COMMUNICATION SYSTEMS NOTES

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Communication Systems Notes, First Edition

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* Normalized Power :- Make $R = 1$. So, original source doesn't matter, its V or I.
So, $P = \frac{V^2}{R} = I^2(R) =$ same
$R \to 1$, $R \to 1$

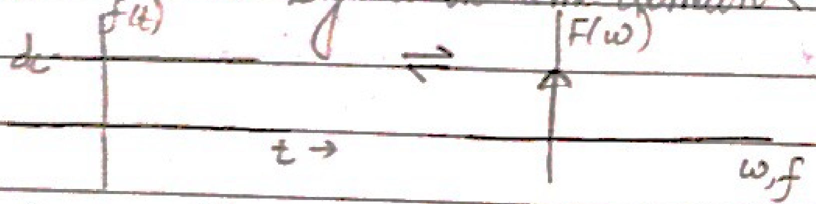
* Fourier series :- $\left[f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \right]$
Representⁿ of periodic signal as a linear combinⁿ of mutually orthogonal fⁿs

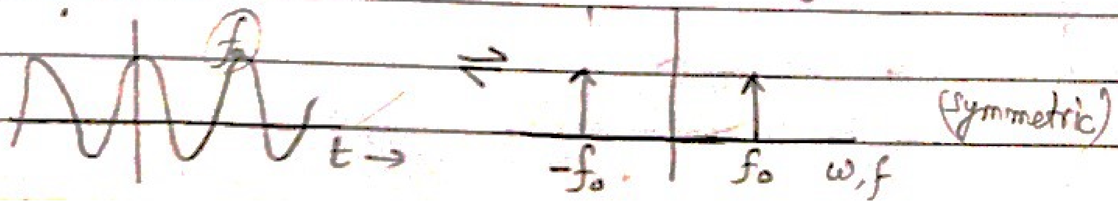
* Fourier transform :- $\left[F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt \right]$
A tool whereby $\exists$ one more domain to define a signal. A transformⁿ of a signal from time domain(t) to complex frequency ($\omega = \frac{2\pi}{T} = 2\pi f$).

* Dirichlet's condⁿ :-
For any periodic fⁿ on which Fourier transform to be applicable, the area under curve of fⁿ should be $< \infty$. Mathematically,
$$\int_{-\infty}^{\infty} |f(t)| \, dt < \infty .$$

* Signum fⁿ :- A fⁿ which indicates sign

* Note : A DC signal in time domain $\rightleftharpoons$ 0 freq. in ω domain

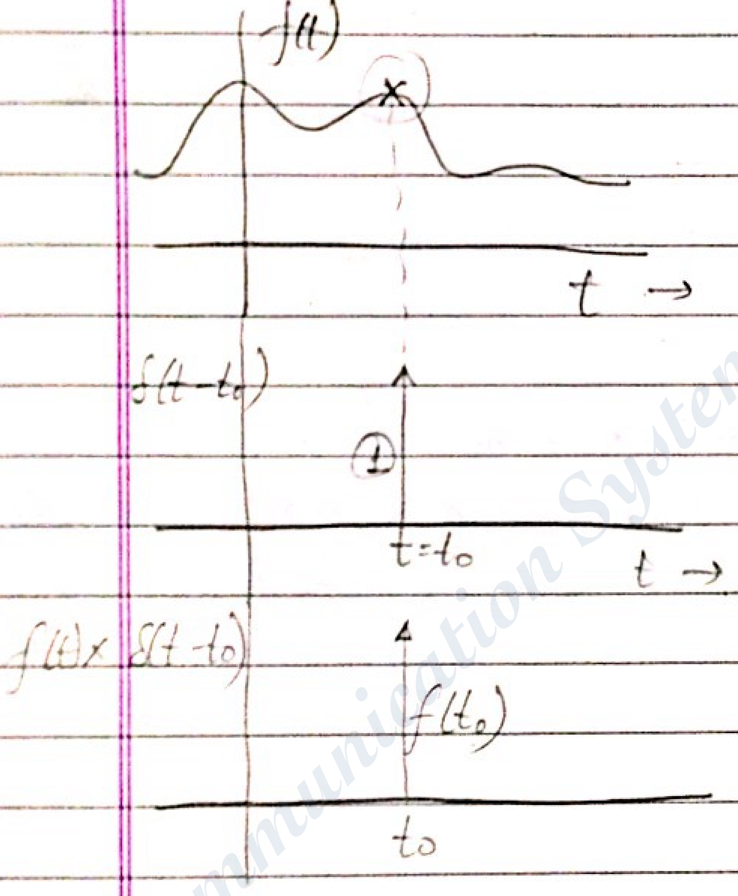




Introduction

* **Sampling property of ~~...~~ impulse f^n.**

$$\int_{-\infty}^{\infty} f(t)\, \delta(t-t_0)\, dt = f(t)\Big|_{t=t_0} = f(t_0)$$



P H Y S I C A L L Y — A f^n multiplied by impulse f^n & integrated equals the value of the f^n at that value of impulse

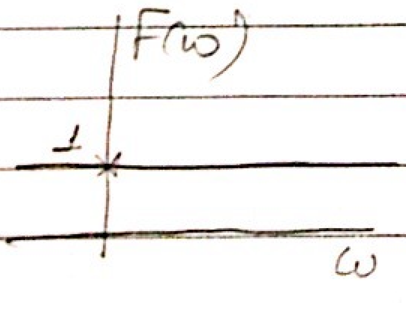
Q. What is Fourier transform of unit impulse

