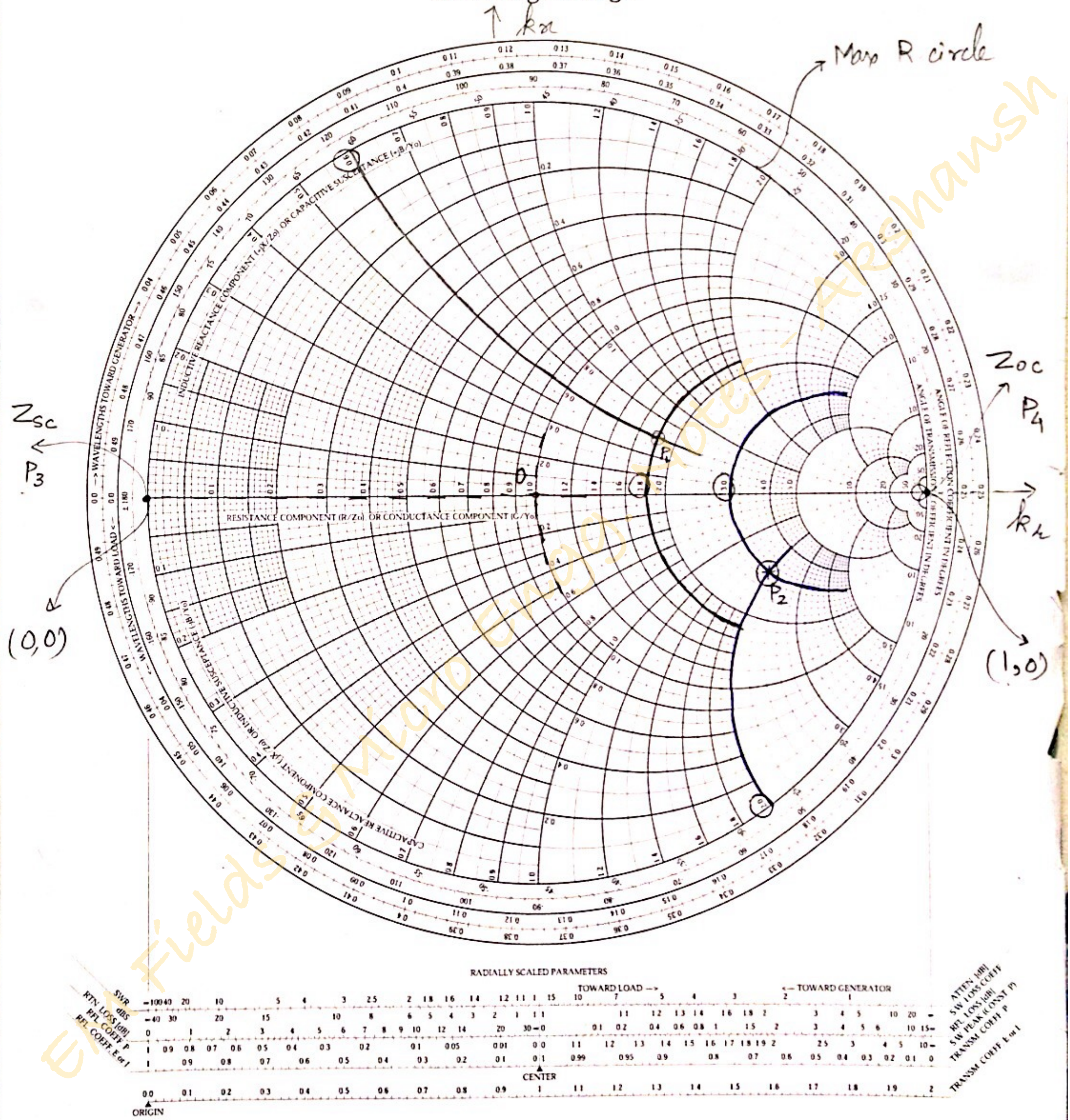
So, plot $Z_L = 1 + j(0)$

R=1, X=0 circle.

Q(A): Smith Chart.

The Complete Smith Chart

Black Magic Design



From the problem,

(a) $Z_r = 1.8 + j(0.6)$

So, find the R-circle at 1.8

& x-circle at 0.6.

Trace it as shown.

Mark the pt. of intersection as P_1.

(b) Similarly, plot $Z_{r_2} = 3 + j(-2)$

Trace it & mark pt. of intersection as P_2

(c) Pt. (0,0) is SC line.

$\because$ R = 0 : biggest R-circle.

X = 0 : k_r axis.

So, pt. of intersection = $P_3(0,0)$

(d) Pt (∞, ∞) is OC line

$\because$ $R \to \infty$, it is a pt at (1,0).

||ly, $X \to \infty$, is a pt at (1,0)

That is pt. of intersection, $P_4(\infty, \infty)$

(e) Matched pt is the pt. with

$Z_r = 1 + j(0)$

i.e, R circle value = 1

X circle value = 0 (k_r axis)

It is shown by dotted lines and point of intersection is marked as O(1,0)

Q. (B) Find reflection coeff. 'k'.

$Z_R = 200 + j(100)\,\Omega$, $Z_0 = 100\,\Omega$

$$k = \frac{Z_R - Z_0}{Z_R + Z_0} = \underline{k_r + jk_x} = K\angle\theta$$

(1) Plot Z_L at point 'P'.

$$Z_L = \frac{Z_R}{Z_0} = \frac{200 + j(100)}{100} = 2 + j$$

So pt $(2, 1)$

(2) Let point $(1, 0)$ be pt O.

Now, for finding reflection coeff, go to bottom of smith chart. Some scales are made. LHS has "REL. COEFF, E or I" is written.

Now,

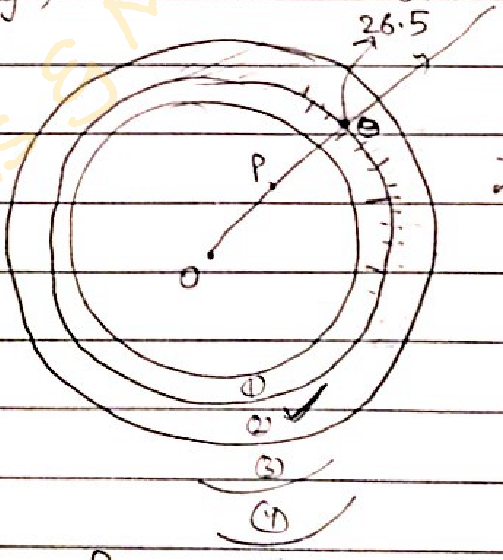
(3) take OP as radius, and mark an arc on reflection coeff E or I scale (as shown)

(4) This arc intersects reflection coeff scale at pt 'k'.

(5) Now, to find angle, draw a line from O through P, towards outer edge of chart. This is extended OP line

This extended line intersects outer scale. Corresponding to 2nd scale, "ANGLE OF REFLECTION COEFFICIENT IN DEGREES" is its name.

(Illy, all the 4 outer circles have their names)



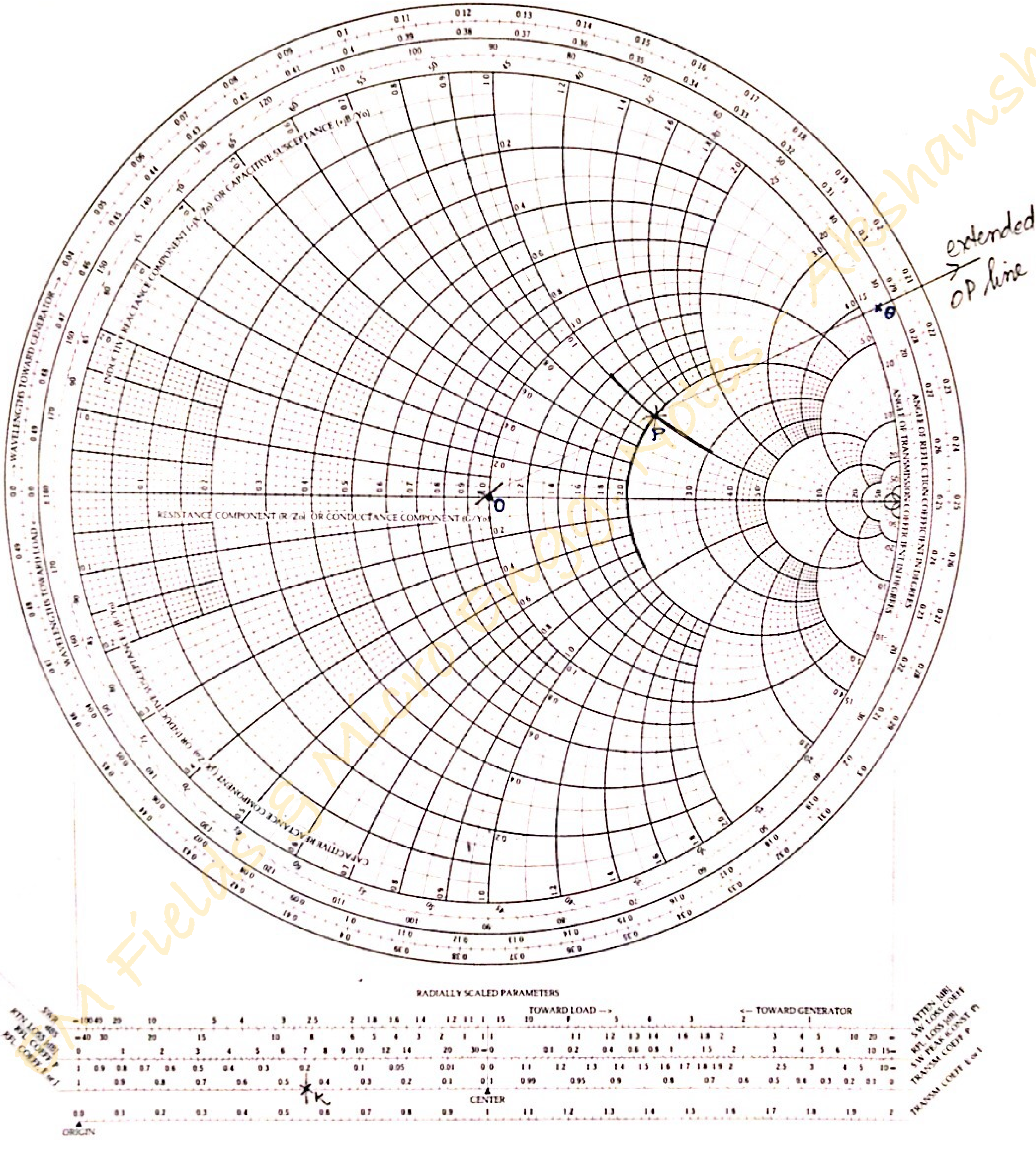
Mark the pt. of intersection as θ (= 26.5) as shown

Now, k = reflection coeff = $0.44\angle 26.5^\circ$.

Q.B

The Complete Smith Chart

Black Magic Design



Q (c) Given : $Z_R = 30 + j50\,\Omega$, $Z_0 = 100\,\Omega$

⇒ Find SWR

① plot Z_r at pt. 'P'

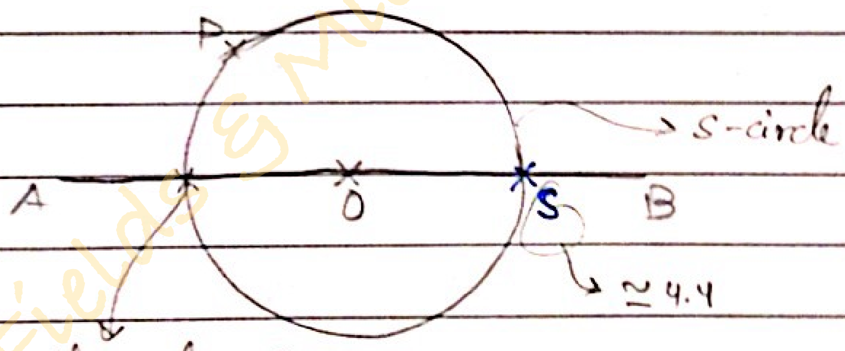
② let pt (1, 0) be O

③ Take OP as radius and 'O' as center. Draw a circle. This circle is termed as 'S' circle (as shown)

① $Z_r = \dfrac{30 + j50}{100} = 0.3 + j(0.5)$

So, $P(0.3, 0.5)$

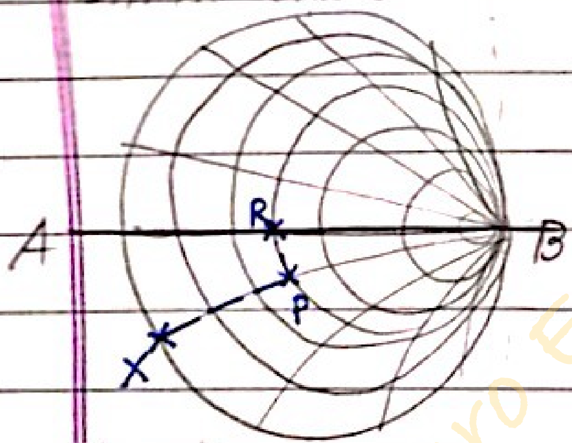
④ Name the horizontal line as AB. S-circle intersects AB line at S (RHS) & 1/S (LHS) as shown.



this value is automatically = $1/S$.

So, its $\dfrac{1}{4.4} \simeq 0.239$ (as shown)

Finding coordinates (R, X), given any pt. on the Smith Chart

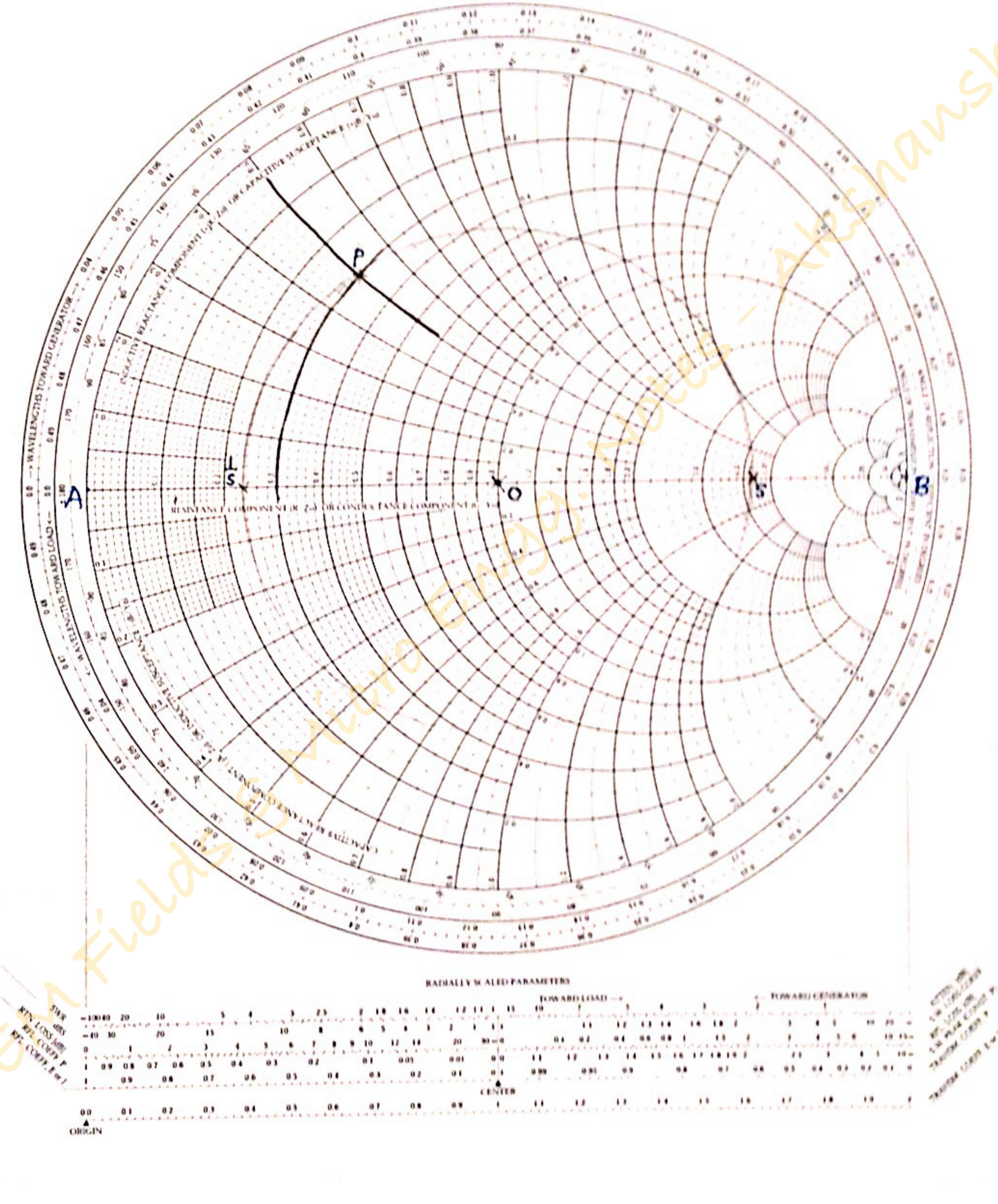


Let pt. P be given.
Then, moving along the R circle that has pt. P in it, the pt. of intersection with AB line gives coordinates of R, or R value.
& moving along the X circle, pt. of intersection with outer circle gives the pt. X
Note: Motion is along circles, not st. lines.

8 (c) $Z_R = 30 + j50\,\Omega$, $Z_0 = 100\,\Omega$

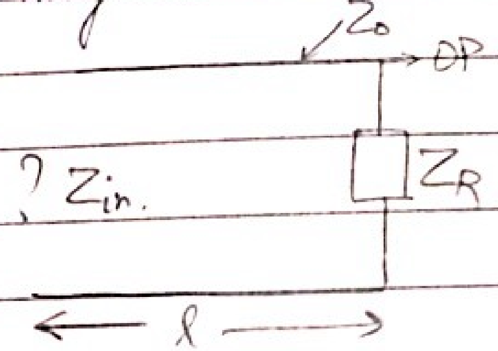
The Complete Smith Chart

Black Magic Design



Q.(D) Finding input impedance of line

Things to be known: (Z_R, Z_0, length of line, 'l')



Find i/p impedance of line using Smith Chart. The line has $Z_R = 60 + j50\,\Omega$, $Z_0 = 100\,\Omega$. The length of the line = 2.33λ,

λ: wavelength of signal passing through the line (If freq given – find λ)

1) Plot Z_r at pt. P. ⇒ We are at the end of line

$$Z_r = \frac{Z_R}{Z_0} = 0.6 + j(0.5)\,\Omega$$

(2) Draw 's' circle ⇒ let the point (1, 0) be 'O' & OP as radius & 'O' as center. Draw a circle. We find, the pt. where it intersects horizontal k_r axis is at $S = 2.2$. This is SWR (Standing wave ratio).

We have to move pt. P of the line

(3) Draw a line from O to P towards outer edge of the chart.

We have to move in "←" dir^n → terminating to i/p impedance.

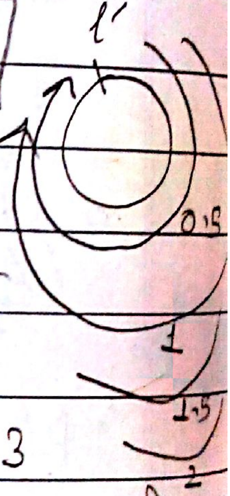
So, in Smith Chart, motion is in clockwise dir^n

(4) Extended OP line intersects λ scale (generator) at l'.
(All Char. exhibited in a line are over a period $\frac{\lambda}{2}$

Complete scale in Smith chart for λ : 0.5
Opposite behaviour => if we travel by $\frac{\lambda}{4}$.

(5) Find $\frac{l}{\lambda}$, where $l = 2.33\lambda$ (given) => $\frac{l}{\lambda} = 2.33$

[Normalising l w.r.t wavelength => $\frac{l}{\lambda}$]



(6) Move pt. l' to l'' by the distance $\frac{l}{\lambda}$.
Integer value gives the same pt
$\therefore$ Ignore the integer value
Consider only the fractional value => 0.33
(ignore 2)

Traveling once in λ circle => $\frac{\lambda}{2}$.