$f(t) = \delta(t)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$= 1$$



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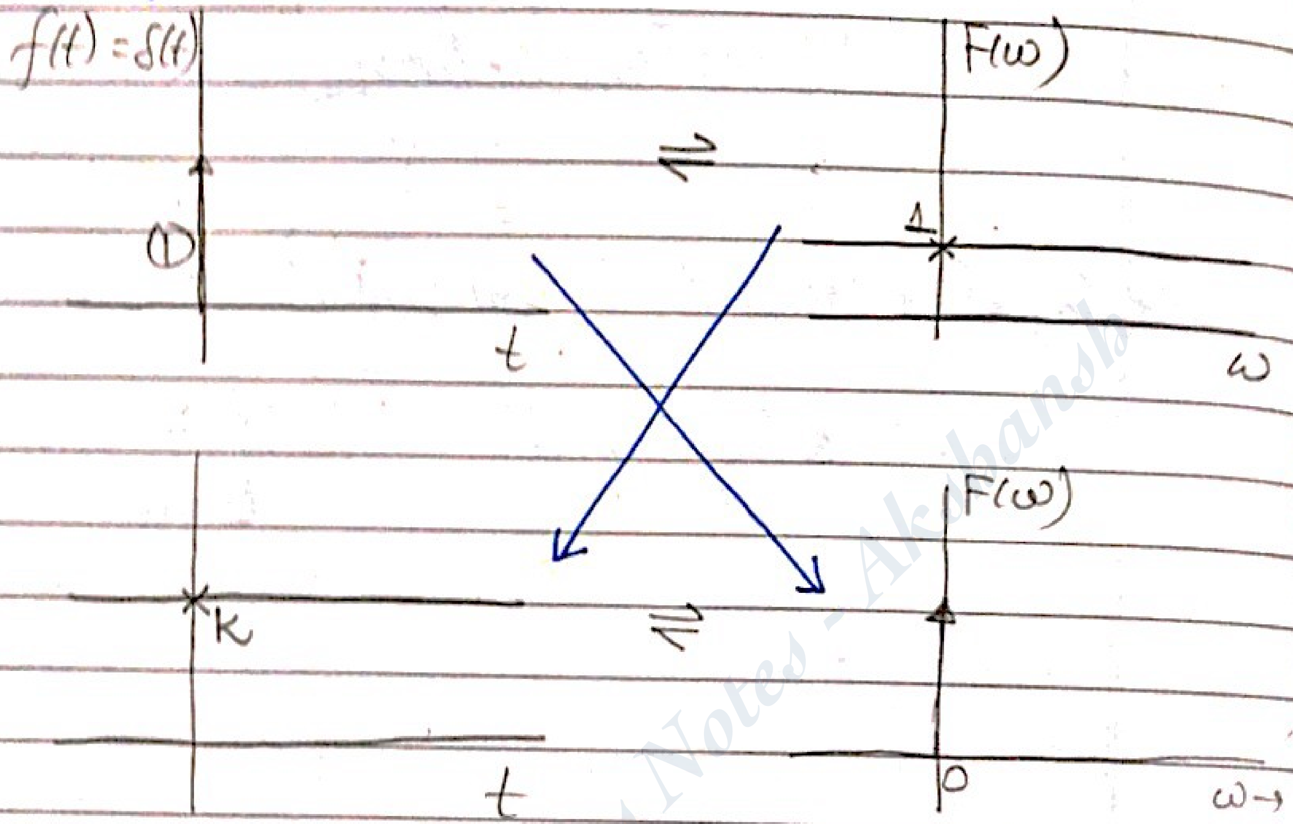
✻ Note :- Symmetry property of FT :



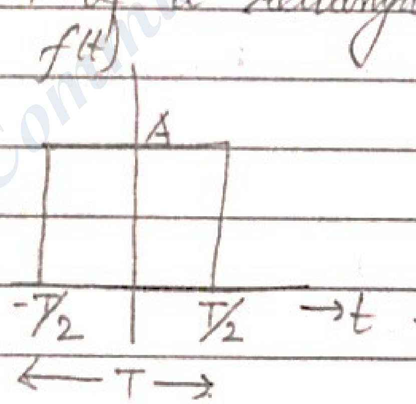
Inference :-

As the width of the f^n in time domain reduces, ~~& its~~ graph of its Fourier Transform expands.

(or) Compression in time $\Leftrightarrow$ Expansion in frequency.

Imp. inference

Q) FT of a rectangular wave :-



$$F(\omega) = \int_{\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{T/2} A\, e^{-j\omega t}\, dt$$

$$= A \left(\frac{e^{-j\omega t}}{-j\omega} \right)_{-T/2}^{T/2}$$

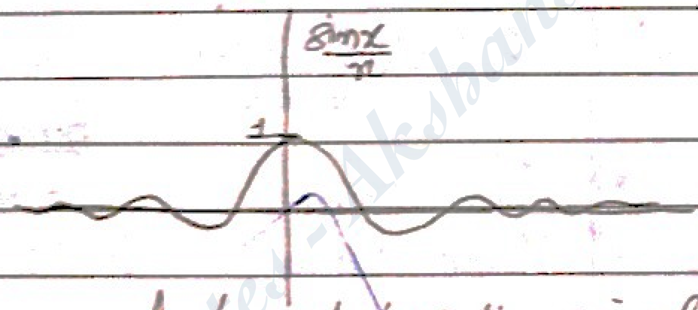
* $\dfrac{e^{j\omega} - e^{-j\omega}}{2j} = \sin\omega$

$$= -\frac{A}{j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

$$= \frac{-A}{j\omega}\left[-2j\sin\left(\frac{\omega T}{2}\right)\right]$$

$$= \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right) \longrightarrow (1)$$

Consider f^n $\dfrac{\sin x}{x}$



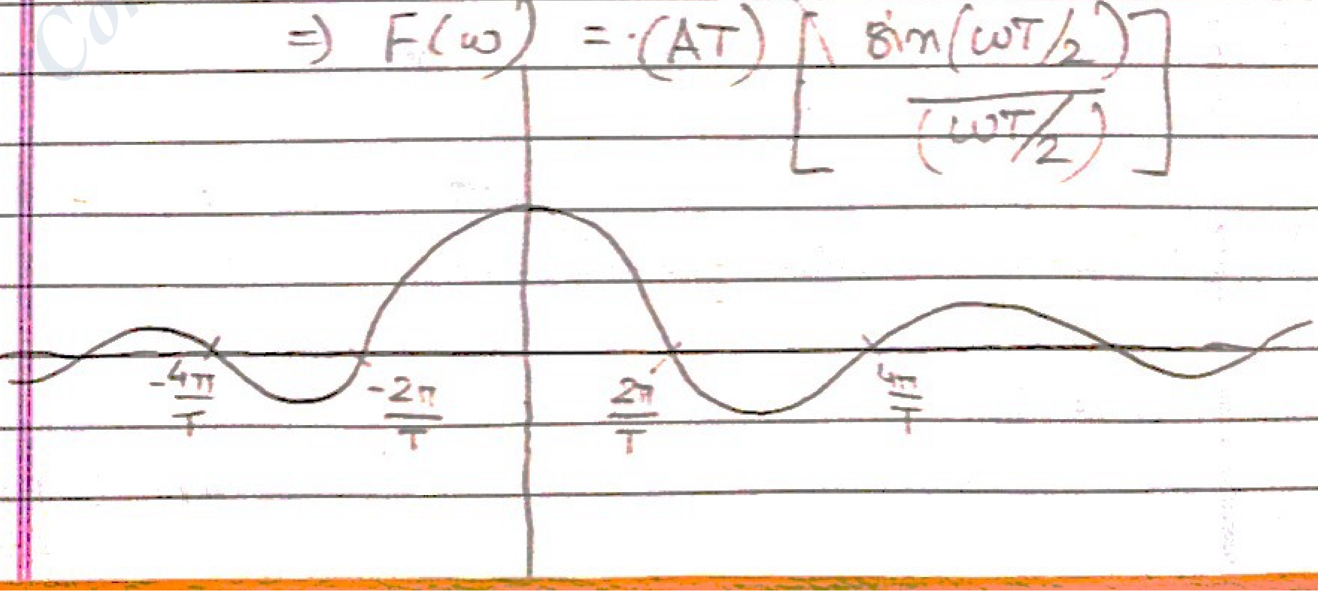
- A damped periodic signal
* Called as Sampling f^n or Sinc f^n.
* $\dfrac{\sin x}{x} = 0$ at $x = n\pi$; $n \in \mathbb{Z}$