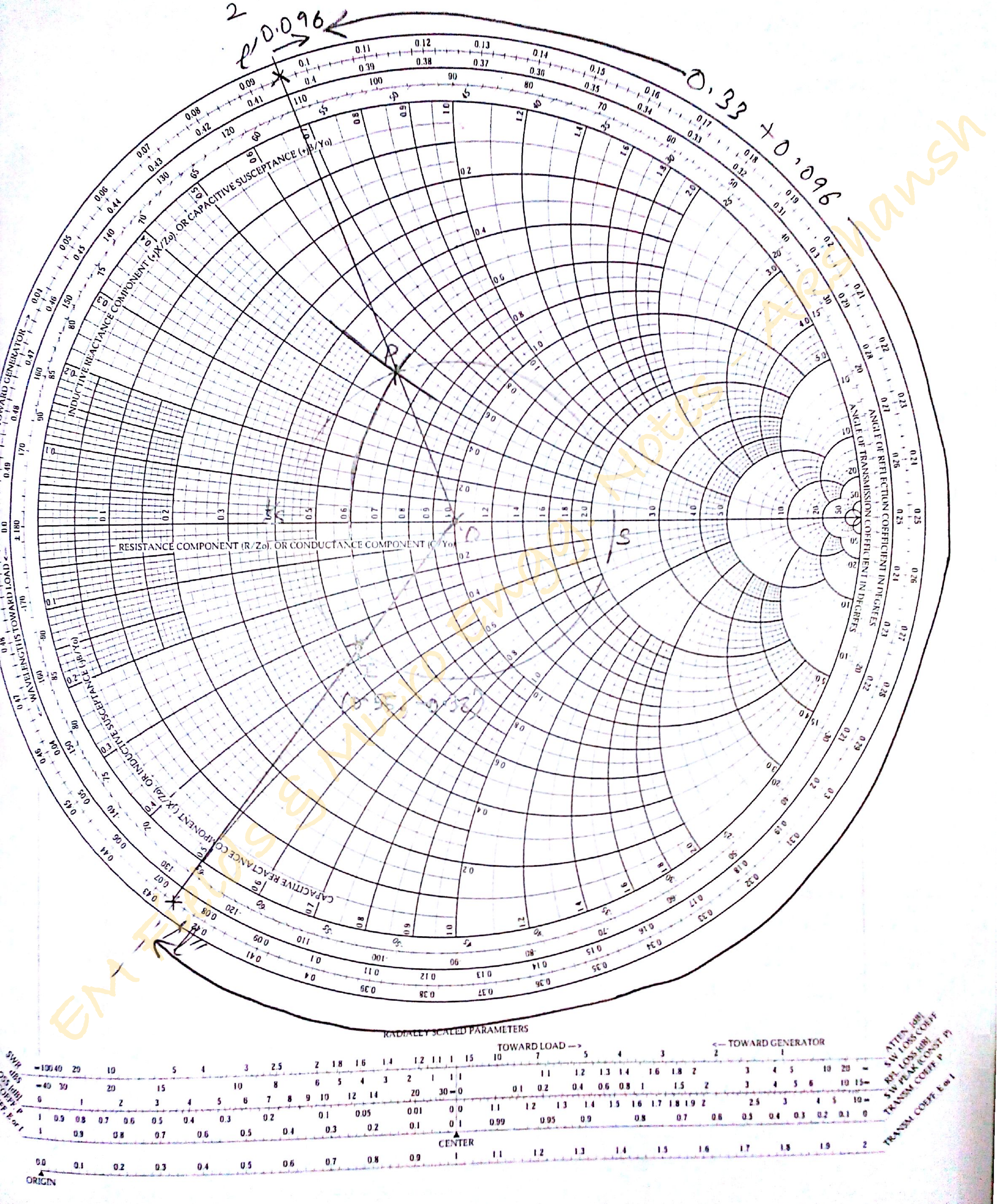
$l' = 0.096$; $l'' = 0.096 + 0.33$
$= 0.426$.

(7) Connect O & l'' (Draw Ol'' line)
(8) Ol'' intersects S circle at pt I
(9) Coordinates of I are unknown value of input impedance in normalised value
(10) To find actual value :- $Z_{in} = I \times Z_0$
=> $I = 0.55 - j\,0.38$
So, $Z_{in} = I \times Z_0 = 55 - j(38)$

Q.2

The Complete Smith Chart

Black Magic Design



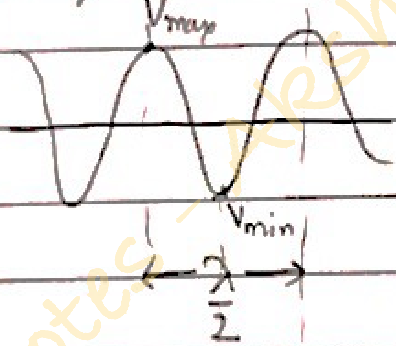
(QUESTION)

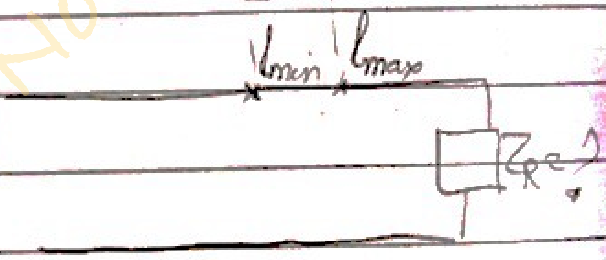
Q(E) Find unknown terminating impedance of the line, if the line has

s data given or l_{max} can be given

SWR = 5.0

$l_{min} = 0.2\lambda$.





(1) Plotting s-circle. All impedances exist on this circle

(2) Make pt O(1, 0)

Plot pt of SWR, s = 5 so (5, 0)

Other pt = $(\frac{1}{s}, 0)$ = (0.2, 0)

(3) Given, $l_{min} = 0.2\lambda$ so, start from pt. A & go down, using scale "Wavelengths toward load". At 0.2, we get the pt. l_{min}.

Join O & l_{min}. It intersects the circle at pt P, as shown. This is Z_r → normalised terminating impedance

Now, $Z_R = Z_r \times Z_0$

$\Rightarrow Z_R = (1.6 - j(2.1)) \times 100 = 160 - j(210)\ \Omega$.

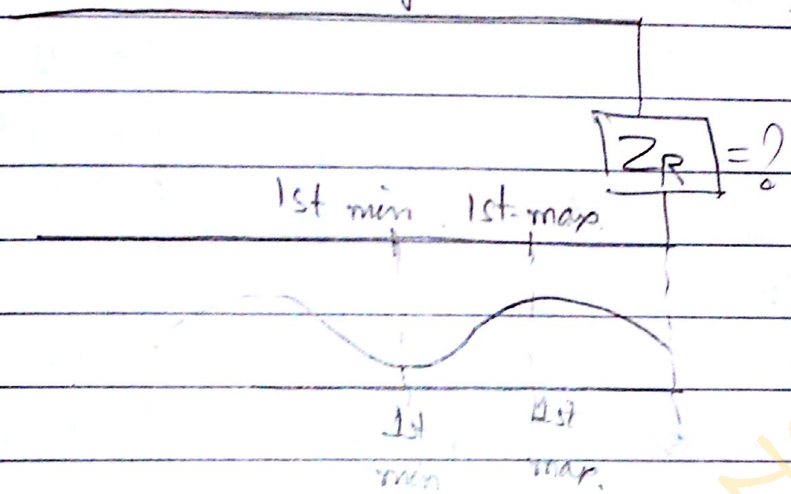
(CONCEPT)

(E) continued — Concept

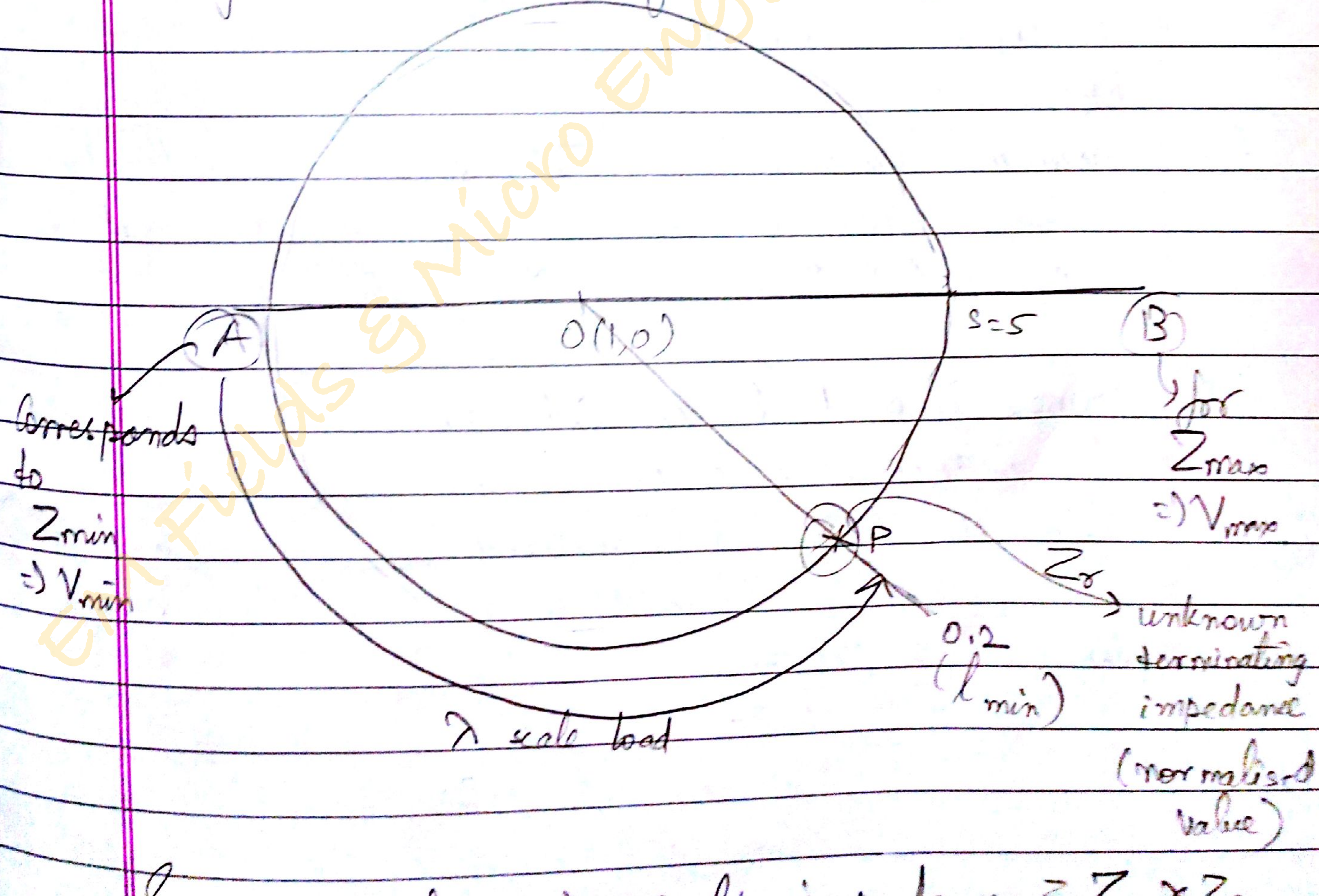
Finding unknown terminating impedance :-

- → we will be given 's' or 'k'.
- → we will be given 1st minima or 1st maxima

Here, we were given 's' - - - -



Seeing smith chart, for line AB,

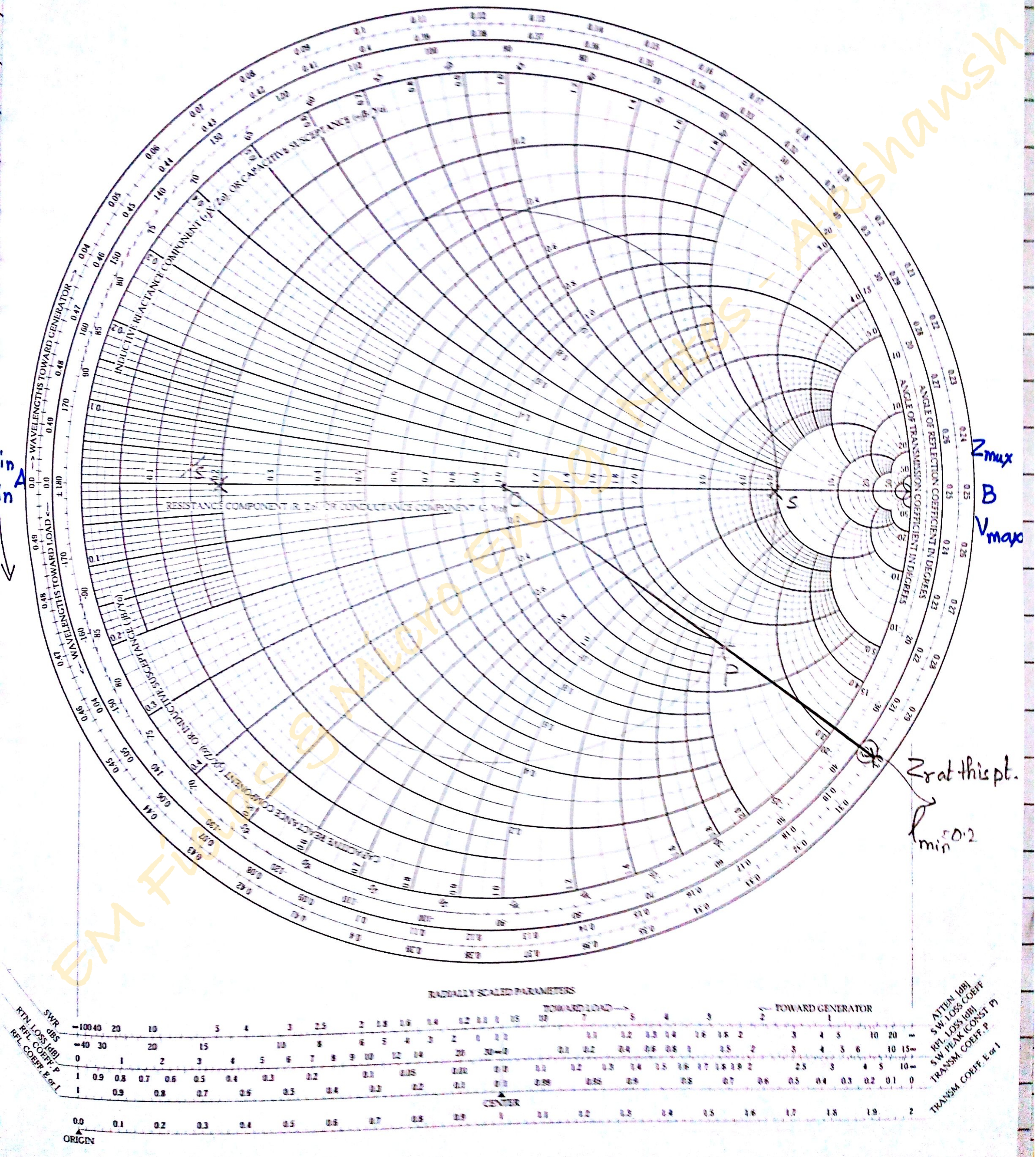


So, Z_R, unknown terminating impedance $= Z_r \times Z_0$

(E) Finding unknown terminating impedance length of line

The Complete Smith Chart

Black Magic Design



Date
Page

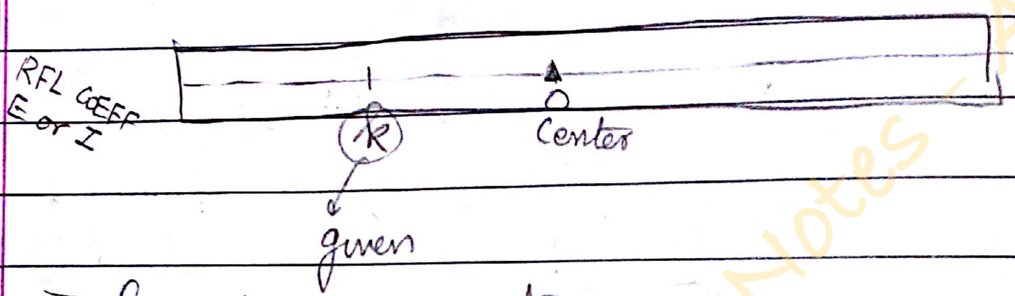
F

(CONCEPT)

Q(F) Suppose $K\angle\theta$ is given instead of S, finding Z_R (unknown terminating impedance)

[Recall: k was found in Q.(B)]

Below smith chart, there is a row of "RFL COEFF E or I"



(1) Take OK as radius.
(2) Go to smith chart & make O(1,0)
(3) Take OK as radius & O(1,0) as center, make a circle. This is S-circle.
Now,
(4) We are given θ also. Mark this on smith chart (Outer circle, under "Angle of Reflection Coeff. in Degrees")

(5) Now, join pt. θ to O(1,0).
It intersects S-circle at pt. P.
This pt. P gives normalized Z_L

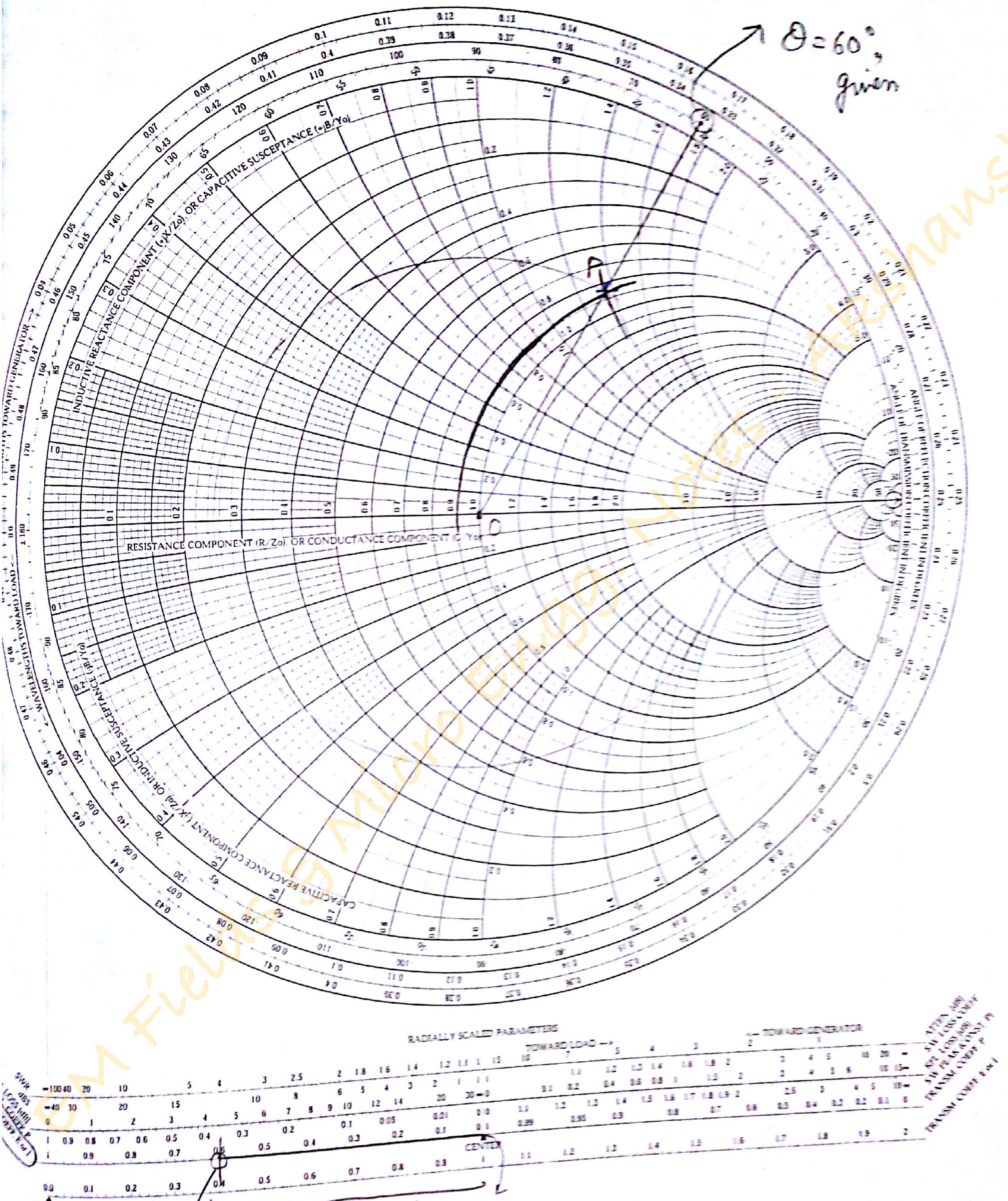
Now, $Z_R = Z_0 \times Z_r$

Ans

(F)

The Complete Smith Chart

Black Magic Design



(QUESTION)

Find Z_R, given $k\angle\theta = 0.6\angle 60°$

As seen from smith chart, using these steps, we got the pt. $P (= R + jX)$

Seeing values of R & X on R-circle & X-circle

Under approximⁿ,

$$R \simeq 0.9, \quad X \simeq 1.35$$

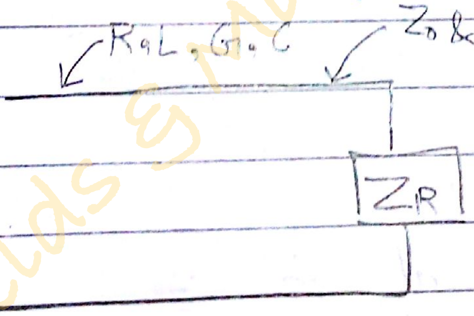
So, $Z_r = 0.9 + j(1.35)$

Hence, $Z_R = 90 + j(135)$ Ans

$(Z_r \times Z_0)$ → 100 Ω, say

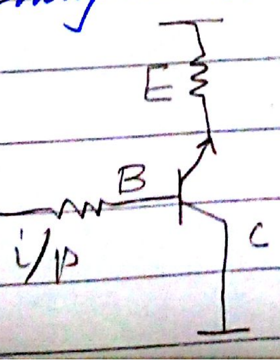
☆ MATCHING

→ reqᵈ to have max. power transfer.



• matching : done using common collector config.

↳ also called Emitter follower

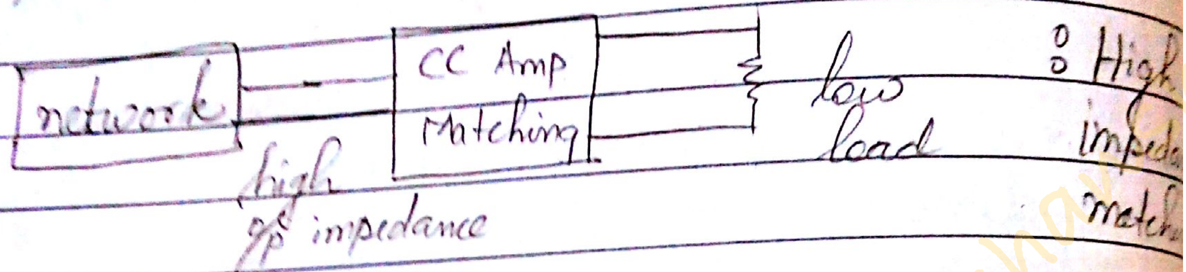


For Transistor operⁿ,

EB junction :- Forward Bias

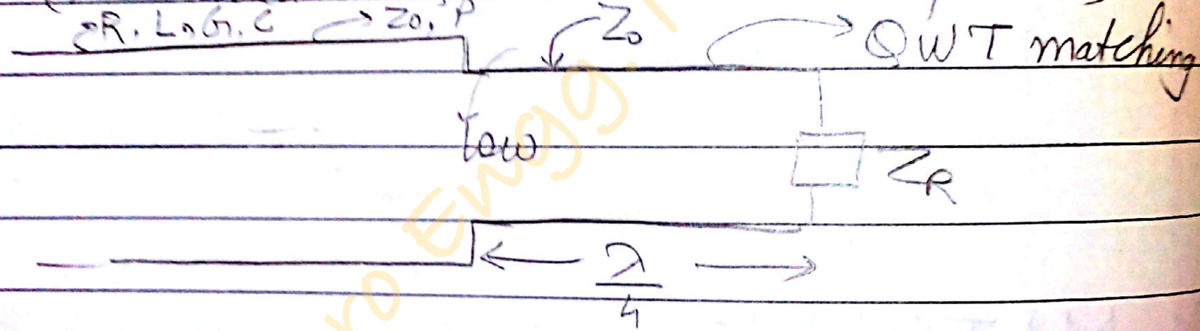
CB junction :- Reverse Bias

At CB junction, impedance is high ($\because$ its RB)
EB " " low ($\because$ its FB)
So, it's something like:



Same can be seen in TL.

Matching (M1): Using Quarter Wave Transformer (QWT)
Introduce a sub transmission line of length $\frac{\lambda}{4}$
(TL called as Quarter wave transformer)



We know,

$$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh pl}{Z_0 + Z_R \tanh pl} \right] \rightarrow (1)$$

Now, $p = \alpha + j\beta$.
For $\underbrace{\text{small length}}_{\frac{\lambda}{4}}$, $\alpha \simeq 0$.

$\Rightarrow p = j\beta$. Now, $\tanh pl = \tanh j\beta l = j \tan$

Also, $\beta = \frac{2\pi}{\lambda} \Rightarrow \beta l = \frac{2\pi l}{\lambda}$, $l \rightarrow \frac{\lambda}{4}$ $= \frac{\pi}{2}$

So, $\tanh pl = j \tan(\frac{\pi}{2}) \rightarrow \infty$

from (i),

$$Z_{in} = Z_0 \left[\frac{\tanh \gamma l}{\tanh \gamma l}\right]\left[\frac{Z_0 + Z_R/\tanh \gamma l}{Z_R + Z_0/\tanh \gamma l}\right]$$

$$\Rightarrow Z_{in} = Z_0 \left[\frac{Z_0 + Z_R/\tanh \gamma l}{Z_R + Z_0/\tanh \gamma l}\right]$$

Now, put $\tan j\beta l \to \infty$

$$\Rightarrow Z_{in} = Z_0 \left[\frac{Z_0 + 0}{Z_R + 0}\right]$$

$$\Rightarrow Z_{in} = \frac{Z_0^2}{Z_R}$$ → (constant for a line)

So, $Z_{in} \propto \frac{1}{Z_R}$

↳ Only at $l = \lambda/4$

At high $Z_R \Rightarrow Z_{in} \to$ low

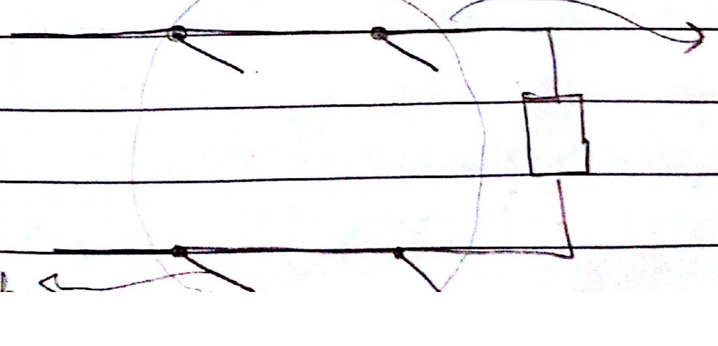
(& vice versa).

So, some kind of matching

Issues: (1) This type of matching adds add^{nal} length.

(2) Can be used only for single freq matching.

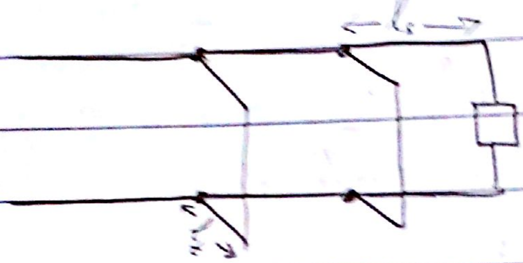
(M2) Stub Matching (connection done in parallel)



* Note :-  : OC line acts as capacitor.

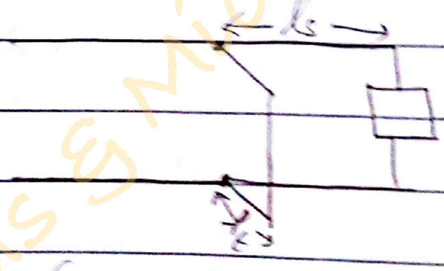
 : SC line acts as inductor

* SC stub is preferred ∵ it gives out less radiation as compared to OC stub.



For matching, how much length l_1 & l_s should be changed of stub : seen from Smith chart.

Q(G). Single stub matching :



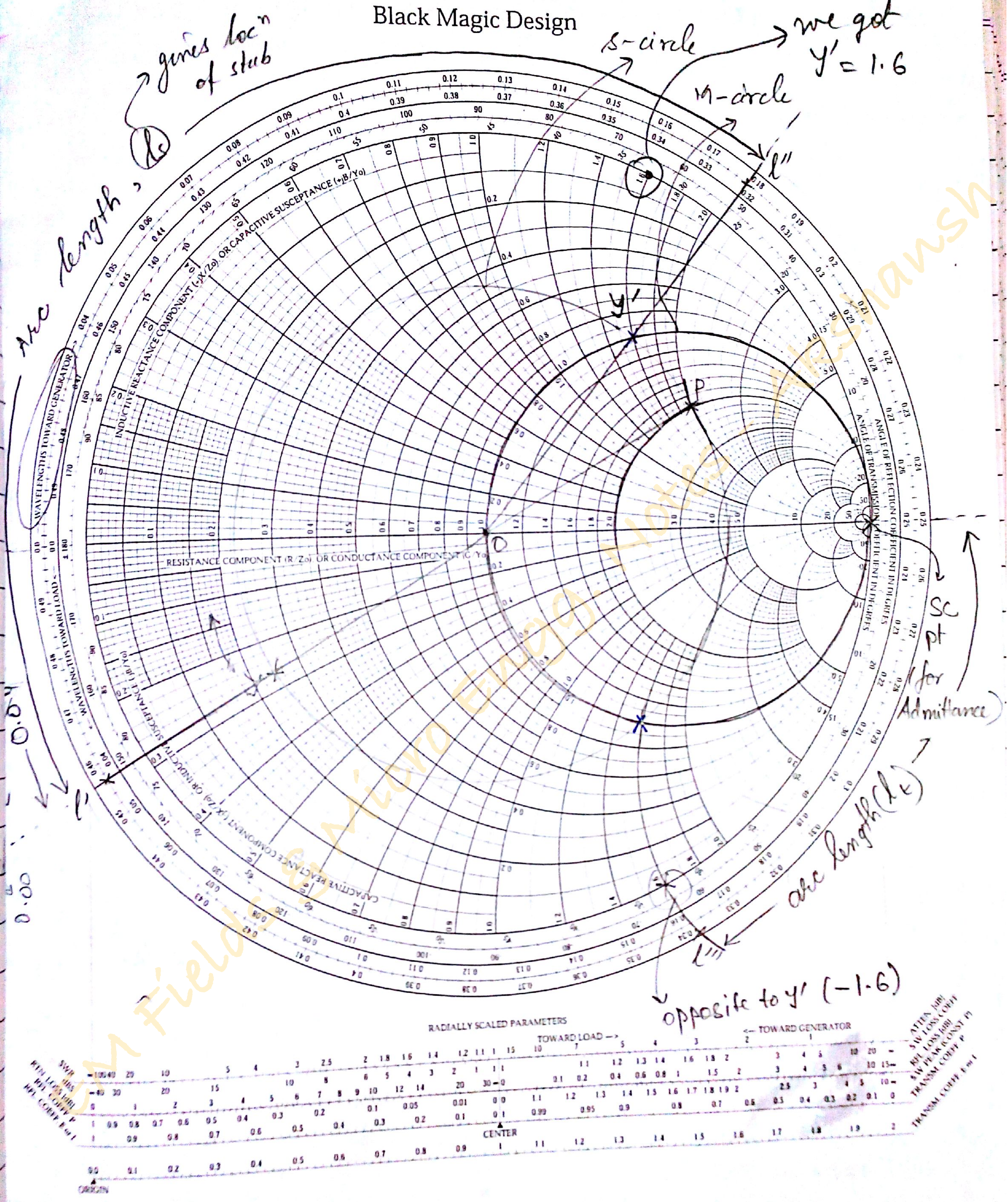
Idea : Find single stub parameters of line having
$Z_R = 200 + j200\ \Omega$
$Z_0 = 100\ \Omega$
$\Rightarrow z_R$ = terminating impedance
$= \frac{Z_R}{Z_0} = 2 + j2$

(Stub is being added in parallel with main TL)

For parallel, for simplicity, choose admittance (Y) instead of impedance (Z).
So, Smith chart is admittance chart

The Complete Smith Chart

Black Magic Design



now, how to convert impedance to admittance?
(done later)

(1) Plot $Z_r = 2 + j(2)$
(2) Plot $O(1,0)$
(3) Taking O as centre & OP as radius, draw s-circle
It intersects R_r axis at s & 1/s
$\simeq s(4.5, 0)$
& $1/s(0.21, 0)$

(4) Now,
finding admittance of P
↳ Find diametrically opp. point of P
i.e, Draw a line from P through O to intersect s-circle at pt. Y. This is admittance

We are trying to match load with line.
Pt. O (1,0) = matching pt
→ R=1, X=0
M-circle (Matching circle)
↳ an R-circle passing through pt (1,0)

(5) Now, s-circle intersects M circle at 2 pts

I got a pt at load (Y). Now, I am travelling towards generator to get stub.
So, From Y, go ↻ & find 1st pt. of intersection
→ towards generator, in λ scale
→ of s-circle & M circle
We find the pt. Y′ & hence its corresponding admittance value

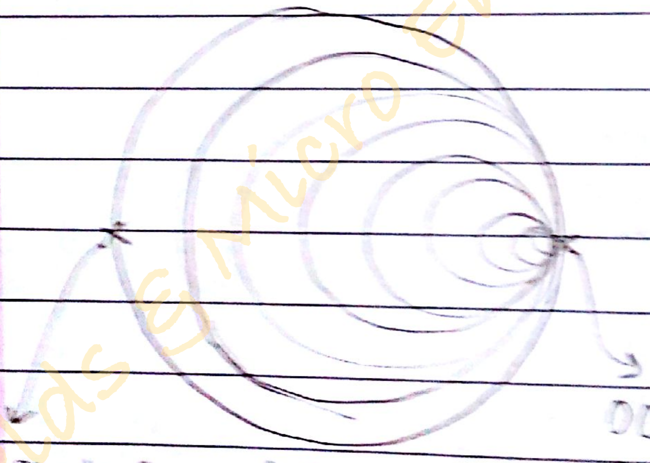
Now, we need to cancel this admittance. So, for that, find locⁿ of STUB :-

(6) Extend $\vec{OY}$ intersecting outer edge at l'
Now, extend $\vec{OY'}$ intersecting outer edge at l''

So, as shown, the arc length, l_s gives locⁿ of stub

l_s = 0.04 (going from l' to A)
+ 0.18 (going from A to l'')
= 0.22

So, locⁿ of stub $\overset{(l_s)}{=}$ arc length × λ
= 0.22 λ



SC pt : In case of Impedance
OC pt : In case of Admittance

OC pt : In case of Impedance
SC pt : In case of admittance

Now, we get 0.16 for Y'. Taking opp. point of that, −0.16. That is l'''.
So, as indicated, we get arc length l_t = 0.3… − 0.… = (0.0…)

So,

l_t = 0.09 × λ

* Smith Chart as Admittance Chart :

1. Plot normalised impedance at point P
2. Take OP as radius and 'O' center, draw "SWR" circle
3) Find diametrically opp. point of pt. P on SWR circle
4) Let this point be Y.
5) Coordinates of Y is normalised admittance of Z_r.

point Y has g_r & b_r

* Stub Matching

Single Stub Matching.

Finding loc" of stub

S1) Plot terminating impedance at pt 'P'.

S2) Draw 'SWR' circle.

S3) Find admittance pt. of P. let it be Y.

S4) Trace M circle.

S5) "SWR" circle intersects with "M" circle at 2 pts.

S6) From pt. Y, travel along SWR circle in λ scale towards generator dir" & find (1) pt of intersection of SWR circle with M circle. let this pt. be Y'.

S7) Draw a line from O through Y' towards outer edge. The extended OY' line intersects λ scale towards generator at M. l'.

S8) Draw a line from O through Y' towards outer edge. The extended OY' line intersects λ scale towards generator at pt l''.

S9) Find arc length l' to l'' = arc length "b". The locⁿ of stub is l_s

$$l_s = (\text{Arc length } l' \text{ to } l'') \times \lambda$$

Finding length of stub

S10) Find susceptance at pt Y'. Let it be "b'". Find opposite b' circle & extend it. Find arc length b/w SC pt. & -b' point.
(-b' → opposite b')

S) This arc length = length of stub.

$$l_t = (\text{Arc length } S/C \text{ and } -b') \times \lambda$$

Q(H) Find single stub matching using Smith Chart. The TL has terminating impedance (Z_R) $100 + j100\ \Omega$

$Z_0 = 100\ \Omega$

① Finding pt. P, $Z_r = \dfrac{Z_R}{Z_0} = \dfrac{100 + j(100)}{100} = (1, 1)$

② Locate pt. O (1, 0)

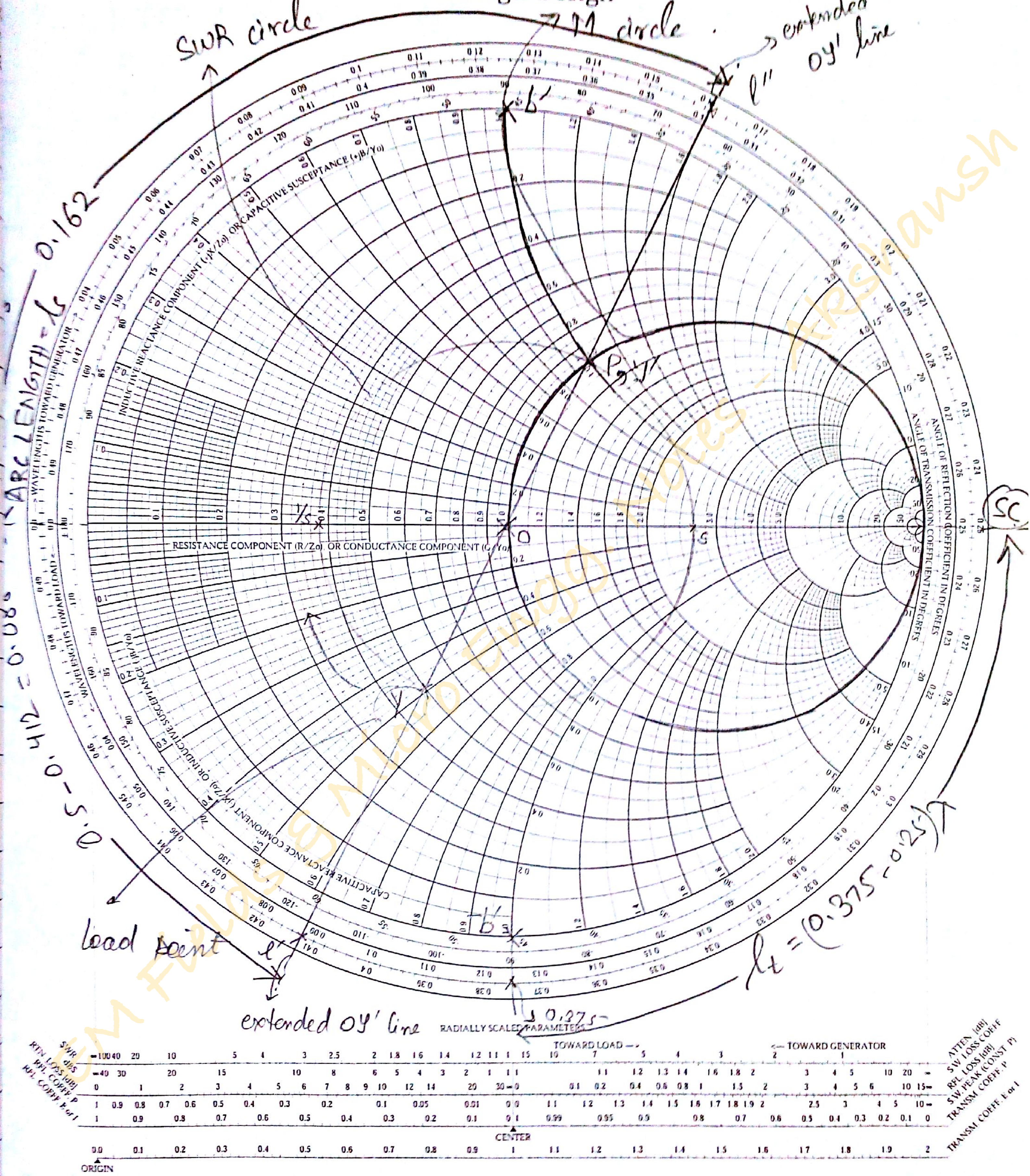
③ Draw SWR circle – OP radius & O center. It intersects R_r axis at S (2.6, 0)

$\frac{1}{S}$ (0.38, 0)

8(H)

The Complete Smith Chart

Black Magic Design



(4) Now, finding admittance of pt P.
i.e, diametrically opp. point of SWR circle. $Y(0.5, -0.5)$

(2) Now finding M circle
i.e, an R circle passing through pt (1, 0)
This circle is already there. Just trace it.

(5) Now, from pt Y, travel along SWR circle towards dir^n as shown. It is found that 1st pt. of intersection is P. Name it as Y'.

(6) Draw extended OY' line as shown
label $\longrightarrow l'(0.412)$

(7) Extend OY'
$l''(0.162)$

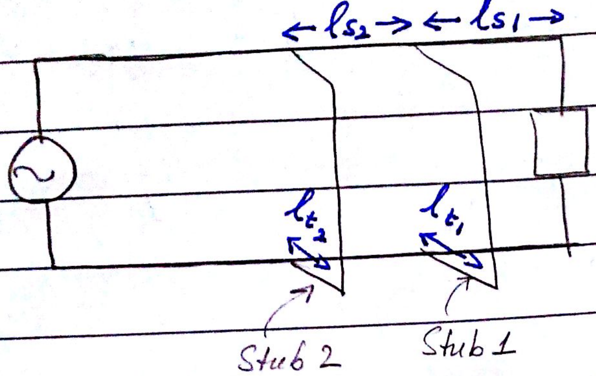
(8) Find arc length l' to l''
$= 0.088 + 0.162 \simeq 0.248$
So, $l_s = (0.248)\times\lambda$

(9) Finding b value :-
See coordinates of Y' (1, 1)
So, G = 1. Move towards X circle corresponding to G = 1
So, b' = 1, as shown.