$\omega T = 2\pi$

$\Rightarrow \omega = \dfrac{2\pi}{T}$

$$= \frac{T \times}{T \times}\ \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right)$$

$$\Rightarrow F(\omega) = (AT)\left[\frac{\sin(\omega T/2)}{(\omega T/2)}\right]$$



SPL : Sound Pressure level = $\underset{20}{\overset{10}{\text{or}}}\log\left(\frac{P_2}{P_1}\right)$
↳ Amplitude measured in dB

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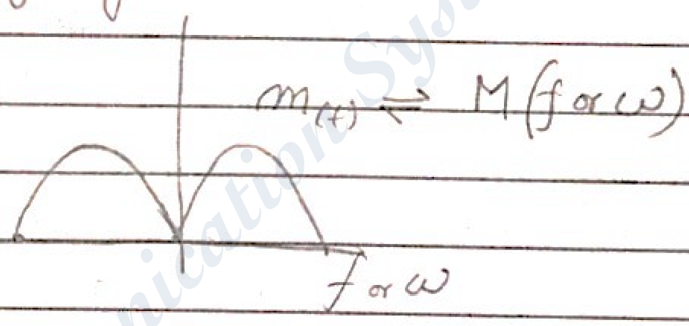
* All practical signals are Time limited & ESSENTIALLY Band limited

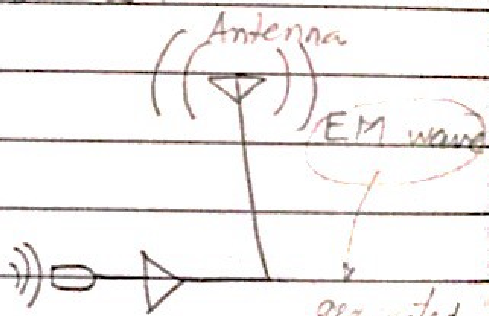
Actually, Time limited signals can never be band limited. But, in practical signals, the band is negligible after sometime. So, it is taken like, Practically, time limited signals are Band limited (limited within range)

✯ MODULATION

	Bandwidth
• Voice signal	~ 5 kHz
• Video Signal	~ 5 MHz
• Music	~ 15 kHz

✯ Any signal will now be made as :-





generated with time varying current

* for effective communicⁿ b/w transmitter & receiver, the height of antenna should be comparable to wavelength of signal wave strength.

Q ✯ Finding height of antenna.

Say, freq = 3 kHz.

$c = f\lambda \Rightarrow \lambda = \frac{3\times10^8 \text{ m/s}}{3\times10^3 \text{ /s}} \Rightarrow 100 \text{ km}$

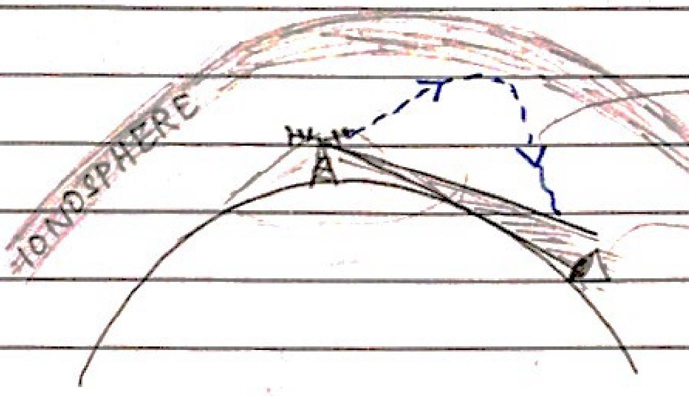
(very large)

Line of sight signal

FM Band width :- 88 MHz – 108 MHz

AM Band :- 3 MHz – 30 MHz

Size of antenna is huge. So, increase in freq. is reqd to reduce λ. For that freq. translation is used in medium.



→ AM works on IONOSPHERE reflection

→ FM stations work on line of sight

① * Message signal ; m(t) (or modulating signal) (or unmodulated signal)

② * Carrier wave :- ≡ very high freq. cosine wave ; C(t)
↳ (generated through oscillator)

③ * Modulated signal (or transmitted signal) ; s(t)

① + ② = ③

i.e $m(t) + c(t) = s(t)$; $c(t) = A_c \cos[\omega_c t + \phi]$

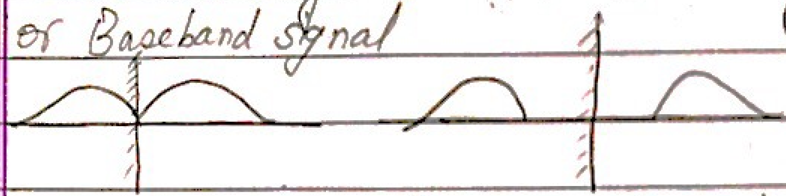
$f_c = \frac{\omega_c}{2\pi}$

→ This is the FM freq. in radios

* General observation :-
Any signal, in general is symmetric about y-axis

* Way to look at some type of signals:-

• low PASS Signals • Band Pass Signals
or Baseband signal

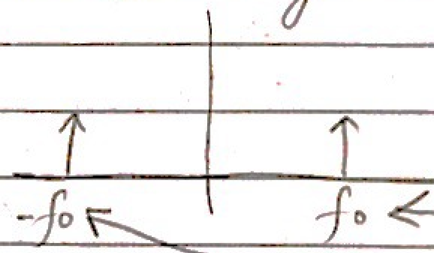


} Always Symmetrical

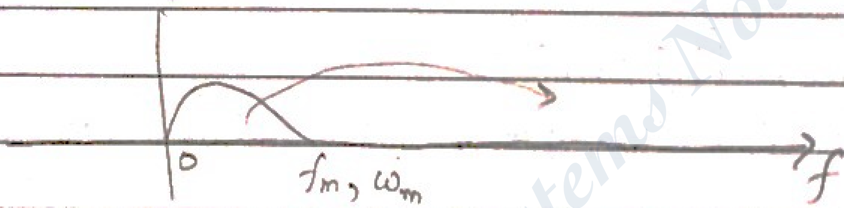
* Consider a signal :- $A = A_c \cos \omega_0 t$

$$= \frac{A_c}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

f_0



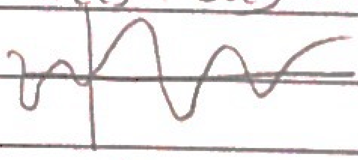
Q.*



Consider a signal with max freq. of f_m. We want to shift this signal to some other value

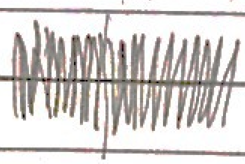
TRY (1): $m(t) + c(t) = s(t)$, if it works.