(10) Find −b' point = at (−1)
extending it below towards λ scale = 0.375

(11) Now, SC scale is as shown
So, l_t is equal to $(0.375-0.25)\lambda = (0.125)\lambda$

✮ Double Stub Matching



(Stubs in parallel)

We have to find :

✓ locⁿ of stub

✓ length of stub

For simplicity, we will fix locⁿ of stub & only find lengths

So, $l_{s_1} = \frac{\lambda}{8}$ & $l_{s_2} = \frac{\lambda}{8}$

Stub 1 gives partial matching to load & stub 2 gives exact matching

✮✮✮

Q.(I) Find double stub matching parameter of a coaxial line having

$Z_R = 100 + j50\ \Omega$

$Z_0 = 50\ \Omega$

1) Plot normalised terminating impedance,

$$Z_r = \frac{Z_R}{Z_0} = 2 + j(1)\ \Omega$$

$$= (2, 1)$$

Let it be pt. P(4,1)

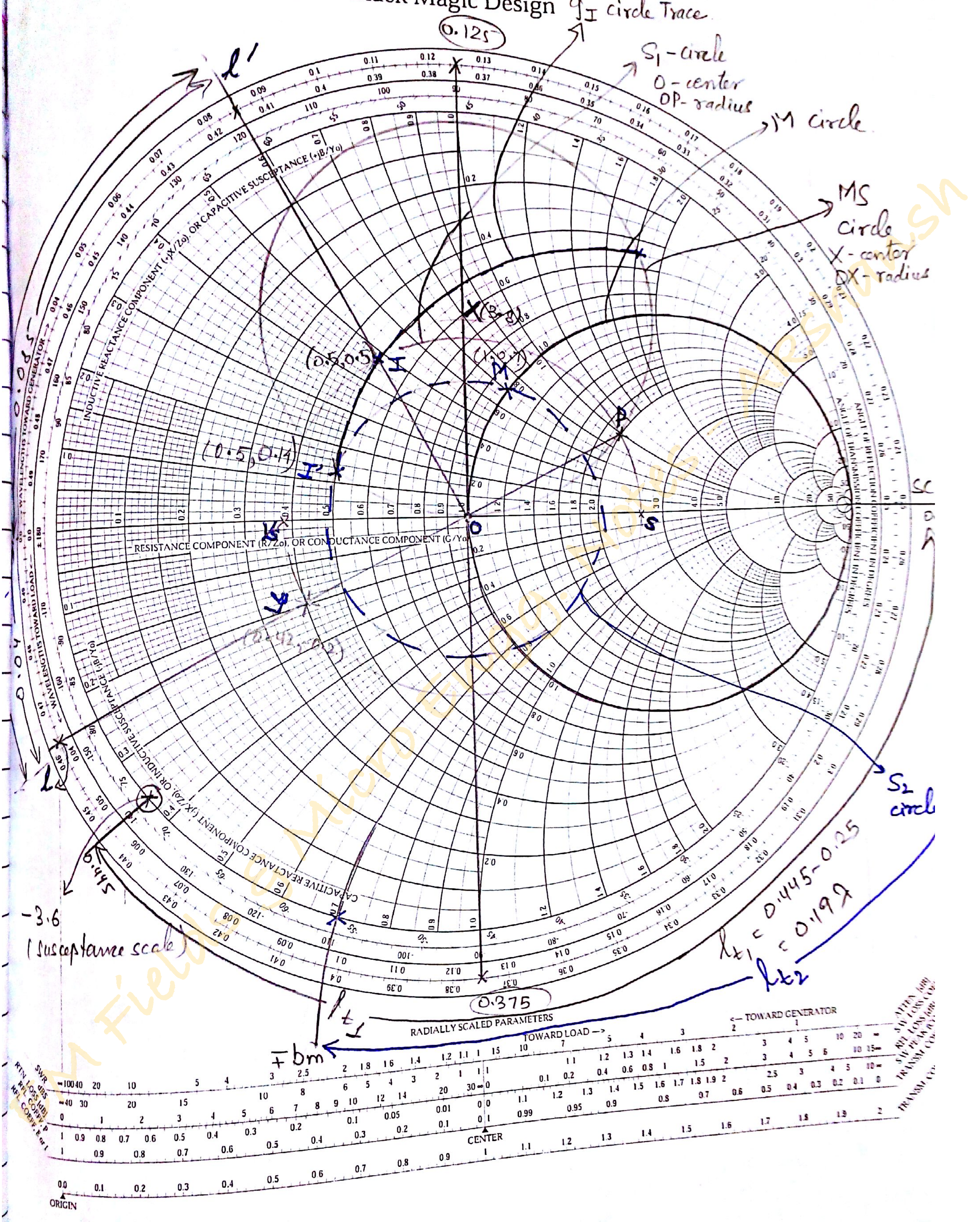
2) Draw s-circle wrt pt. P & O(1,0).

Let circle be S_1 - circle

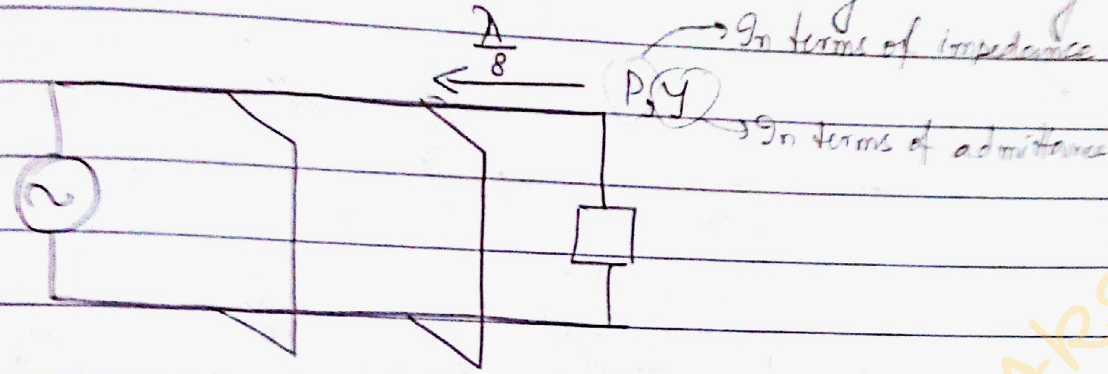
Q.12.

The Complete Smith Chart

Black Magic Design



3. Find admittance pt. of P on $\to S_1$-circle
Mark diametrically opp. pt. (0.42, −0.2)
Idea: We have to reach pt. O by matching.



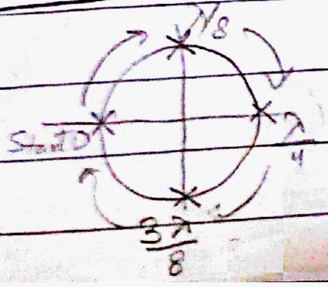
(4) Draw line from O through Y towards outer edge of chart. This is extended OY line.
See pt of intersection with "λ scale towards generator". Let it intersect at pt l (0.46)

(5) Move pt. l to pt. l', a distance of λ/8.
So, 0.46 + 0.125
= (0.585) pt (l')

(6) Draw line segment Ol'.

(7) To obtain complete matching, shift matching-circle (M-circle) by λ/8 :-
↳ passes through O(1, 0) & (∞, ∞)
For that find diameter & radius of matching circle
↳ M-circle
diameter :- 7.6 cm
radius :- 3.8 cm.

*Note :-



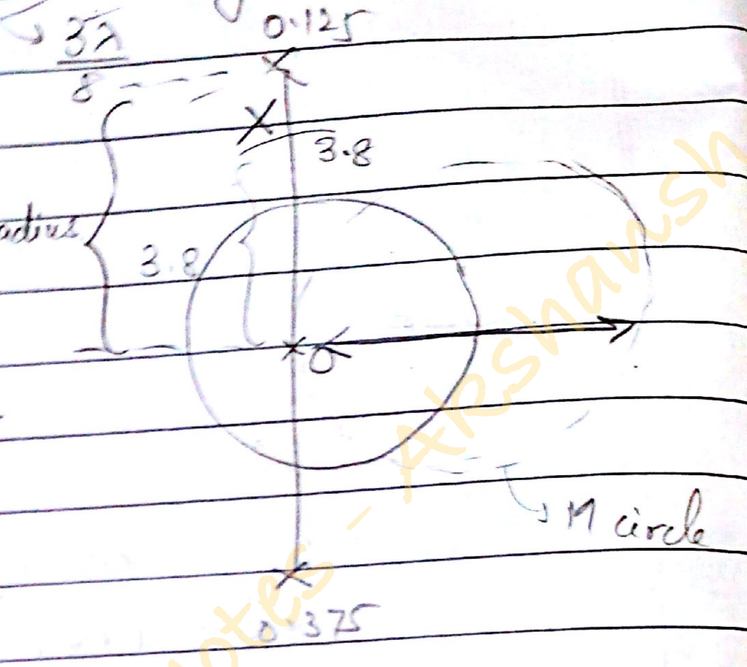
Travel from one pt & reach it back again → λ/2 length ($\frac{\lambda}{2}$) covered.

Now, draw vertical line corresponding to $\frac{\lambda}{8}$ shift

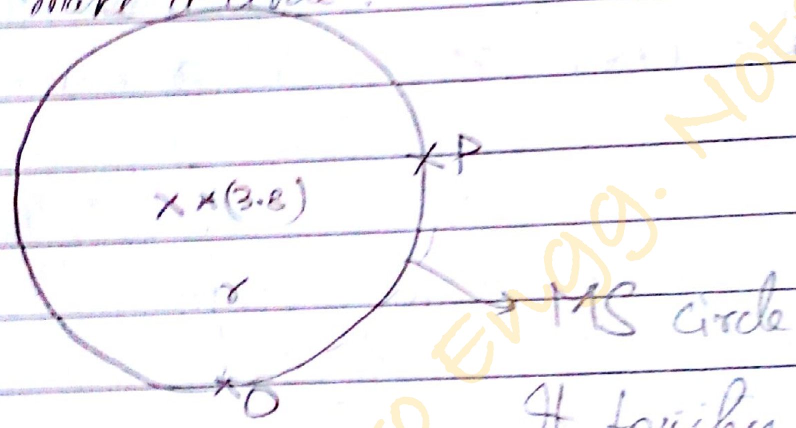
& mark 0.125 & 0.375 & join them

It passes through pt. O.

Mark distance 3.8 from O.

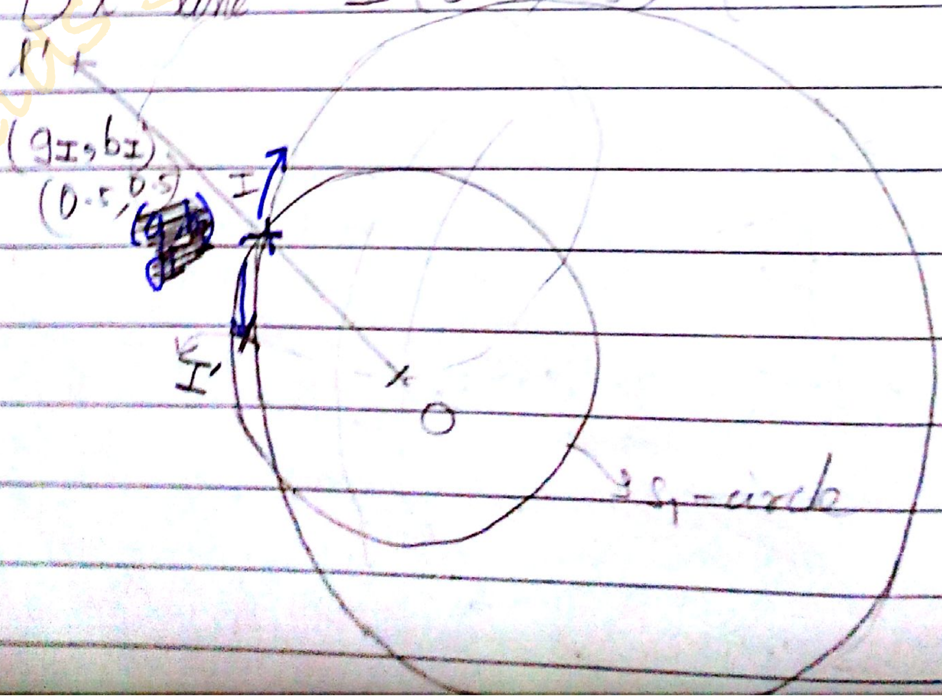


Take X(3.8) as center & 3.8 as radius, make a circle.



It touches O & P point.

(8) Now, find pt. of intersection of g_1-circle & OP' line $I(0.5, 0.5)$ (coordinates $I(g_I, b_I)$)



... by $\frac{\lambda}{8}$: Partial matching

now, move g_I circle (shift by $\lambda/8$) : Complete matching

(9) Travel along g_I - circle (Trace it) & choose the nearest pt. of intersection with MS circle (intersection will be seen on 2 pts.)

Take nearest pt. & mark it $I'(g_I', b_I')$

$(0.5, 0.14)$

(10) Now, find the diff $b_I - b_I'$

$= 0.5 - 0.14 = 0.36$

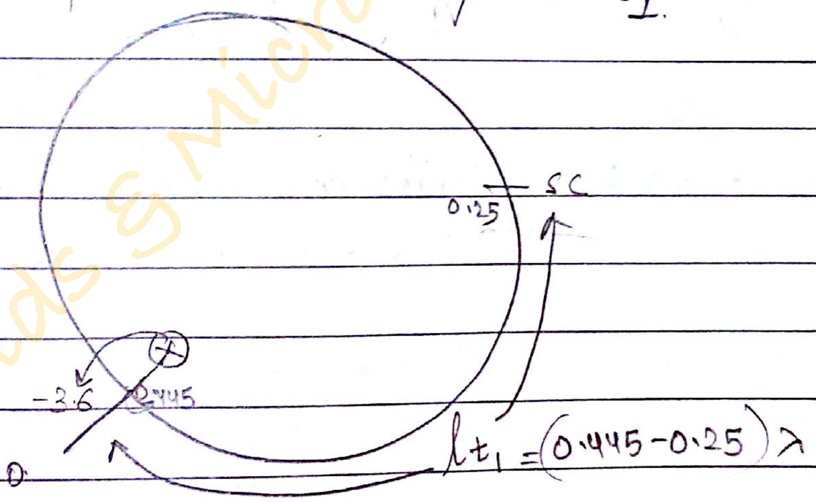
(diff of susceptance)

Now, plot -0.36 on susceptance scale (now, opposite type of susceptance should be added (i.e, add -0.36) to cancel susceptance of 0.36)

(11) Now, find arc length from susceptance $\underbrace{(-0.36)}$ to SC point. This length = l_{t_1}

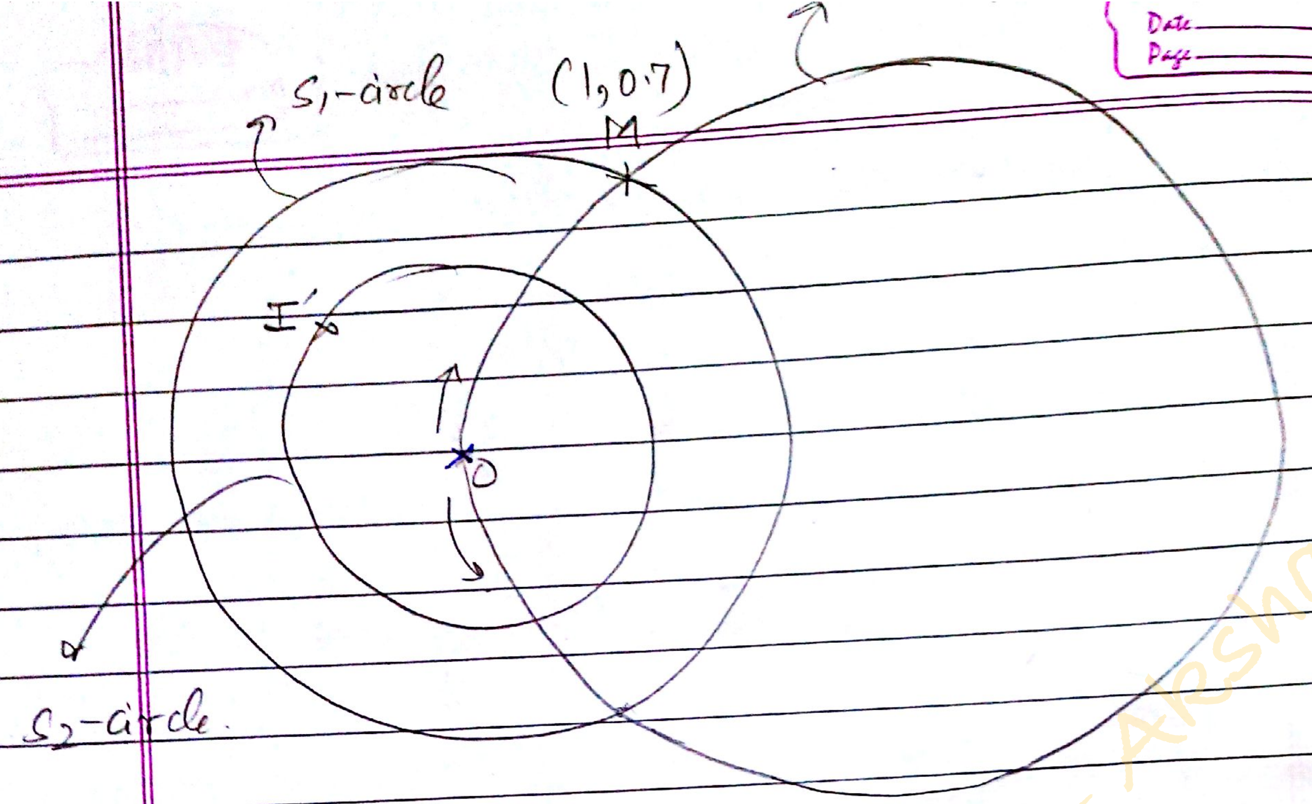
opposite $\underbrace{(b_I - b_I')}$

$-(b_I - b_I')$



Now, for stub 2, $\underbrace{\text{load + Stub 1}}_{= I'}$ becomes impedance to it.

(12) Take OI' as radius & O as center, draw another circle. This is S_2-circle.



(13) From pt. O, move in dir^n of y_a circle. It intersects at 2 pts. (Symmetrically at 2 pts. here).

Choose nearest pt., mark it M (1, 0.7)

Real part is matched, imaginary part has to be cancelled.

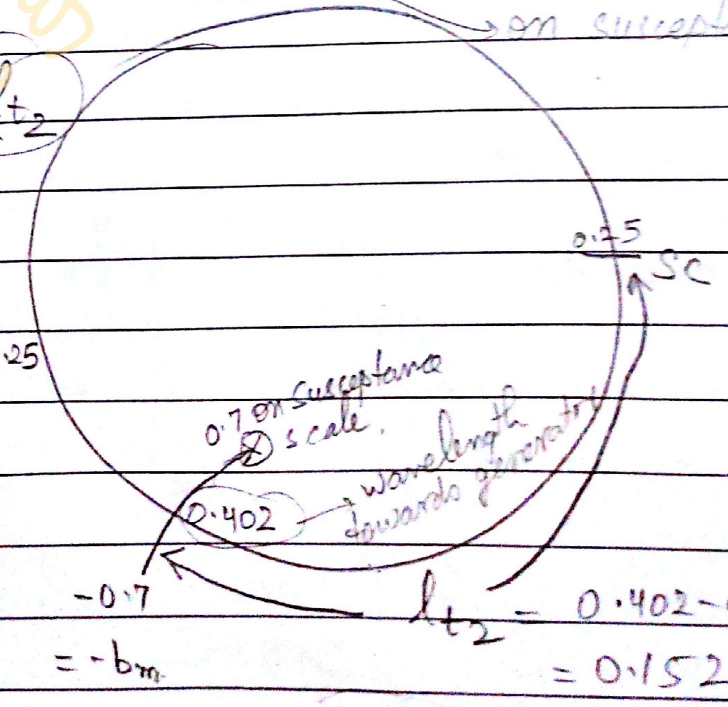
Coordinates of : M (g_m, b_m)

So, opposite (0.7) = −0.7 ($-b_m$)

(14) locate −0.7 → on susceptance scale

(15) Find l_{t_2}

arc length b/w 0.402 & 0.25



l_{t_2} = 0.402 − 0.25
= 0.152

$$\begin{array}{r} 0.402 \\ -\ .250 \\ \hline 0.152 \end{array}$$

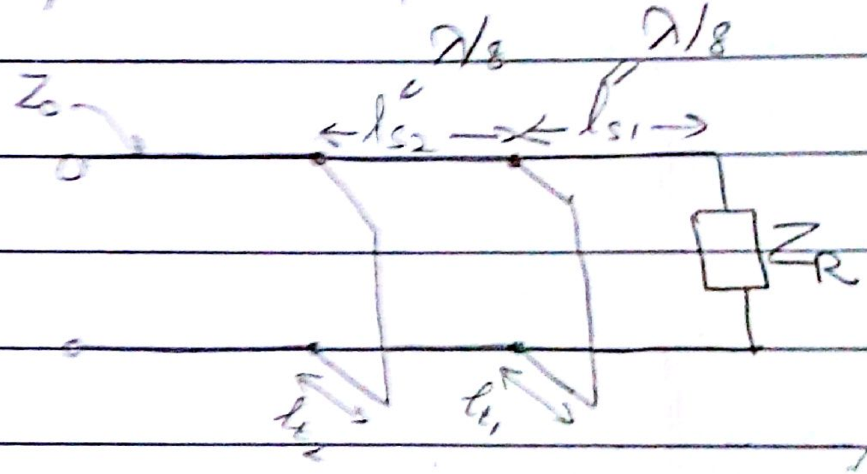
So, length of stubs are req^d :-

$$l_{t_1} = 0.19\lambda$$

$$l_{t_2} = (0.152)\lambda$$

Ans

Self
Q(J) Find double stub matching parameter for the line having $Z_R = 150 + j100\,\Omega$
$Z_0 = 100\,\Omega$



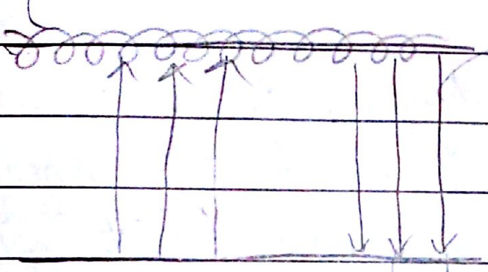
(Ans to be given as :-)

8 Antennas

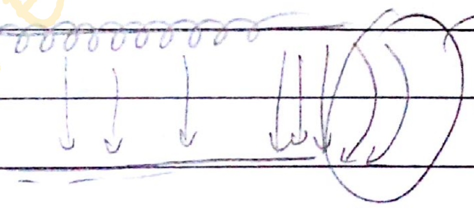
- **principle of radiation**
  - OC TL has tendency of radiation.
    - Only time varying current (AC) can radiate.

magnetic field

E field

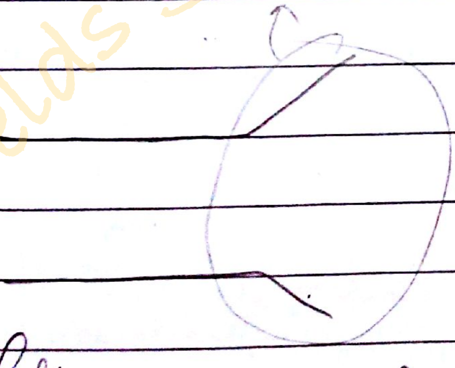


They are like charges & repel each other.
So, as a result, we get



FRINGING effect. (also seen in capacitances)

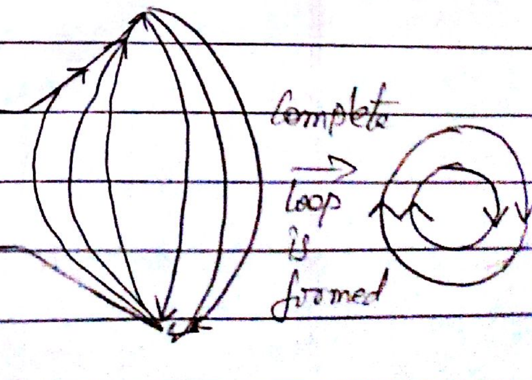
Now, we FLARE EDGE



As a result, impedance of free space matches TL impedance.

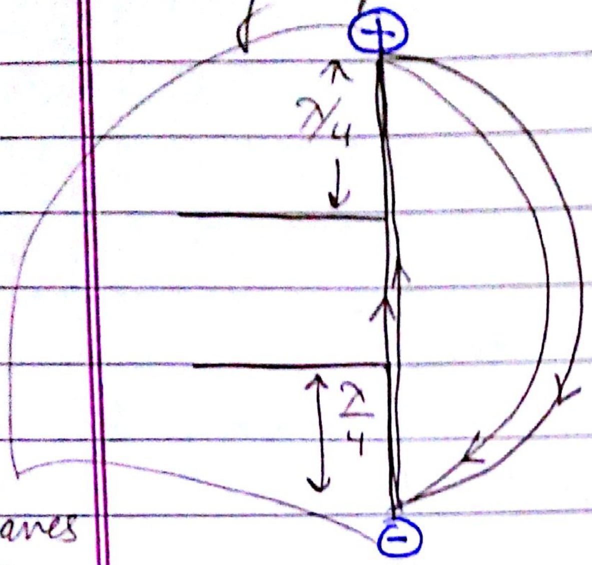
So, fields are seen as:

repulsion from other E-fields

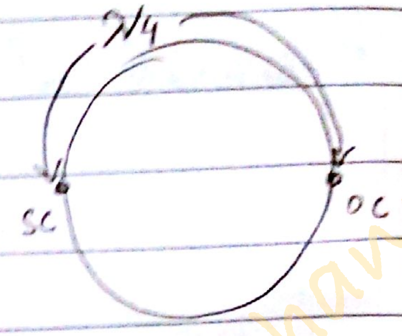


complete loop is formed

Taking dipole (i.e., doing more flaring)

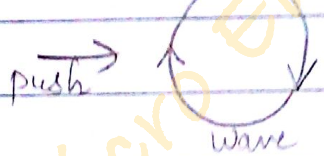


For impedance



behaves as a dipole

If we oscillate the charges of dipole, (wrt time), dipole gets tendency of radiation. hence then, it can push these waves out



Antenna:
Device that connect electronic circuit with free space (for transmission) & vice versa (for receiving)

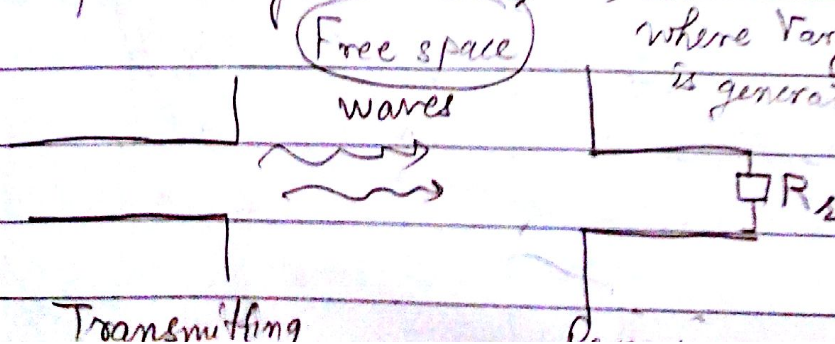
★ EMF generation: → electromotive force

eg: generator operⁿ: rotⁿ (moving)

eg②: transformer operⁿ: static

For EMF subject, Antenna's operⁿ is seen as Transformer sys:

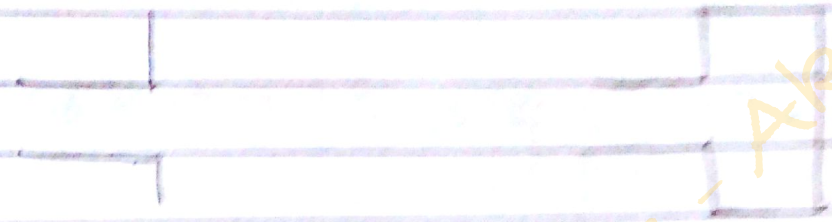
Free space → behaves as a medium where varying magnetic field is generated



- Marconi antenna : has length $\lambda/4$.
- Types of Antenna :

(1) Wire Antenna : Constructing structures only using wire

eg : Dipole antenna :   Loop antenna :



(2) Reflector Antenna :

$\exists$ reflector structure behind antenna. Reflector antenna follows parabolic / hyperbolic structure

(3) Array Antenna : We have group of antenna using same char.

» Based on Operating Freq :

(a) Low freq. antenna
(b) Medium freq. antenna
(c) High freq. antenna.

} have their own char.

★ RADIATION ~~PATT~~ PATTERNS :- → In which dir^n does the antenna transmit max. energy

→ In dir^n we get the receiving power.

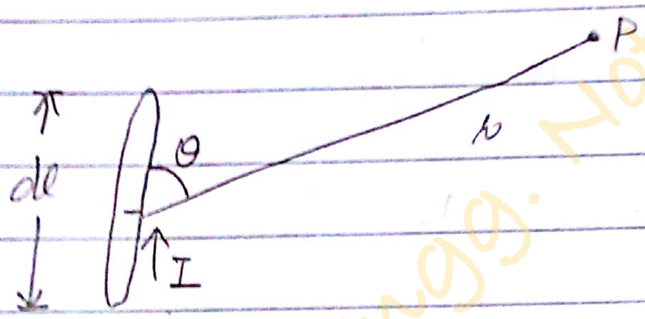
To study radiation patterns, we use POLAR CHAR[T]

- taking small current element $(I.dl)$
  - ↳ source of magnetic field

> Electric field depends on : Scalar potential
> Magnetic field : Scalar & vector potential

- Retarded Vector Potential : $[A]$

∵ ∃ delay



expression, $[A] = \frac{\mu}{4\pi} \int_V \frac{[\hat{J}]}{r} dV$

↳ considering magnitude only :-

signifies time delay
c : speed of light

$$[A] = \frac{\mu}{4\pi} \int_V \frac{J_m e^{-j\omega(t - \frac{r}{c})}}{r} dV$$

$$= \frac{\mu}{4\pi} \int_V \frac{J[t - r/c]}{r} dV$$

$dV \rightarrow ds \times dl$

$$= \frac{\mu}{4\pi} \int_V \frac{J[t - r/c]}{r} ds\, dl$$

$-j\omega(t - r/c)$

$J \cdot ds = I$

$$\Rightarrow [A] = \frac{\mu}{4\pi}\int \frac{I[t - r/c]}{r}\, dl$$

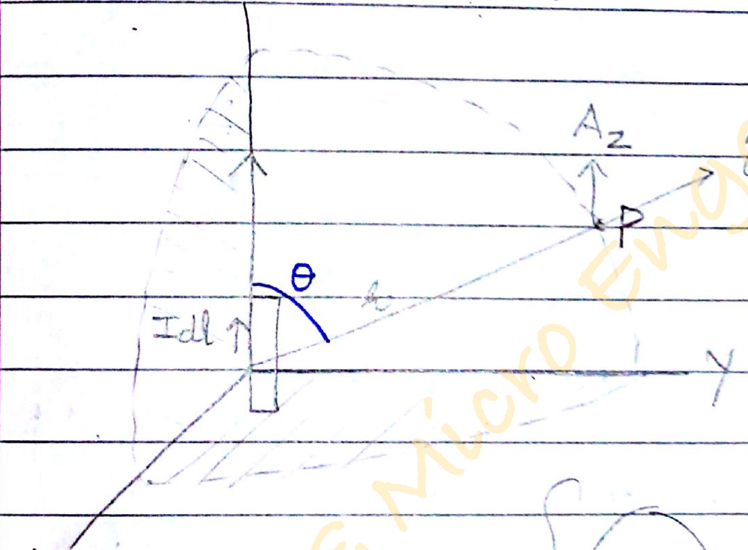
$$[A] = \frac{\mu}{4\pi}\int \frac{I_m \sin\omega(t - r/c)}{r}\, dl$$

or

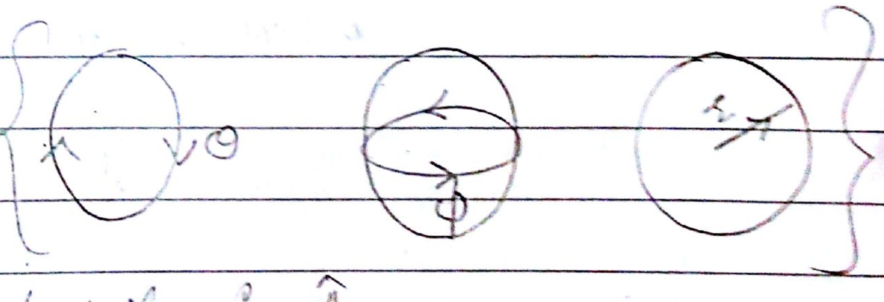
$$[A] = \frac{\mu}{4\pi}\int \frac{I_m \cos\omega(t - r/c)}{r}\, dl$$

retarded vector potential

★ Radiation by strong current element :



∴ transmitting antenna is at earth. So, changing to spherical coordinates



Finding vector potential : $\hat{A}$

↳ we will get B, $\boxed{\hat{B} = \nabla \times \hat{A}}$

★ Assume TM wave.

↳ Magnetic field won't exist in dirn of propagn

$\Rightarrow H_z = 0$.

Now, say, E is along θ dir^n

$\Rightarrow \because H_r = 0$ & E & H cannot be in same dir^n $\Rightarrow$ H is along ϕ (H_ϕ)

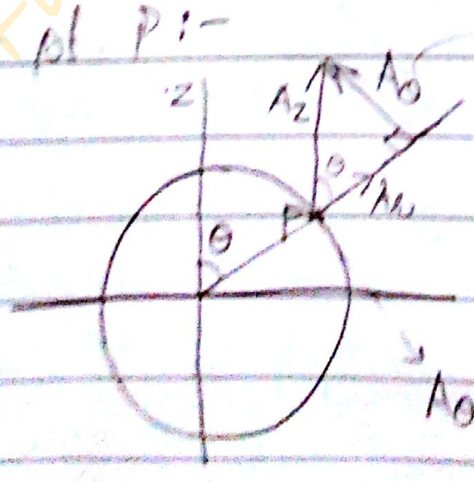
Now, $\hat{B} = \nabla \times \hat{A}$

$\hookrightarrow$ let $A = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

$$\Rightarrow \hat{B} = \begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{r\partial\theta} & \frac{\partial}{r\sin\theta\,\partial\phi} \\ A_r & A_\theta & A_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

At pt P :-



$A_r = A_z \cos\theta$

$A_\theta = (-) A_z \sin\theta$ *

Finding magnetic field, H_ϕ

($H_r = H_\theta = 0$, as told before)

$$\nabla \times A = \mu H = [\cancel{H_r}, \cancel{H_\theta}, H_\phi]$$

with $H_r = 0$ and $H_\theta = 0$.

So, $$(\nabla \times A)_\phi = \frac{1}{r^2 \cancel{\sin\theta}} \left(\cancel{r\sin\theta}\right)\left(\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial \theta} A_r\right)$$

$$= \mu H_\phi$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z \sin\theta\, r) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right]$$

Puffin
Date
Page

Finding magnetic field, H_ϕ

($H_r = H_\theta = 0$, as told before)

$$\nabla \times A = \mu H = [H_r, H_\theta, H_\phi]$$

($H_r \to 0$, $H_\theta \to 0$)

$$\text{So, } (\nabla \times A)_\phi = \frac{1}{r^2 \cancel{\sin\theta}} \left(\cancel{r \sin\theta}\right)\left(\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial \theta} A_r\right) = \mu H_\phi$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z \sin\theta\, r) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right]$$

$$\Rightarrow H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z \sin\theta\, r) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right]$$

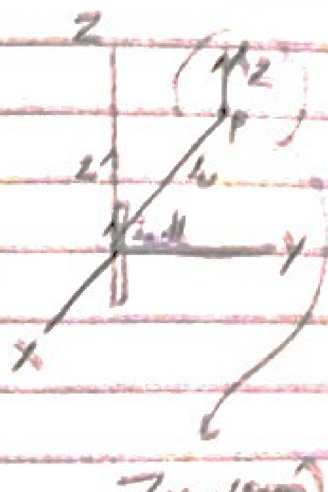
BIG DERIVATION --

☆ Radiation by small current element ($\ll \frac{\lambda}{2}$)

We know,

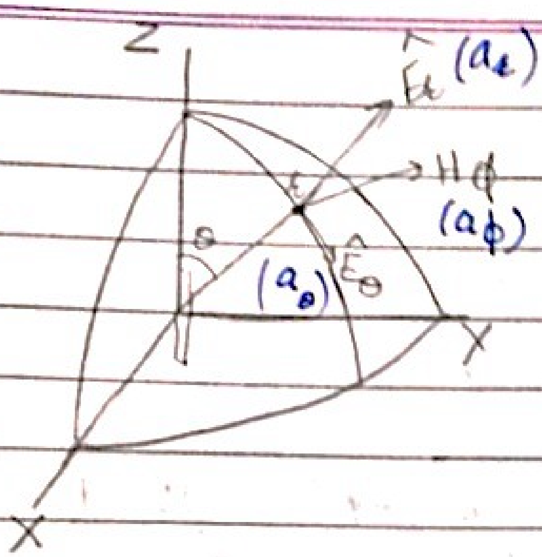
Retarded vector potential, at pt. P due to current element.

$$[A_z] = \frac{\mu I_m \, dl \cos \omega(t - r/c)}{4\pi r}$$



Assume we are transmitting TM wave,
So, the components are E_r, E_θ, H_ϕ.

Zoom (PTO)

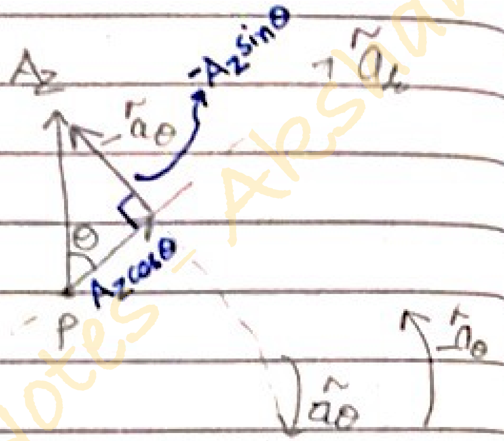


Steps:

S① Find $[A_z]$

S② Find H_ϕ from $[A_z]$

S③ Find E_r & E_θ from $[H_\phi]$



Now,

S2

$$\vec{B} = \mu \vec{H} = \nabla \times \vec{A}$$

Now, $\vec{H} = [\hat{a}_r H_r + \hat{a}_\theta H_\theta + \hat{a}_\phi H_\phi]$

||^{ly}

$$\vec{A}_z = [\hat{a}_r A_r + \hat{a}_\theta A_\theta + \hat{a}_\phi A_\phi]$$

For TM wave, E field : $\hat{E}_r \checkmark$, $\hat{E}_\theta \checkmark$, $\hat{E}_\phi = 0$.

H field : $\hat{H}_r = 0$, $\hat{H}_\theta = 0$, $H_\phi \checkmark$

So, $\nabla \times \vec{A} = \mu H_\phi a_\phi$ $\quad (\because \hat{H}_r = \hat{H}_\theta = 0)$

$$\frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix} = \mu H_\phi a_\phi$$

$(a_r = a_\theta = 0$, by TM wave$)$

So,

$$\frac{1}{r^2\sin\theta}\left(r\sin\theta\,\hat{a}_\phi\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial}{\partial\theta}(A_r)\right]\right)=\mu H_\phi\,\hat{a}_\phi$$

Taking only magnitude

$$\Rightarrow \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)}_{1} - \underbrace{\frac{1}{r}\frac{\partial}{\partial\theta}(A_r)}_{2} = \mu H_\phi \longrightarrow A$$

Now, $\hat{A}_z\Big|_{\text{along }\theta} = A_\theta = -A_z\sin\theta$

$\hat{A}_z\Big|_{\text{along } r} = A_r = A_z\cos\theta$.