Practically, $m(t)$



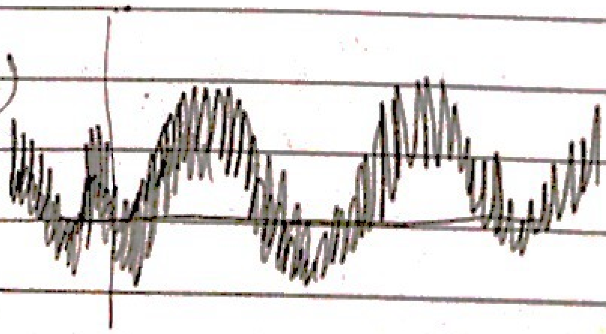
+

$c(t)$



=

$s(t) = m(t) + c(t)$

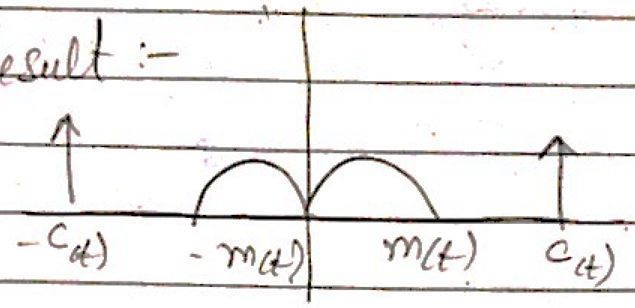


- Fourier transform satisfies superposition principle :-

$m(t) \rightleftharpoons M(\omega)$

$c(t) \rightleftharpoons C(\omega)$ $\Rightarrow$ $m(t) + c(t) \rightleftharpoons M(\omega) + C(\omega)$

Result :-



∴ By simple add^n, the waves didn't mix. We wanted sth like :-

DIDNT WORK

- FREQUENCY SHIFT PROPERTY of Fourier Transform :-

If $m(t) \rightleftharpoons M(\omega)$

then,

$$m(t)\, e^{j\omega_0 t} \rightleftharpoons M(\omega - \omega_0)$$

Illy, $m(t)\, e^{-j\omega_0 t} \rightleftharpoons M(\omega + \omega_0)$

TRY ② :- $\underline{m(t) \times c(t)} = s(t)$

$$s(t) = m(t) \times c(t)$$

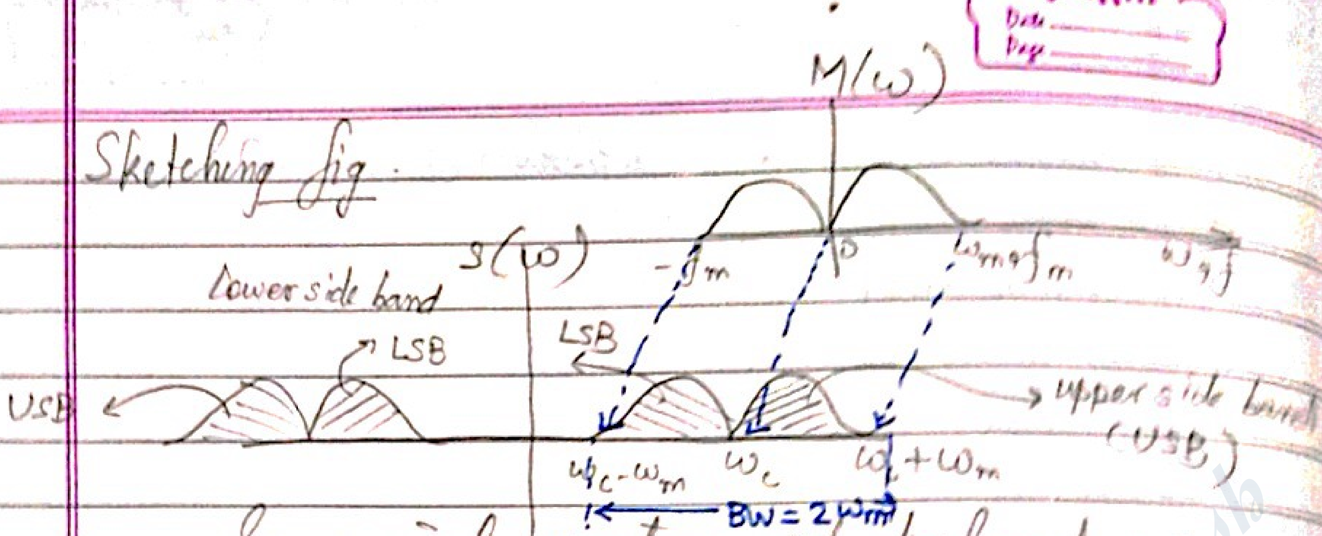
$$= m(t) \times A_c \cos\omega_c t \rightarrow \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$$

$$= \frac{A_c}{2} m(t)\, e^{j\omega_c t} + \frac{A_c}{2} m(t)\, e^{-j\omega_c t}$$

By FT :

$$S(\omega) = \frac{A_c}{2}\Big[\underbrace{M(\omega - \omega_c)}_{\text{Spectrum centered at } \omega = \omega_c} + \underbrace{M(\omega + \omega_c)}_{\text{Spectrum centered at } -\omega_c}\Big]$$

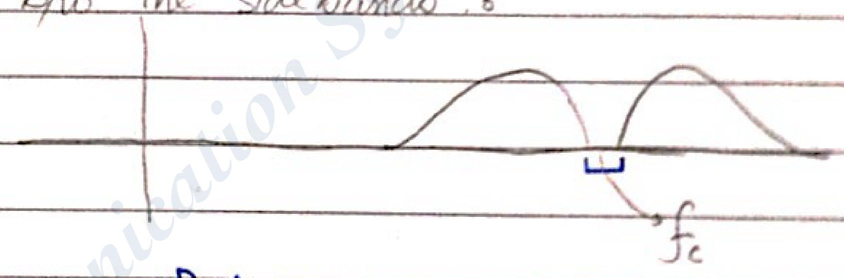
Sketching fig.



So, signal spectrum got displaced, as desired (by multiplying)

- **Double-sideband suppressed carrier (DSB-SC) Modulation** : The process of modifying the amplitude & shifting a signal to make a signal with 2 side bands.

for audio carriers :- ∃ an f_c (compressed freq.) b/w the sidebands :

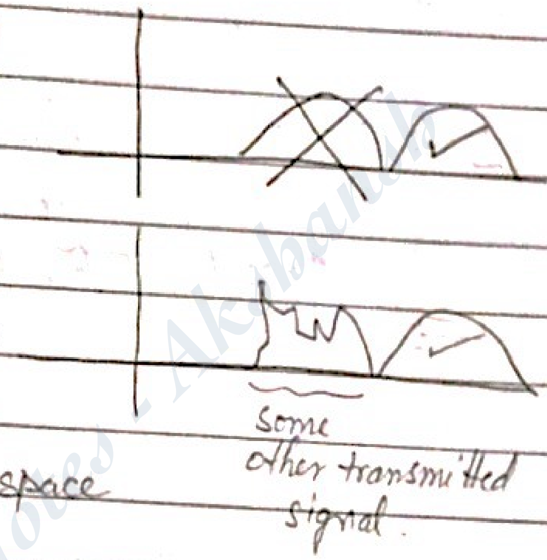


MODULATION

- ANALOG
  - Amplitude modulⁿ
    - Double sideband suppressed carrier (DSB-SC) modulⁿ
    - Conventional AM (DSB) — full carrier
    - Single side band suppressed carrier (SSB-SC)
    - Vestigial sideband modulⁿ (VSB) — in picture transmⁿ
  - Angle Modulⁿ
    - Frequency modulⁿ
    - Phase modulⁿ
- DIGITAL

* In DSB-SC : ∃ 2 sidebands. So, BW is wide ($= 2W_m$)
By filtering, one of the sidebands (LSB or USB) can be removed so that BW is reduced.
Then, some other signal can be used in removed region

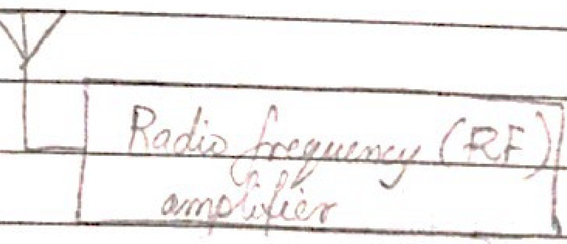
So, space is saved in BW. One station doesn't interfere with other

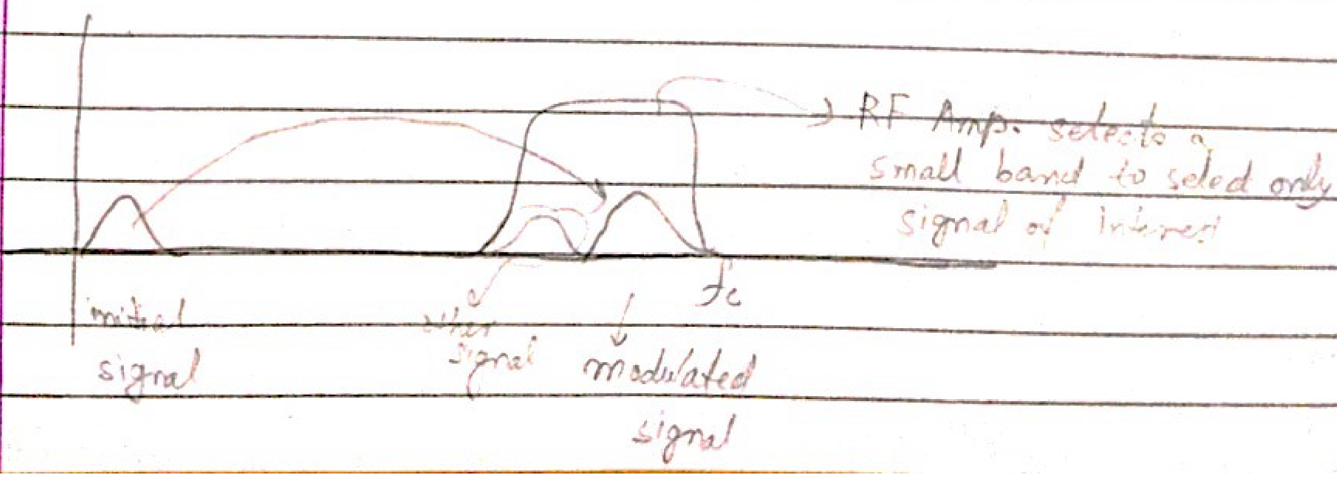


* Many signals share same space

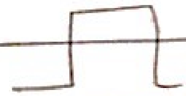
* 88 MH_z – 108 MH_z is the BW of FM stations ($= 20\ MH_z$)
The allowance for each station = 0.2 MH_z.
So, $\frac{20}{0.2} = 100$ stations can be accomodated.

* All receivers follow the principle of SUPERHETERODYNE receiver



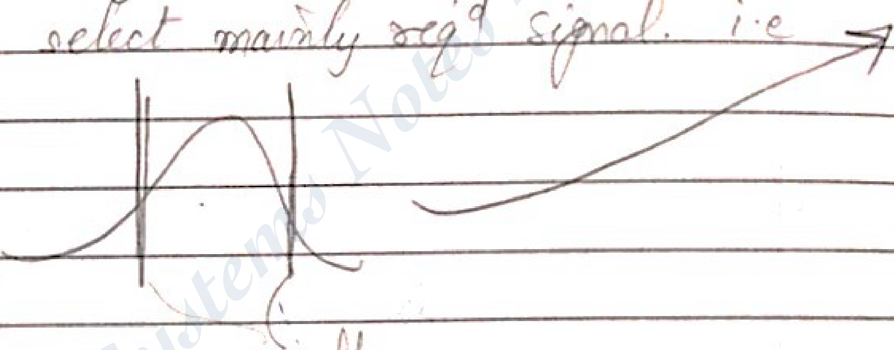


RF Amp. should have a large band so that any station can be selected.