$\hookrightarrow A_z = \mu\dfrac{I_m dl}{4\pi r}\cos\omega(t-r/c)$

Solving 1

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(\cancel{r}\left(-\mu\frac{I_m dl}{4\pi \cancel{r}}\cos\omega(t-r/c)\sin\theta\right)\right)$$

$$= \frac{1}{r}\left(-\mu\frac{I_m dl\sin\theta}{4\pi}\right)\frac{\partial}{\partial r}\cos\omega(t-r/c)$$

$$= -\mu\frac{I_m dl\sin\theta}{4\pi r}\left(\cancel{-}\sin\omega(t-r/c)\times\left(\cancel{-}\frac{\omega}{c}\right)\right)$$

$$= -\mu\frac{I_m dl}{4\pi r}\sin\theta\sin\omega(t-r/c)\times\frac{\omega}{c}$$

$$= -\mu\frac{I_m dl\,\omega}{4\pi r c}\sin\theta\sin\omega(t-r/c) \longrightarrow 1$$

Solving (2)

$$= -\frac{1}{r}\frac{\partial}{\partial\theta}\left(A_z\cos\theta\right)$$

$$= -\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{I_m \mu\, dl}{4\pi r}\cos\omega(t - r/c)\cos\theta\right]$$

$$= -\frac{1}{r}\left(\frac{\mu I_m dl}{4\pi r}\right)\cos\omega(t-r/c)\left[-\sin\theta\right]$$

$$= \frac{\mu I_m dl}{4\pi r^2}\sin\theta\,\cos\omega(t-r/c) \rightarrow (2)$$

Combining (1) & (2) & put in (A)

$$\Rightarrow -\frac{\mu I_m dl\,\omega\sin\theta\sin\omega(t-r/c)}{4\pi r C} + \frac{\mu I_m dl\sin\theta\cos\omega(t-r/c)}{4\pi r^2}$$

$$= \mu H_\phi$$

$$\Rightarrow \mu\left[\frac{I_m dl\sin\theta}{4\pi}\left\{-\left(\frac{\omega}{C}\right)\sin\omega\left(t-r/c\right)\left(\frac{1}{r}\right) + \cos\omega\left(t-\frac{r}{c}\right)\left(\frac{1}{r^2}\right)\right\}\right]$$

$$= \mu H_\phi$$

$$\Rightarrow \mu\left\{\frac{I_m dl\sin\theta}{4\pi}\left[\sin\omega(t-r/c)\left(\frac{-\omega}{Cr}\right) + \cos\omega(t-r/c)\left(\frac{1}{r^2}\right)\right]\right\}$$

$$= \mu\left\{H_\phi\right\}$$

far field region

$$\Rightarrow H_\phi = \frac{I_m\, dl \sin\theta}{4\pi}\left[-\frac{\omega}{c}\left(\frac{1}{r}\right)\sin\omega(t - r/c) + \left(\frac{1}{r^2}\right)\cos\omega\left(t-\frac{r}{c}\right)\right]$$

(B)

near field region

(S3) We have

→ 0, for free space (signal transmitted in free space)

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t}$$

$$\Rightarrow \vec{\nabla}\times\vec{H} = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial \hat{E}}{\partial t}$$

For (TM)

$$\vec{H} = \hat{a}_r H_r + \hat{a}_\theta H_\theta + \hat{a}_\phi H_\phi$$

$H_r \to 0$, $H_\theta \to 0$

$$\vec{E} = \hat{a}_r E_r + \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$E_\phi \to 0$

Finding E fields, E_r, E_θ

So, $\vec{\nabla}\times H_\phi = \epsilon_0\left[\frac{\partial}{\partial t}\left(\hat{a}_r E_r + \hat{a}_\theta E_\theta\right)\right]$

$$\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & r(0) & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= \frac{\hat{a}_r}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}\left(r\sin\theta\, H_\phi\right) - 0\right]$$

$$- \frac{r\hat{a}_\theta}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r\sin\theta\, H_\phi\right) - 0\right]$$

Comparing LHS & RHS.

$$\frac{\hat{a_r}}{r^2 \sin\theta} \frac{\partial}{\partial\theta}(r\sin\theta\, H_\phi) = \epsilon_0\, \hat{a_r}\left(\frac{\partial}{\partial t} E_r\right)$$

Taking only magnitude

$$\Rightarrow \frac{1}{r^2\sin\theta}\left\{\frac{\partial}{\partial\theta}\left[\frac{r\sin\theta\, I_m dl}{4\pi}\left(\frac{-\omega}{Cr}\sin\omega(t-r/c) + \frac{1}{r^2}\cos\omega(t-\frac{r}{c})\right)\right]\right\}$$

$$= \frac{1}{r^2 \cancel{\sin\theta}}(r)\left(\frac{2\cancel{\sin\theta}\cos\theta . I_m dl}{4\pi}\right)\left[\frac{-\omega}{Cr}\sin\omega(t-r/c) + \frac{\cos\omega(t-\frac{r}{c})}{r^2}\right]$$

$$= \frac{2\cos\theta\, I_m dl}{4\pi r}\left[\frac{-\omega}{Cr}\sin^{\omega}(t-r/c) + \frac{\cos\omega(t-r/c)}{r^2}\right]$$

$$= \epsilon_0 \frac{\partial}{\partial t} E_r$$

$$\Rightarrow \int\left(\frac{2\cos\theta\, I_m dl}{4\pi\epsilon_0 r}\left[\frac{-\omega}{Cr}\sin^{\omega}(t-r/c) + \frac{\cos\omega(t-r/c)}{r^2}\right]\right)dt = \int \partial E_r$$

$$= \frac{2\cos\theta\, I_m dl}{4\pi\epsilon_0 r}\left[\left(\frac{-\omega}{Cr}\right)\left(\frac{-\cos(t-r/c)}{\omega}\right) + \frac{1}{r^2}\frac{\sin\omega(t-r/c)}{\omega}\right] = E_r$$

$$\Rightarrow E_r = \frac{2 I_m dl}{4\pi\epsilon_0}\cos\theta\left[\frac{\cos\omega(t-r/c)}{Cr^2} + \frac{\sin\omega(t-r/c)}{\omega r^3}\right]$$

IIIly, finding

$$E_\theta = \frac{I_m dl \sin\theta}{4\pi\epsilon_0}\left[-\frac{\omega}{c^2 r}\sin\omega(t-r/c) + \frac{\cos\omega(t-r/c)}{\omega r^3} + \frac{\sin\omega(t-r/c)}{\omega r^3}\right]$$

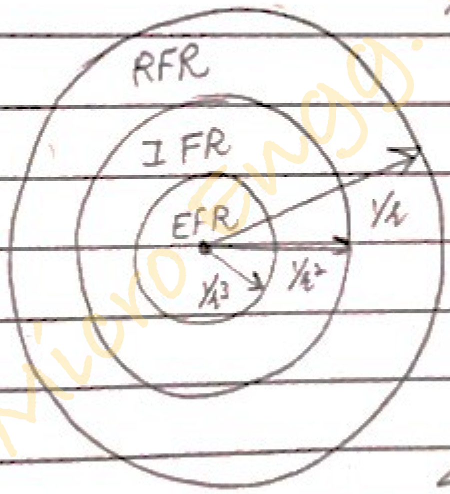
Seeing E_r & E_θ, we have terms of $\frac{1}{r}$, $\frac{1}{r^2}$ & $\frac{1}{r^3}$

$\frac{1}{r^3}$ terms very sharply with $r \uparrow$

$\frac{1}{r^3}$ term : called as Electrostatic field region ~~EFR~~ EFR

$\frac{1}{r^2}$ term : called as Inductive field region IFR

$\frac{1}{r}$ term : called as Radiation field region ~~EFR~~ RFR



We mainly consider $\frac{1}{r}$ term

So,

$$H_\phi = -\frac{I_m dl\, \omega \sin\omega(t - r/c)}{4\pi C r} \sin\theta$$

$$E_\theta = \frac{-I_m dl}{4\pi \epsilon_0} \frac{\omega}{C^2 r} \sin\omega(t - r/c) \cdot \sin\theta$$

(E_r doesn't have $\frac{1}{r}$ term)

- E_θ & H_ϕ are the fields present in Radiation region.

Now, keeping r as constt & varying θ, we get to see a radiation pattern

Now,
for a constt r, t, E_θ, H_ϕ only depend on $\sin\theta$

So, let

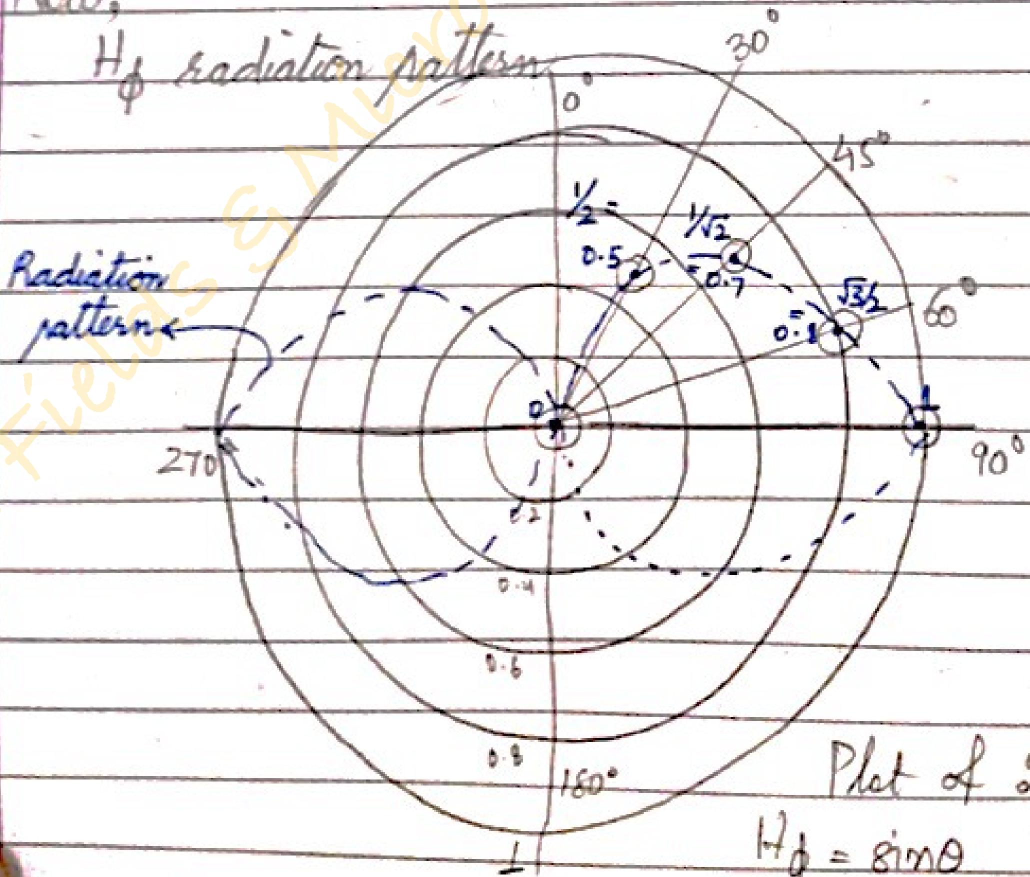
some constt = 1, for analysis below, say

$$H_\phi = H_M \sin\theta$$

$$E_\theta = E_M \sin\theta$$

Now, drawing radiation pattern. It is plotted on a Polar Graph
- Graph of concentric circles
- We can choose radii of circles & posⁿ of 0° & other angles by ourselves.

Now,
H_ϕ radiation pattern

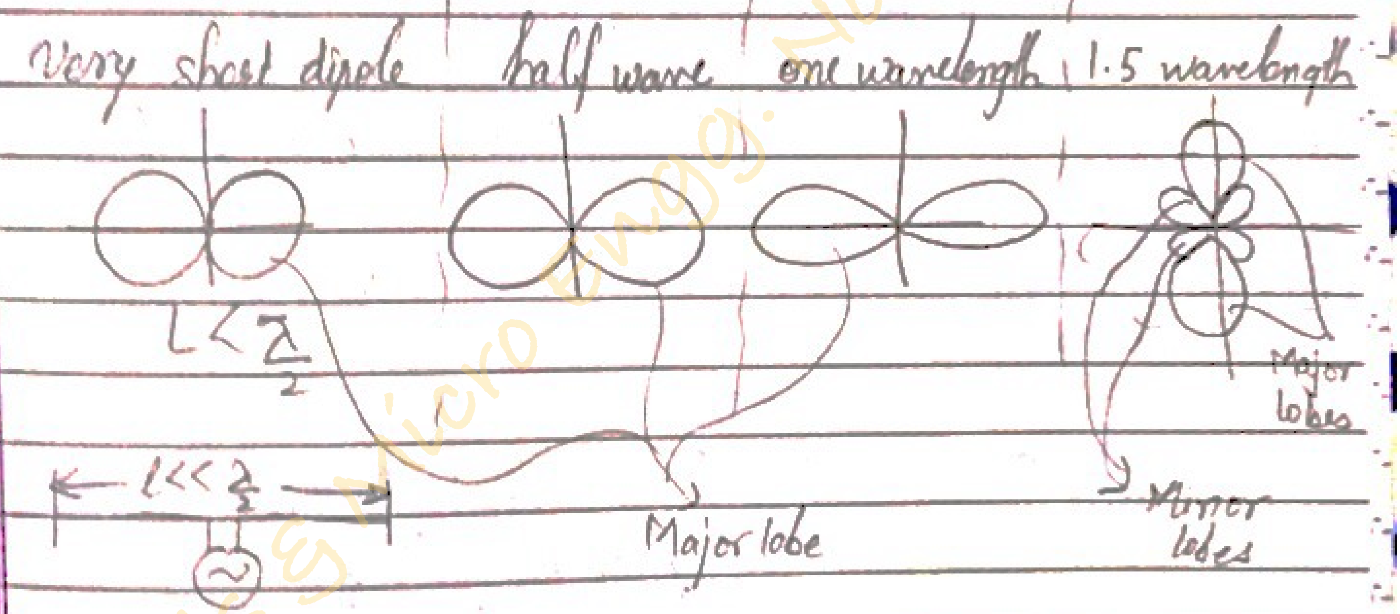


Plot of :
$H_\phi = \sin\theta$

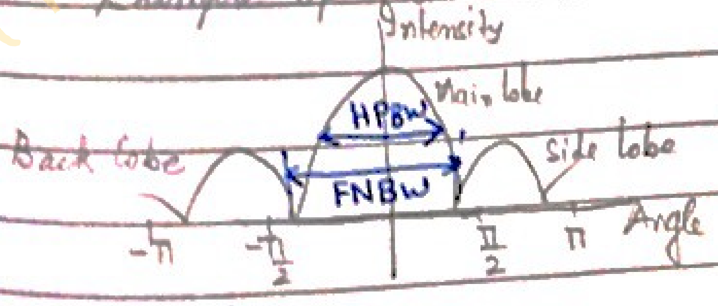
Inference from graph :
We get max radiation pattern at $\theta = 90°$.

- Note: Spherical coordinates
  Z : zenith
  θ : elevation
  ϕ : azimuth.

✱ Changes in radiation pattern :
By changing length of antenna (L)



✱ Sample of Radiation Pattern :



HPBW : Half Power Beam Width

FNBW : First Null Beam Width

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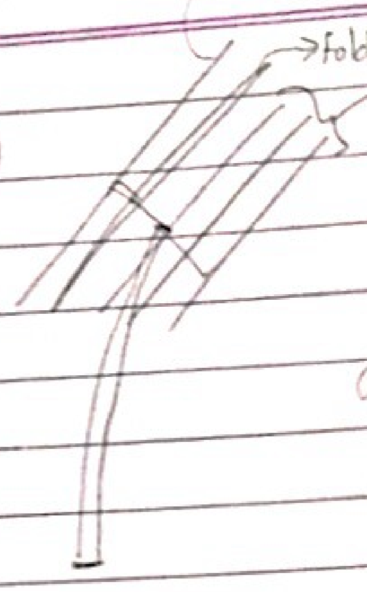


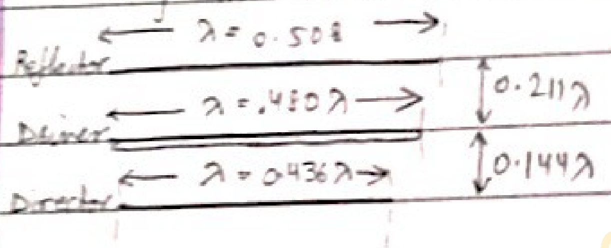
∘ Yagi's Antenna (6m & 10m)

for signals : 57MHz – 64 MHz.

$$f_c = 60.5\ MHz$$

$$\lambda = \frac{c}{f_c}.$$

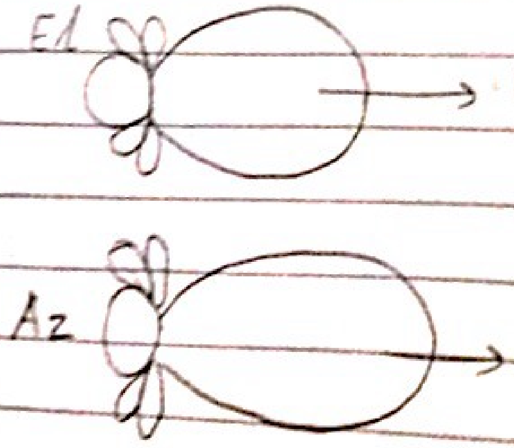
Making this antenna



In this, current flows only on skin of conductor. So, it reduces our cost ∴ we use hollow conductor

* Omnidirectional antenna : Antenna which can transmit signal in all dir^n.

In Yagi's antenna, signal is transmitted in dir^n of reflector



dir^n of Antenna as per major lobe loc^n.

Date: 9/12/13

✫ Power Density, S (Pointing Vector)

Instantaneous power density, $w = Re(E \times H^*)$

in W/m²

Averaged Power Density: $\boxed{w = Re(S) = Re\left(\frac{1}{2} E \times H^*\right)}$ Pointing vector, S

✫ Total power, P.

Avg. power through area ds: $dP = w \cdot ds = Re(S) \cdot ds$

$$dP = Re\{S\} \cdot d\sigma = Re\{S\} \cdot (r^2 \sin\theta \cdot d\theta\, d\phi)$$

Total avg. power P radiated is:

$$P = \int_0^{2\pi}\int_0^{\pi} Re(S) \cdot (r^2 \sin\theta)\, d\theta \cdot d\phi$$

✫ Radiation resistance, R_r

↳ value of a hypothetical resistor which dissipates power equal to power radiated by antenna when fed by same current I.

mathematically,

$$\frac{1}{2} I^2 R_r = P$$

✫ Antenna Impedance, Z_A

ratio of voltage at feeding pt. ($V(0)$) of the antenna to the resulting current flowing in the antenna I.

* EM wave $\longrightarrow$ (Conductor
complete reflection

* Total solid angle of antenna

✓ Area projected on sphere having 1 unit radius
✓ expressed in steradian
✓ Used to find **Radiation Intensity** (U)

$$U = \frac{dP}{d\Omega} \quad \text{(Power per solid angle)}$$

* Directivity : (D)

Ratio of max. radiation intensity (U_{max}) of antenna under study to that of isotropic radiator (U_0) radiating same amount of power in all dir^{ns}

$$D = \frac{U_{max}}{U_0} = 4\pi \frac{U_{max}}{P}$$

* Radiation efficiency (e)

Input Power = Radiator power (P) + loss (P_{Loss})

$$e = \frac{P}{P_{input}} = \frac{P}{P + P_{Loss}}$$

Type of antenna :

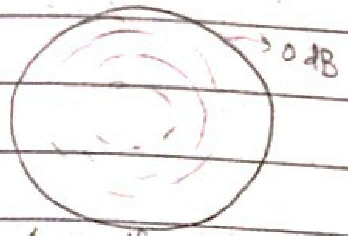
① ✓ INVERTED L Antenna — a vertical monopole that has been folded so that some portion runs horizontally

② ✓ Vertical antenna (creates image antenna on earth's surface)

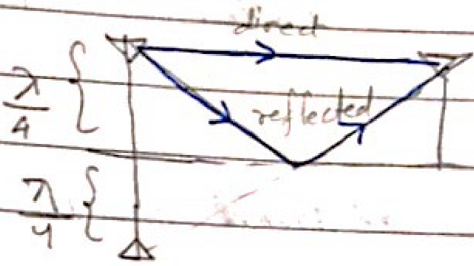
↳ has 2 paths of communic^n with receiving antenna : Direct & after reflection

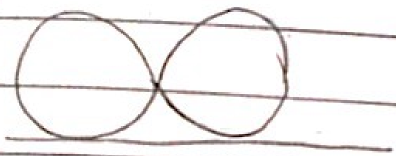
Polarizⁿ : Dirⁿ of E field wrt earth

★ Analysing $\frac{\lambda}{4}$ Vertical monopole :



Azimuth plot





★ Also seen for $\frac{\lambda}{4}$ & other lengths. (Book)

(3) ✓ loop antenna
- ↳ rectangular loop : to have wider beamwidth antenna
  - ✓ We can find aspect ratio & directivity of all antennas
- ↳ Delta loop

(4) ✓ Log Periodic Antenna
- ↳ can operate over wide range of freq
- ↳ directional antenna → gives max. gain in director's dirⁿ.

(5) ✓ Helical Antenna
- ↳ ✓ has a circular loop & rectangular loop. ∴ so, antenna creates circularly polarised wave.
- ↳ E field is circular.

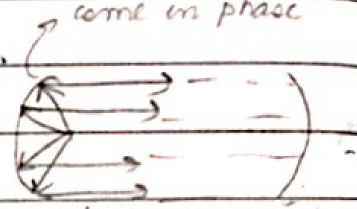
Horizontally Polarised wave : ⇉ field ∥ earth surface (Earth)

Vertically Polarised wave ⇊ field ⊥ to earth surface (Earth)

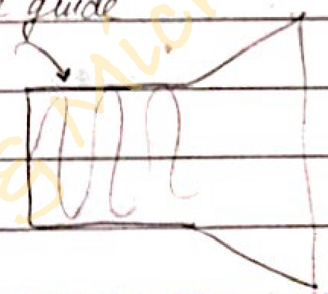
We have to match polarizⁿ.
If horizontal antenna at sender, horizontal should be there at receiver.

* When EM wave passes through charged medium, change of polarizⁿ takes place. This is called FARADAY ROTATION.
  → eg: ionosphere.
  ✓ Using circularly polarized wave works & doesn't have any effect & detected by Helical antenna.

⑥ ✓ Parabolic dish antenna
has point to point communicⁿ with sender & receiver.