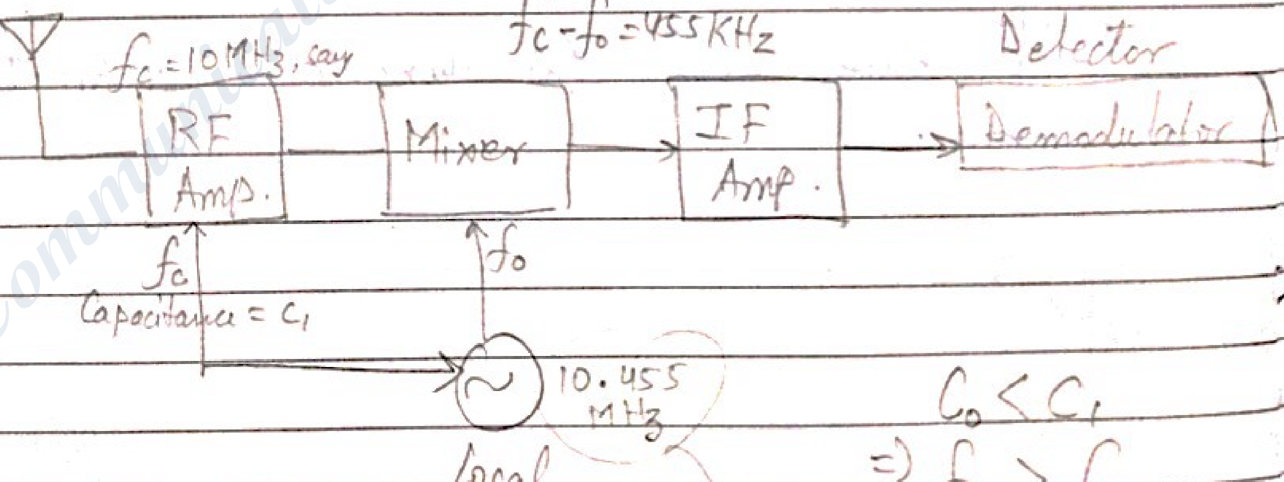
Practically,  not possible

So,  is the way how RF amp. selects the station. So, due to this, neighbouring signals also come.

For removing neighbours, we have **IF AMPLIFIER** (intermediate freq. amp.) (: 455 KHz & BW = 10 KHz). So, this creates a wall to amplify & select mainly req'd signal. i.e



Now, we have.

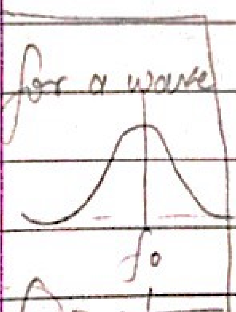


$C_o < C_1$

$\Rightarrow f_{C_o} > f_{C_1}$

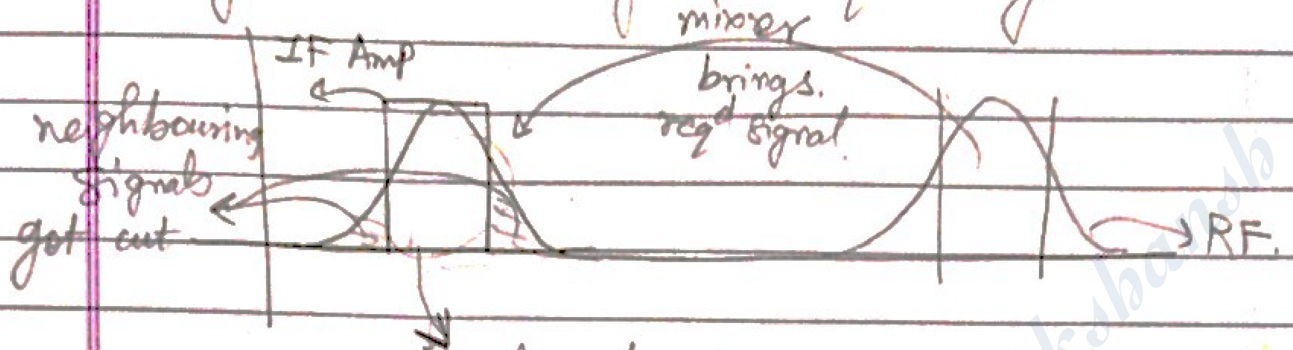
$\Rightarrow f_c > f_o$

& $f_c - f_o = 455\,KHz$



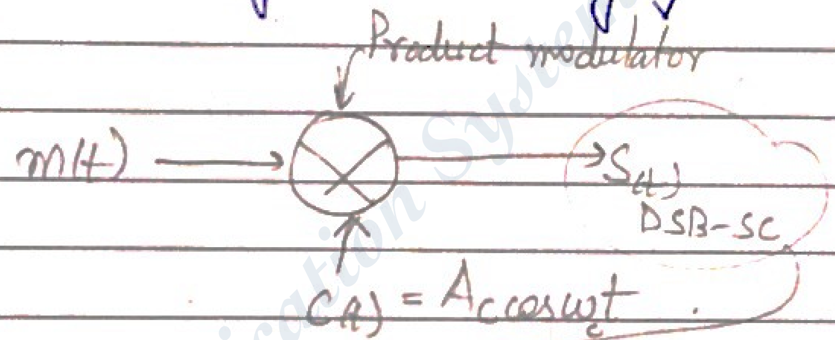
Center freq. ← $f_o = \frac{1}{2\pi\sqrt{LC}}$

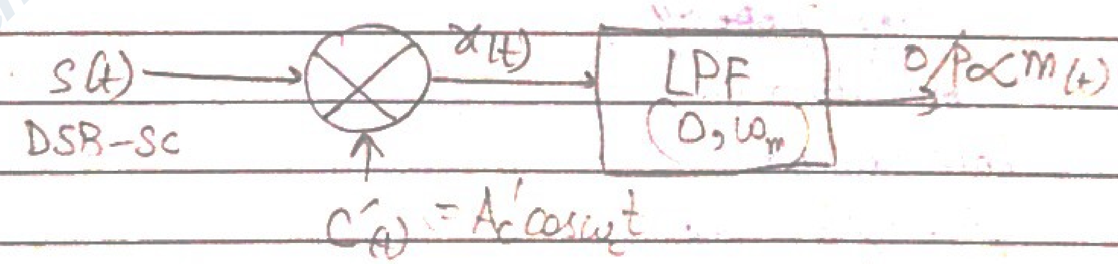
Mixer will take λ of interest $(f_c - f_0)$ & bring it to IF Amp. IF Amp. removes neighbours & signal req^d is got



This is principle of Superheterodyne receiver

✩ DSB-SC signal — Bringing back message signal





$$x(t) = s_{DSB-SC}(t) \times \underbrace{c'(t)}_{\text{locally generated carrier}} = [m(t) \cdot A_c \cos\omega_c t] A_c' \cos\omega_c t$$