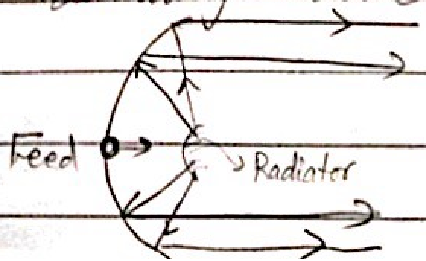
⑦ ✓ Horn Antenna



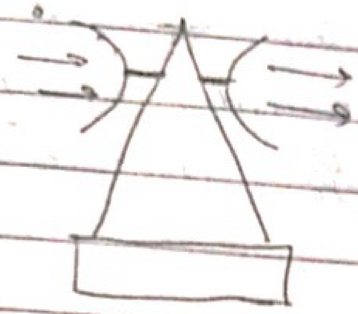
✓ provides unidirectional transmission.

Continued

⑧ ✓ Parabolic reflector antenna.
✓ meant for satellite communicⁿ.
✓ provides high gain & directivity
✓ has reflecting structure at back of radiating source



Structures like :

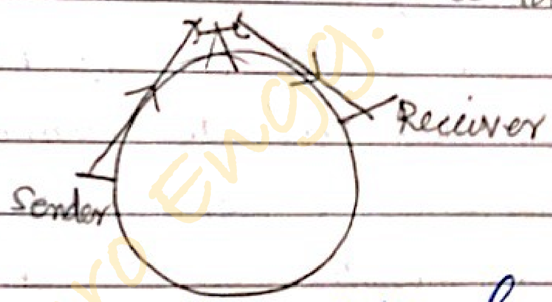


} req^d for transmitting over long distance

Microwave Relay Station } put every 50-70 kms.

Even if we create large amount of power, we are not able to send signals over large distances due to :

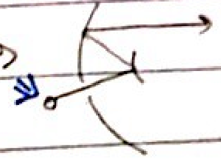
1. Earth troposphere (absorbs EMF energy)
2. Curvature of earth (straight signals cannot see the destination)



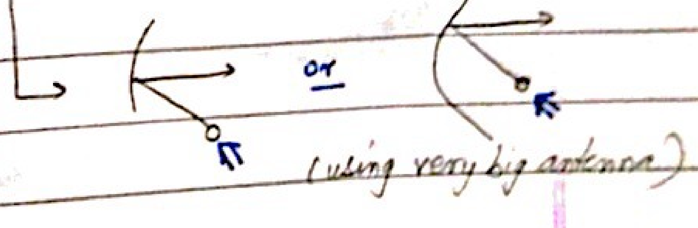
* laser communic^n not possible from earth's surface to satellite : ∵ Dispersion takes place in atmosphere.

- Parabolic antennas have an area of blockage at focal point. This blockage has to be reduced. how? ∵ Use Parabolic Reflectors :

✓ Cassegrain feed →



✓ Form feed



(using very big antenna)

Gain of parabolic antenna :-

or Directivity (D) $= \frac{4\pi}{\lambda^2} A_e$ → Area of reception.

→ $A_e = \eta A_p$

→ $D \simeq \frac{4\pi}{\lambda^2} (\pi \cdot r^2)$

dish antenna seems like circle

☆ In ships, we see an antenna round & round, that is meant to detect any flight nearby. That rotational electromagnetic beam can be achieved by using Array of antenna.
→ used in remote sensing also.

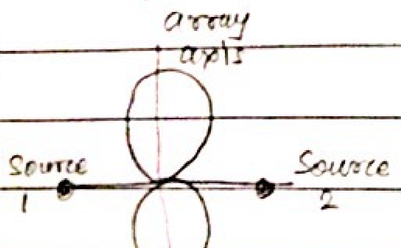
☆ ANTENNA ARRAY

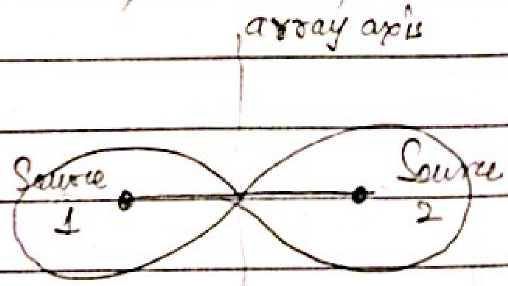
Broadside array

Current through all elements are in phase

End fire array

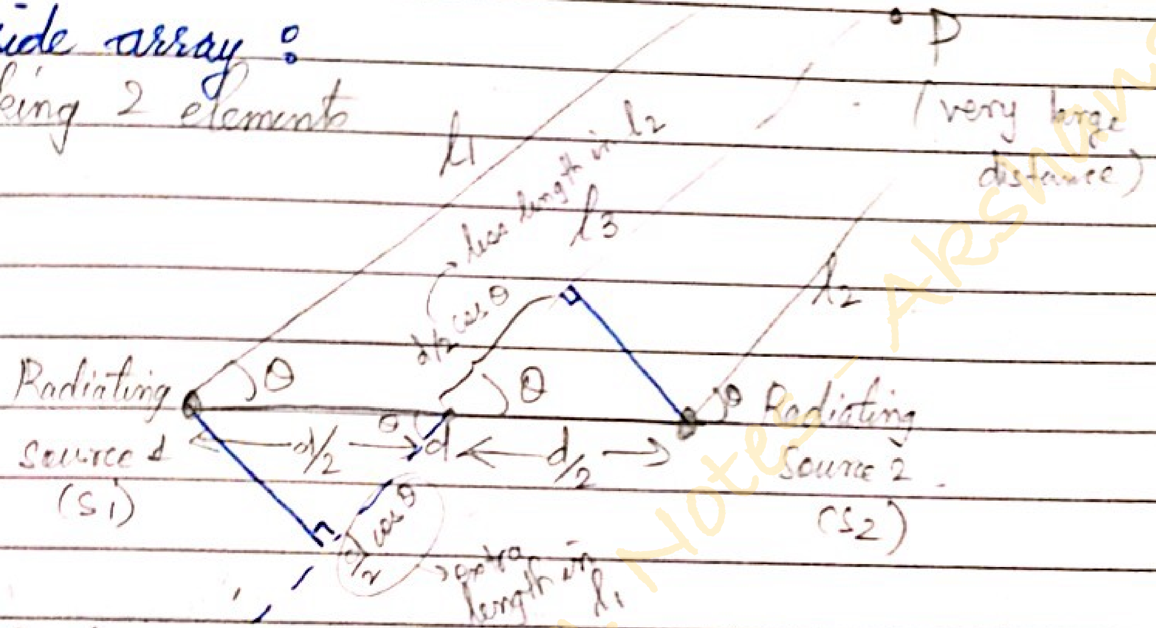
Successive elements have 180° phase shift current.





Idea : If we change phase (without moving anything), we have tilted EM beam.
So, this can be used for steering

⟨⟨1⟩⟩ Broadside array :
Taking 2 elements



l_3, l_1 & l_2 reach point P & $l_1 \parallel l_2 \parallel l_3$
(P is very far away)

length wise $l_1 > l_3 > l_2$ (l_1 has some extra path)
So,

$$l_1 - \frac{d}{2}\cos\theta = l_3$$

$$\& \; l_2 + \frac{d}{2}\cos\theta = l_3$$

So, Total path difference $= \frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta$

$$\text{Path difference} = d\cos\theta$$

for path along l_3, some phase gets added
So, for taking that effect, $\times \beta = \times \frac{2\pi}{\lambda}$
So,

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times d\cos\theta$$

Suppose point S_1 gives E_1 field at P &
S_2 gives E_2 field at P.

Total E at P $= E_1 + E_2$

for l_1, phase delay $= \psi \rightarrow \frac{\theta}{2}$.

Illy, l_2 $= \psi \rightarrow \frac{\theta}{2}$

Adding that to total E

$$\Rightarrow E = E_1 e^{j\psi} + E_2 e^{-j\psi}$$

↳ for simplificⁿ, assume $E_1 = E_2 = E_0$

$$\Rightarrow E = E_0 \left(e^{j\psi} + e^{-j\psi} \right)$$

$$\Rightarrow E = \underbrace{2E_0}_{\text{magnitude}} \underbrace{\left(\cos(\psi)\right)}_{\text{Phase}}$$

↳ $\psi = \beta \left(\frac{d}{2} \cos\theta \right)$

$$\Rightarrow E = 2E_0 \cos\left(\beta \frac{d}{2} \cos\theta \right)$$

$$= 2E_0 \cos\left(\frac{2\pi}{\lambda} \frac{d}{2} \cos\theta \right)$$

↳ assume $d = \frac{\lambda}{2}$.

$$\Rightarrow E = 2E_0 \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \times \frac{\cos\theta}{2} \right)$$

$$\Rightarrow E = 2E_0 \cos\left(\frac{\pi}{2} \cos\theta \right)$$

Removing further complexity, let $2E_0 = 1$

$$\Rightarrow \boxed{E = \cos\left(\frac{\pi}{2}\cos\theta\right)}$$ ☆

↳ This relⁿ establishes radiation pattern.
by using values of E at diffᵗ values of θ.

⇒ Usually, we see the angle at which E =
a) max
b) min
c) corresponding to ½ power.

(a) Maxima dirⁿ.

↳ $\Rightarrow \cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$

$$\Rightarrow \frac{\pi}{2}\cos\theta_{max} = n\pi, \; n \in Z^+$$

$$\Rightarrow \theta_{max} = \cos^{-1}(2n) \; ; \; n = 0, 1, 2, \ldots$$

or

$$\cos\theta_{max} = 2n.$$

↳ $n = 0$

$\cos\theta_{max} = 0$

↳ $n = 1$

$\cos\theta_{max} = 2$ (not possible)

Now, $\cos\theta_{max} = 0$

$$\Rightarrow \theta_{max} = (2n+1)\frac{\pi}{2} \; ; \; n \in Z^+$$

$$\Rightarrow \theta_{max} = 90^\circ, 270^\circ$$

& Max. value $= \pm 1$

b. Minima dir^n

$\hookrightarrow \Rightarrow \cos\left(\frac{\pi}{2}\cos\theta\right) = 0$

$\Rightarrow \frac{\pi}{2}\cos\theta = (2n+1)\frac{\pi}{2}$; $n = 0, 1, 2, \ldots$

$\Rightarrow \cos\theta_{min} = (2n+1)$

$\hookrightarrow n = 0, \cos\theta_{min} = 1$

$n = 1, \cos\theta_{min} = 3$ ✗ not possible.

$\cos\theta_{min} = 1$

$\Rightarrow \theta_{min} = 0, 180°$

& min. value $= 0$

c. Half power point

(finding $\frac{P}{2}$)

half power, $\frac{P}{2} = \frac{E^2}{2} = \left(\frac{E}{\sqrt{2}}\right)\cdot\left(\frac{E}{\sqrt{2}}\right)$

finding this expression ∵ we are plotting E, not E^2

So, $\cos\left(\frac{\pi}{2}\cos\theta_{HP}\right) = \pm\frac{1}{\sqrt{2}}$.

$\Rightarrow \frac{\pi}{2}\cos\theta_{HP} = (2n\pm1)\frac{\pi}{4}$; $n = 0, 1, 2, \ldots$

$\Rightarrow \cos\theta_{HP} = n \pm \frac{1}{2}$

$\hookrightarrow n = 0, \cos\theta_{HP} = \pm 1/2$ ✓

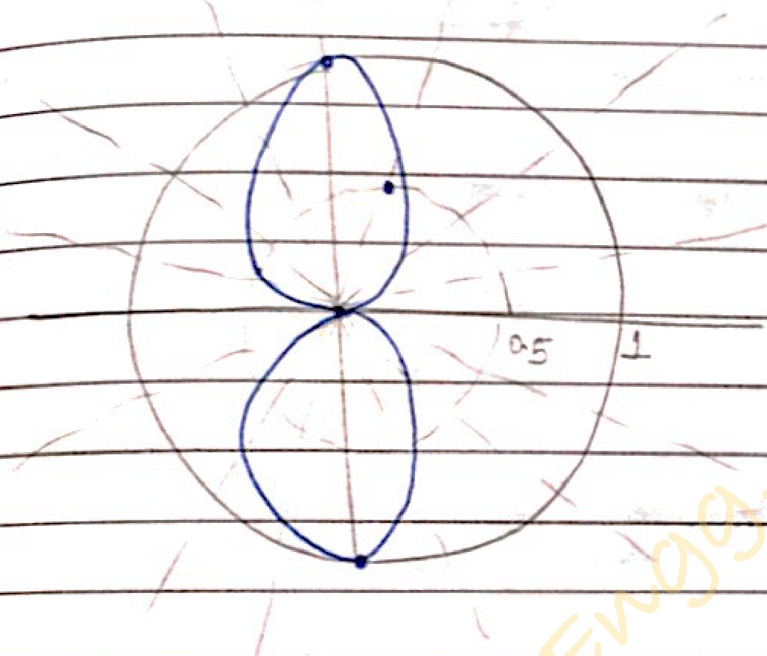
$n = 1, \cos\theta_{HP} = 1.5$ ✗

So, $\cos\theta_{HP} = \frac{\pm 1}{2}$

$$\Rightarrow \theta_{HP} = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

& HP value $= \pm \frac{1}{\sqrt{2}}$

Now, Plotting these values in the chart



• Radiation pattern

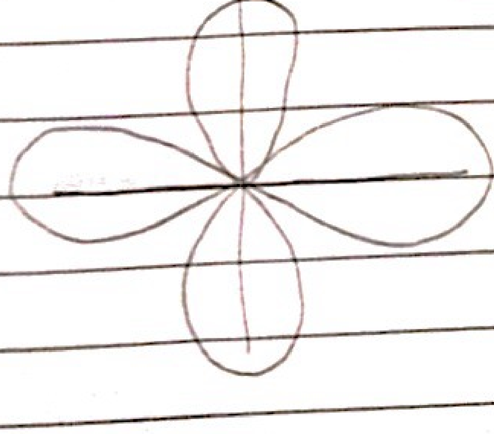
Now, I had, $E = 2E_0 \cos\left(\beta \frac{d}{2} \cos\theta\right)$

Increasing length as $d = \lambda$ $\left(, \beta = \frac{2\pi}{\lambda}\right)$, we get

$E = 2E_0 \cos(\pi \cos\theta)$

Now, finding θ_{max}, θ_{min} & θ_{HP} (self)

Radiation pattern becomes



(Just by increasing distance)

(On doubling length, everything gets doubled)

⟨⟨2⟩⟩ End fire - array :

Current through successive elements differ by 180°.

$$\bullet\longleftrightarrow\bullet$$

	0°	180°
current	$\pm I$	$\mp I$

Consider 2 fields → E_1 & E_2. So, we are taking resultant

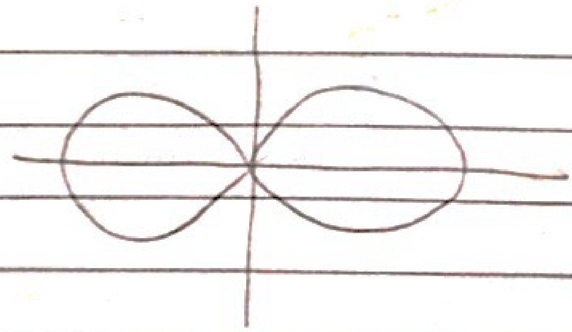
$$E = E_1 \pm E_2$$

Solving just as in prev. case

$$\Rightarrow E = 2E_0 \sin\left(\frac{\beta d}{2}\cos\theta\right)$$

Finding θ_{max}, θ_{min} & θ_{HP}, we get $d = \frac{\lambda}{2}$.

Radiation pattern becomes



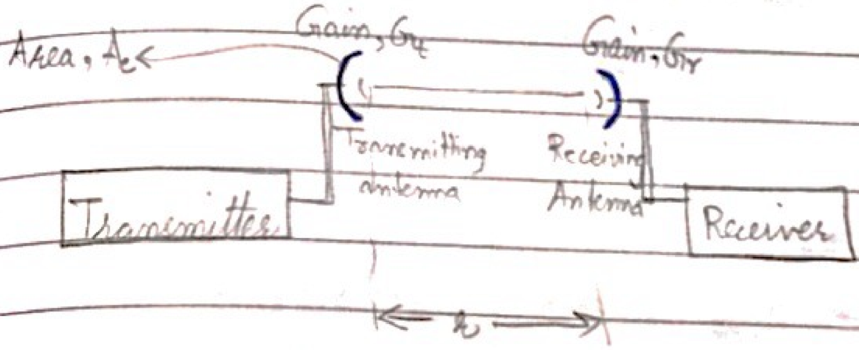
along horizontal axis
(array axis)

eg: Using instantaneous orientation of satellite

* Transmitting antenna is Omni-directional antenna.

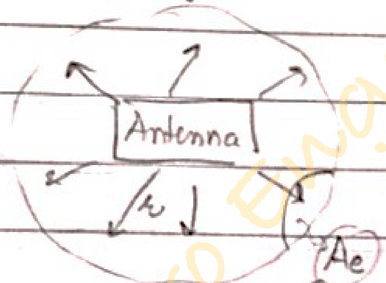
FRISS FORMULA:

↳ for Radio Link.



Finding received power at distance r, P_r:

For transmitting antenna,



Including antenna gain

$$\text{Power density, } S = \frac{P_t \times G_t}{4\pi r^2}$$

$$P_r = S \cdot A_e = \frac{P_t G_t A_{e_r}}{4\pi r^2}$$

on receiver's side

Area of Apperture

Area used for receiving signal

For any antenna, finding Area of Apperture: Gain

$$\cancel{A_e}\, G = \frac{4\pi}{\lambda^2} \cancel{G_r}\, A_e$$

$= A_{e_r} \rightarrow$ receiver

$= A_{e_t} \rightarrow$ transmitter

So, $G_r = \frac{4\pi}{\lambda^2} A_{e_r}$, $G_t = \frac{4\pi}{\lambda^2} A_{e_t}$

$$\Rightarrow P_r = \frac{P_t G_t G_r \lambda^2}{4\pi r^2 (4\pi)} = P_t G_t G_r \left(\frac{1}{\left(\frac{4\pi r}{\lambda}\right)^2} \right)$$

called EIRP
Effective Isotropic Radiated Power

called FSL
Free space Loss

Received power

$$\Rightarrow \boxed{P_r = \frac{EIRP \times G_r}{FSL}}$$

→ not in our control. This loss has to occur.

→ Including Noise added by Receiver & not CHANNEL (called SYSTEM NOISE)

* Note:- Terminal noise,

Noise power ← $P_n = kTB$ (Hz → GHz)

Boltzmann's const. ; Equivalent noise Temp. = T_s (sys. noise temp)

Now, $$\boxed{\frac{P_r}{T_s} = \frac{EIRP}{FSL} \times \left(\frac{G_r}{T_s}\right)}$$

sys. noise temp. → FIGURE OF MERIT

Also,

$$P_r = (S.A_{et}) \times A_{er}$$

$$= \frac{P_t}{4\pi r^2} \times A_{et} \times A_{er}$$

$$\Rightarrow \boxed{\frac{P_r}{P_t} = \frac{A_{et} \times A_{er}}{4\pi r^2}} \rightarrow \text{FRISS formula}$$

end of Antenna

Structure, principle of opr^n, applic^ns — No Derivations
(only theory questions)

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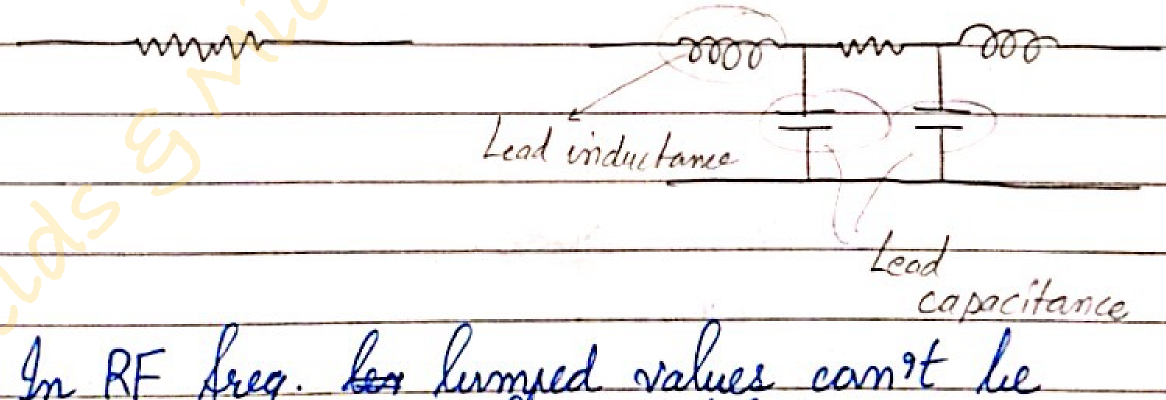
Ch: Microwave Devices.

* Electronic components work at lower frequency
* If RF freq. is applied :-
  Transit time comes into picture & doesn't allow device to operate in RF freq
  time taken by charged particle to go from Emitter to collector (of the order $\times 10^{-6}$)
  If nano second signal is sent, it is not received

Convention resistor | Resistor at RF freq



* In RF freq. ~~for~~ lumped values can't be applied. So, go for distributed components.
  for using distributed components, use TL
  (lead inductance, capacitance effect are removed then)

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☆ Active Components :
↳ Vacuum Tube microwave devices
↳ Semi conductor devices

- Vacuum tube microwave device are :
Klystron, TWT, Magnetron
↳ used for handling High Power
↳ used for transmitting μ wave signal to distance pt.
- Semi Conductor devices
↳ Si, Ge can't be used to make them
↳ InP, GaAs are used to manufacture such devices

Generate μ wave Signals

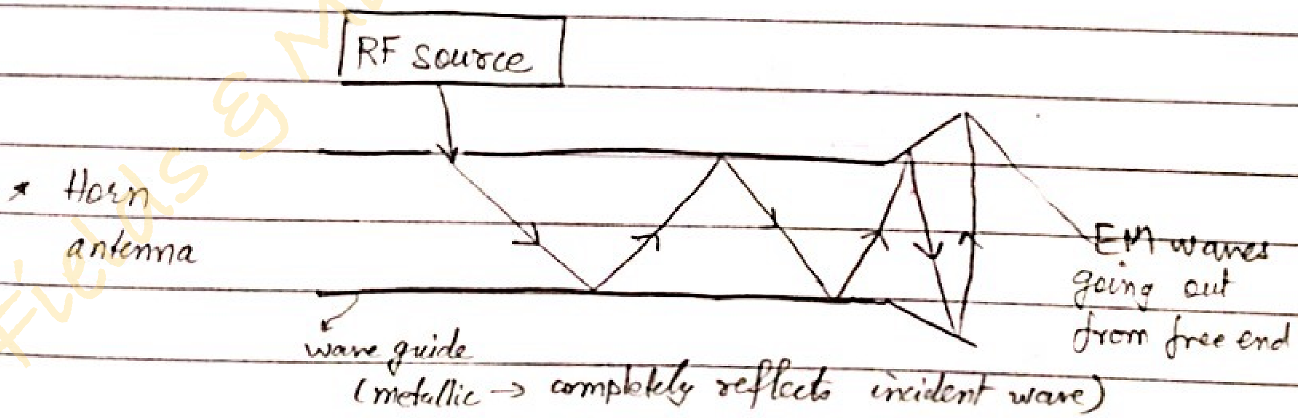
☆ Gunn Diode :
Generates microwave signal.
☆ Varacter Diode :
Used to amplify
☆ IMPATT Diode
↳ Transit time
☆ TRAPATT Diode
☆ Tunnel Diode
☆ RF Transistor.
☆ RF Mosfet.

☆ Generate microwave signals

☆ Passive component.
That component doesn't require any external energy source for its action. eg. Resistor

* **Active component:**
  External energy source is req^d to have its action. eg: transistor.

- **Components in Passive microwave device:**
  1) Wave Guide
     A medium which transmits TE & TM wave. (doesn't transmit TEM wave ∵ In TEM, ∃ no E or B field in dir^n of propag^n)
     * TEM wave is carried by TL (∵ TL is analysed by V & I not waves)
     * If ∃ OC o/p or open ended o/p, there are chances of EM radiations.



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✱ Microwave Devices :

∵ of transit time, conventional components can't be used for microwave devices

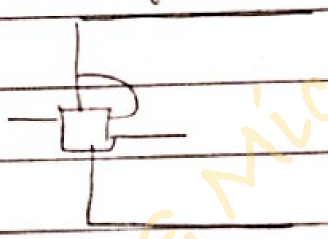
✱ Structure of waveguides is seen in Rectangular, Circular & Elliptic.

✱ Small value of λ : Transmission ✓ from waveguide
higher value of λ : " ✗ " ".

✱ **Waveguides :**

↳ dielectric ~~mat~~ material can be used as air.
(∴ dielectric loss is very less)
→ minimal losses in conductive walls
↳ No radiation losses, as E & H fields are contained.

✱ Energizing wave guide :

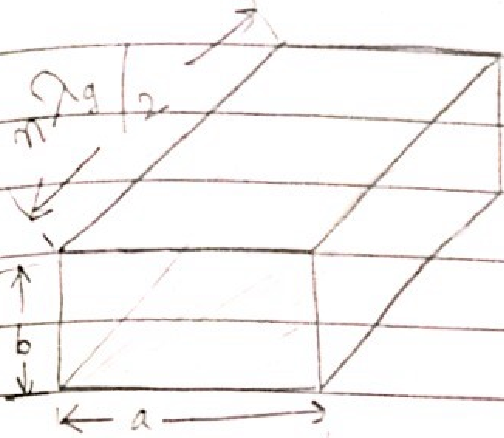


; we get transverse electric & magnetic wave

Wave guide structure

- Bends in Waveguides
  - ↳ E-plane bend → Bend in dirⁿ of E field
  - ↳ H-plane bend → " " " H field
- T shapes in waveguides (Tees)
  - ↳ E-plane Tee → o/p is out of phase with i/p (Series tee)
  - ↳ H-plane Tee → o/p is in phase with i/p (Shunt tee)

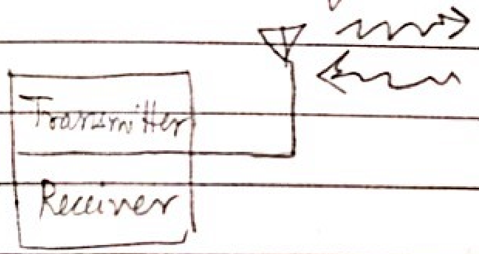
✱ Hybrid Tee → Tapping signal at diff^t points, we get in phase & out of phase i/p & o/p

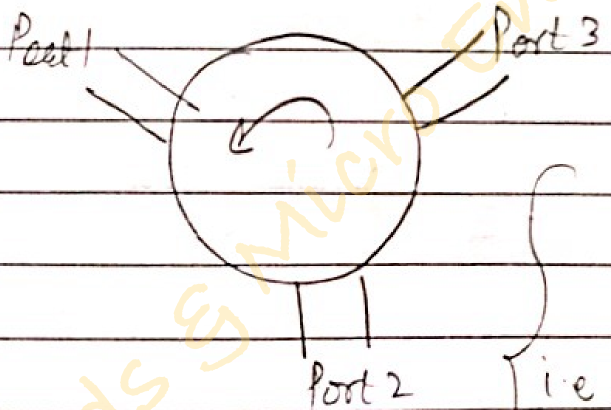


: Waveguide

• CIRCULATOR

In communicatio^n sys, like mobile phones, we have same antenna for transmission & reception





Port 1 connected to Port 2 only

Port 2 ⟶ Port 3

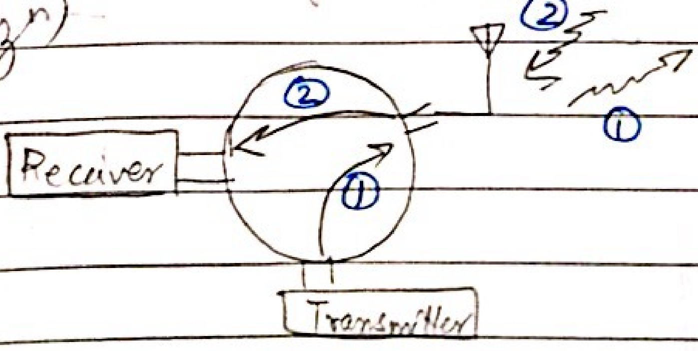
Port 3 ⟶ Port 1

i.e, any i/p to port 1, can't go in port 3. (goes to port 2)

possible due to use of ferrite material (provides isolation)

As EM wave enters, it undergoes FARADAY Rot^n (Change of polariz^n)

Idea :

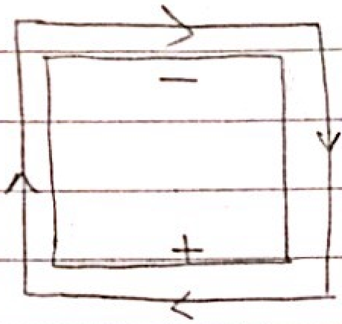


TWT: Travelling Wave Tube

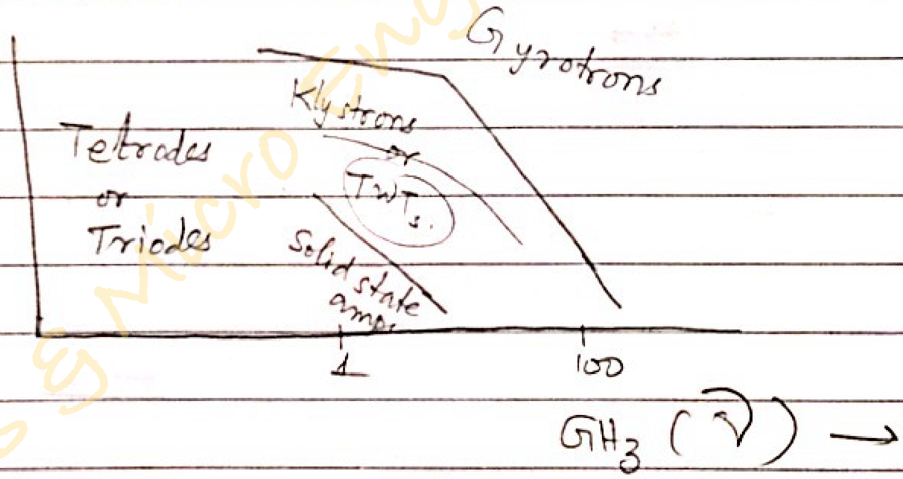
* hollow metallic structure (is) resonance circuit

* RF structure :

hollow metallic structure acts as tuning element
When magnetic field flows across conductor, I is generated



* Microwave tubes .



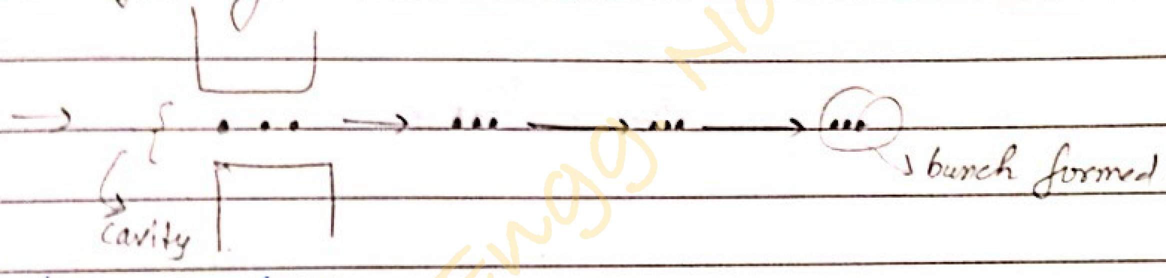
• Klystron: Tube version of device.

cathode (emits e^-) → Add^nal Anode (Accelerating e^-'s speed) → Hollow metallic structure (i/p cavity) signal to be amplified is applied → Energy of e^- bunch released at Output Cavity → Collector or Anode (collects e^-)

(cavity → resonant circuit)

* In picture tube, 18000 V is generated b/w cathode & anode
  ↳ TV's

* Velocity modul^n :
  > Change in vel. of e^- on passing through cavity.
  > Change of vel is changed w.r.t i/p signal magnitude
  > When -ve cycle of RF i/p : retardation
    +ve cycle : accl^n
    no change : constt vel
  → This forms e^- bunch



• Klystron Applic^{ns}
  - radars tracking, aerial survey.
  - missiles
  - satellites.
  - industrial microwave heating.

• Carrier : generated using ~~carrier~~ Oscillator.
• Klystron : An Amplifier
  Amplifies RF input signal.
  ↳ another structure : REFLEX Klystron :
    i.e., reflection happens at collector.
    Modific^n : Make collector plate -vely charged

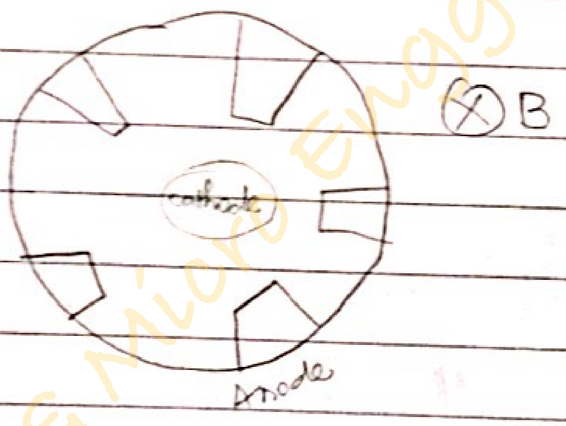
> On making collector plate, act as reflector, e^- bunch start forming in the cavity (depending on repeller voltage)

☆ **MAGNETRON** → a source, giving out microwave signal

↳ has Magnetic field & Electric field $\perp$ to each other. So, called as CROSS FIELD DEVICE

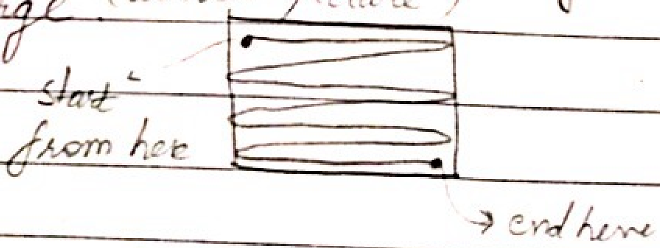
eg: magnetron present in Microwave devices

✓ Inherently efficient.



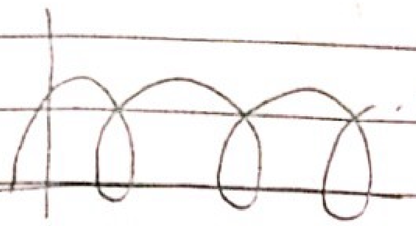
Scanning of e^- beam
(used in CRO also)

→ e^- beam is moved according to value of charge (across picture)



→ Scanned from top left to bottom right

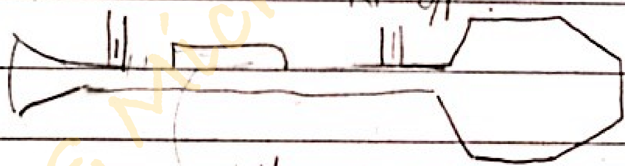
Path of e^- b/w cathode & anode :- (for magnetron)



. These days, ∃ ~~hardware~~ hybrid ~~def~~ devices.

* TRAVELING WAVE TUBE
  ↳ Simpler structure.

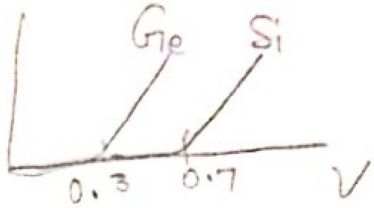
* Using Vacuum tube :
  ∃ cathode & anode
  ∃ attenuator circuit



✓ High Energy e^- beam $\xrightarrow{\text{given to}}$ RF output
✓ Microwave : big devices : can have value from milli Hz to few megaGHz)

* Devices that can generate * wave signals.
* Geo-synchronous satellite
* Uplinks freq & Down links flex.

Conduction :



* Band design" vs freq range.

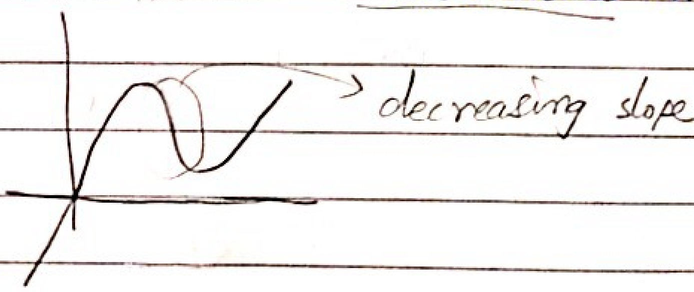
✲ TUNNEL DIODE

↳ Tunneling : a quantum mechanical ~~formula~~ phenomena with no analog in classical physics.

↳ Occurs when an $\bar{e}$ passes through a potential barrier without having energy to do so.

↑ voltage, I (current) is ↓ (Decreasing)

↳ Device exhibits -ve resistance char.



✲ Amplifier with +ve feedback = Oscillator

↳ generates freq

- Diffusion process due to lower magnitude of Forward bias.

✓ Diode can act as switch

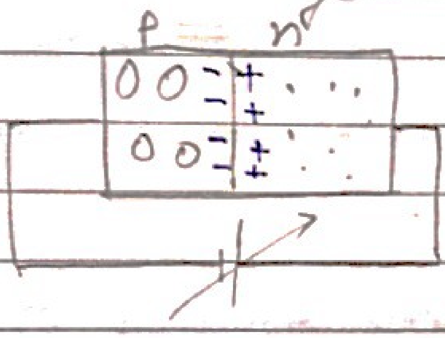
Variable Capacitor diode

✲ VARACTOR DIODE :

According to amount of capacitance wanted, this diode is designed.

Symbol :

Previous knowledge



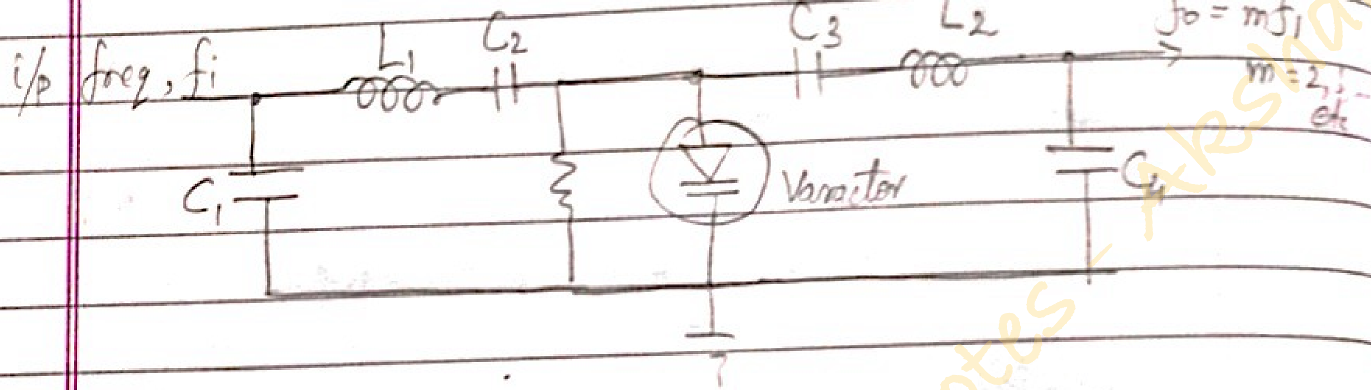
Note : P type & N type are NOT charged
They are Neutral atoms
One has more e^- in valence $\Rightarrow$ N type
less e^- in valence $\Rightarrow$ P type

n-type material has elements with 5 e^- in valence. Some atoms with more Energy will lose e^- & make a +ve ion.
This e^- goes through the junction & joins with p-type atoms, making them -ve ion.
So, At junction we have +ve & -ve ions, making depletion region. When p-n junction is RB, we are supplying donor & acceptor e^- on both sides. So, ions start forming, which are stationary.
So this stable sys. is basically STORING Energy.

Consider V_0 : o/p voltage
V_1 : i/p voltage.

If $V_0 = a_0 + a_1 V_1 + a_2 V_1^2 + a_3 V_1^3 + \dots$

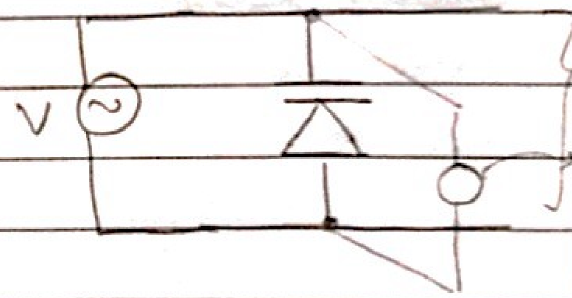
- Varactor diode as freq. multiplier :



- LNA : Low Noise Amplifier or
  Low Noise Boost.
  ↳ Say i/p from satellite (pWatt range). So, for preventing further power reduction during transmission, amplificⁿ is done at the receiver end itself → done using varactor diode

☆ $V = \frac{Q}{C}$

- Parametric amplifier.



gives amplified o/p

pump source (used to change capacitance of varactor) (an energy source)

Zener breakdown : Meant for voltage stabilizn

Puffin
Date
Page

GUNN DIODE :

→ also called Transferred electron devices (TEDs)
→ used to generate Microwave signals.

→ high voltage → [GaAs or InP material] → -ve resistance property leads to genern of Microwave signals

IMPATT DIODE

→ Impact Ionization Avalanche Transit Time
→ 2 terminal multilayer device.
→ many layers of p & n (unlike other diodes which have only 1 or 2)

Due to avalanche process, ∃ impact on one part of semiconductor. Due to impact, ionizn occurs. Due to ionizn, e^-s start moving & hence, transit time is low now. So, -ve resistance is created.
This -ve resistance helps in voltage variation.