* Convolution operⁿ, $*$ = $\int$: in analog & = $\sum$: in discrete

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

$$\Rightarrow x_{(t)} = A_c A_c' \, m_{(t)} \cos^2 \omega_c t$$

$$= (A_c A_c') \, m(t) \left[\frac{1}{2}(1+\cos 2\omega_c t) \right]$$

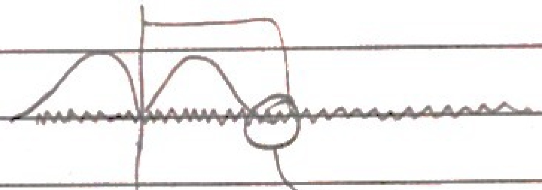
$$= \underbrace{\left(\frac{A_c A_c'}{2}\right) m_{(t)}}_{\text{Desired term}} + \underbrace{\left[\frac{A_c A_c' \cos 2\omega_c t}{2}\right] m_{(t)}}_{\text{unwanted component}}$$

Desired term. (except m(t), no other fⁿ of time (lower freq))

unwanted component (has very high f)

* higher freq (unwanted component) can be removed by Low Pass filter (LPF)

Why need to filter :- Natural noise exists in signal by default. So, actually, we have



extra part that comes into ear if the signal is heard from headphone. This noise can be removed by LPF

Conversion from time to ω domain done by taking FT

V. Imp. property
Convolution in one domain = Multiplicⁿ in other domain.

time domain

freq. domain

Suppose I want to do
$y(t) = x(t) * h(t)$
↳ convert to ω domain
$Y(\omega) = X(\omega) \cdot H(\omega)$
↳ convert back to time
$y(t) = (\quad)$

* **Unit impulse response, $h(t)$** :- If i/p = $\delta(t)$, o/p = $h(t)$
↳ It characterises a sys. Diff. b/w sys. can be found.

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \qquad y(t) = x_{(t)} * h(t)$$

convolution sign

* Spectrum = Amp. vs. freq. graph

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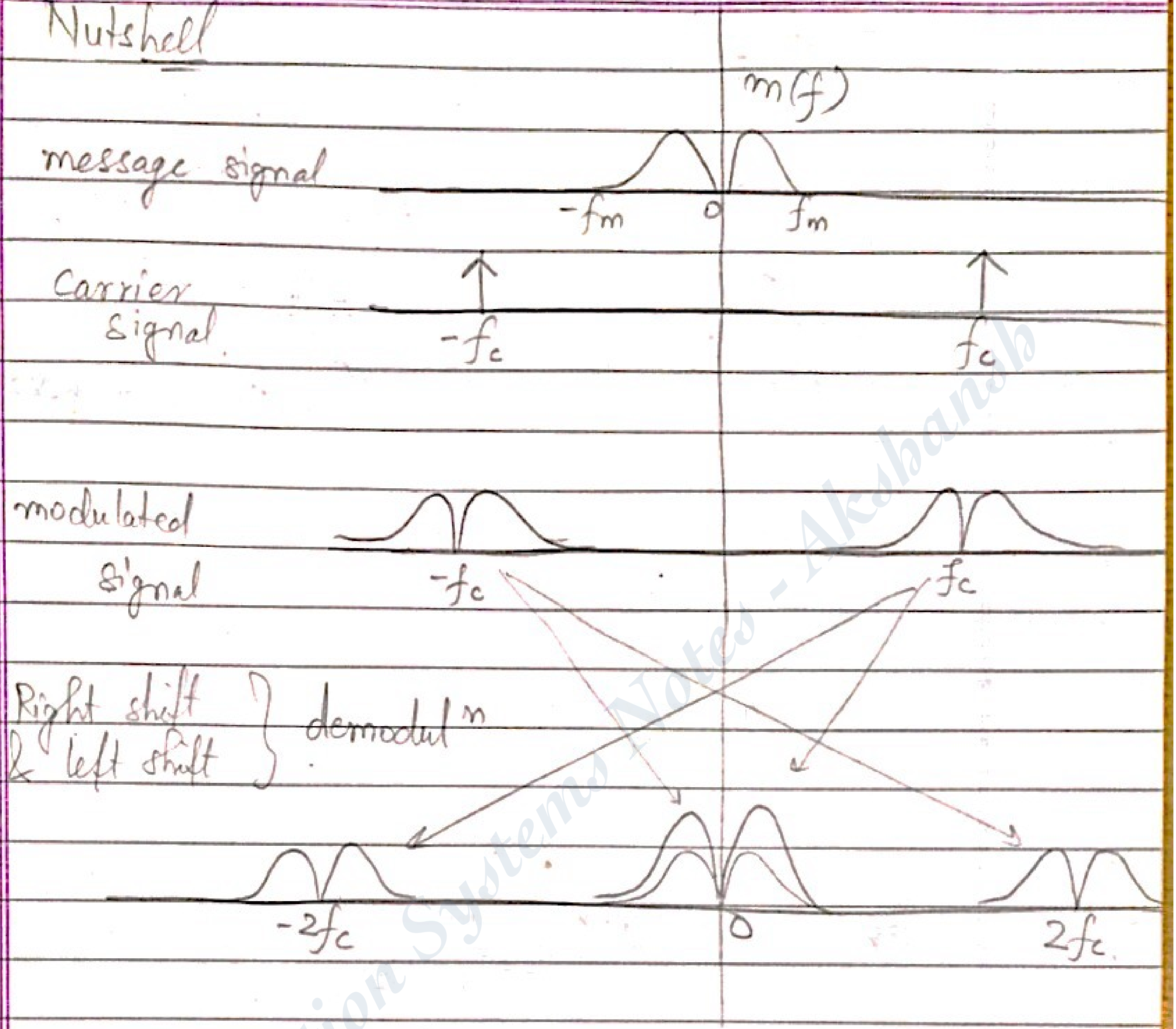
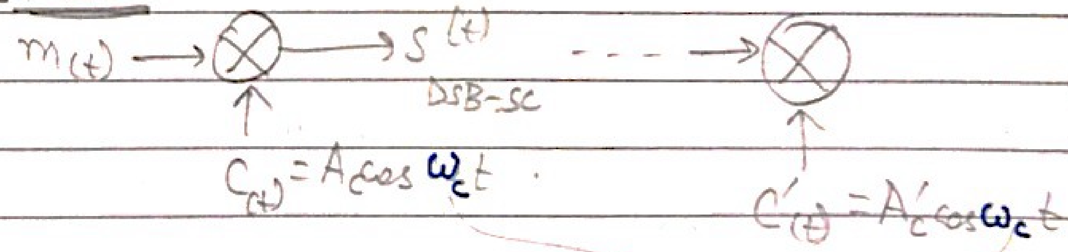
Nutshell



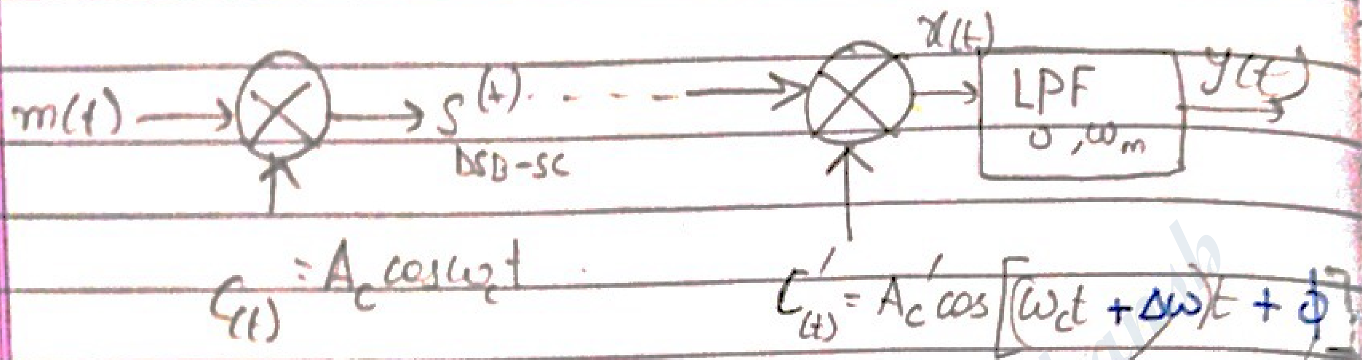
☆ EFFECTS OF FREQ. & PHASE ERRORS IN DSB-SC MODULATION.

IDEAL



same freq. & phase should be there reqd (difficult to achieve actually

ACTUAL



Will proper demodul^n be successful?

We have

$$x_{(t)} = A_c\, m_{(t)} \cos\omega_c t \times A_c' \cos\left[(\omega_c + \Delta\omega)t + \phi\right]$$

$$\left[\begin{array}{l}\text{We know} \\ 2\cos A \cos B = \cos(A+B) + \cos(A-B)\end{array}\right]$$

$$\Rightarrow x_{(t)} = \frac{A_c A_c'}{2} m_{(t)} \left\{ \cos\left[(2\omega_c + \Delta\omega)t + \phi\right] + \cos(-\Delta\omega t - \phi) \right\}$$

$$= \frac{A_c A_c'}{2} m_{(t)} \left\{ \cos\left[(2\omega_c + \Delta\omega)t + \phi\right] + \cos(\Delta\omega t + \phi) \right\}$$

no significance ∵ of its high freq.
It'll be removed by LPF.

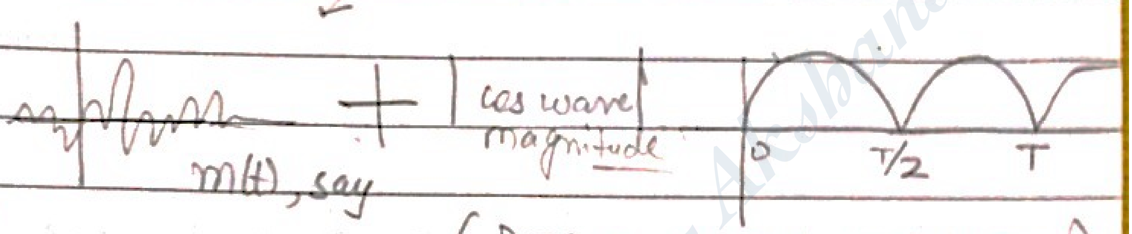
$$\Rightarrow x(t) \propto m_{(t)} \cos\left[(\Delta\omega)t + \phi\right]$$

Analysing the problems in the use of this signal.

Principle of Superposition

↳ Case a) Freq. error only $\Rightarrow \phi = 0$

So, o/p $\propto$ $m(t)\cos[(\Delta\omega)t]$



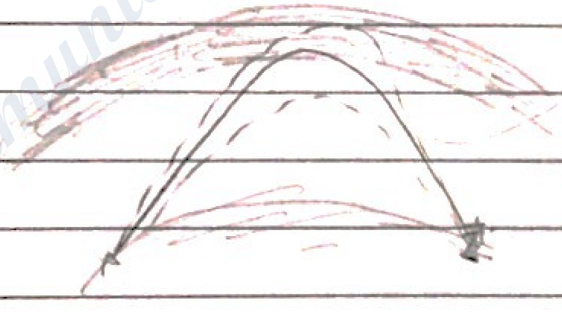
(BEAT PHENOMENON)

Case b) Phase error only, $\Delta\omega = 0$

So, o/p $\propto$ $m(t)\cos(\phi)$

→ not constt. Its a f^n of t.

Consider ionosphere transmission of signal. That varies acc. to time of day



→ as time of day varies, distance of transmission varies

ϕ depends on time & time varies with distance. So, sometimes signal received, sometimes not. $\Rightarrow$ Not reliable.

Hence, $\exists$ need of Conventional AM

(PTO)

✡ CONVENTIONAL Amplitude Modul^n

(or Standard AM
or Normal AM
or DSB - Full Carrier
or DSB - Large Carrier
or DSB - Total Carrier)

General expression :-

$$s_{AM}(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

$A_c[1 + k_a m(t)] \rightarrow A(t)$ = Envelope

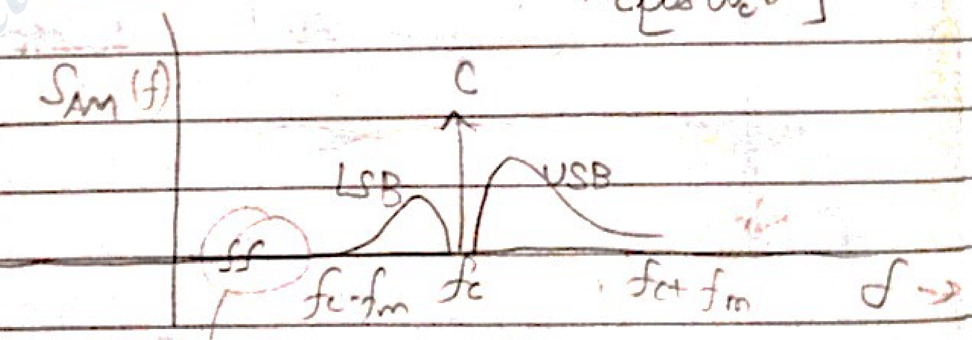
↳ where $|k_a m(t)| \leq 1$ } to ensure that over modul^n doesn't take place.

↳ k_a : constt

$$\equiv s_{AM}(t) = \underbrace{DSB-SC}_{LSB+USB} + \text{Carrier}$$

Carrier $\rightarrow A_c[\cos \omega_c t]$

So, now it -

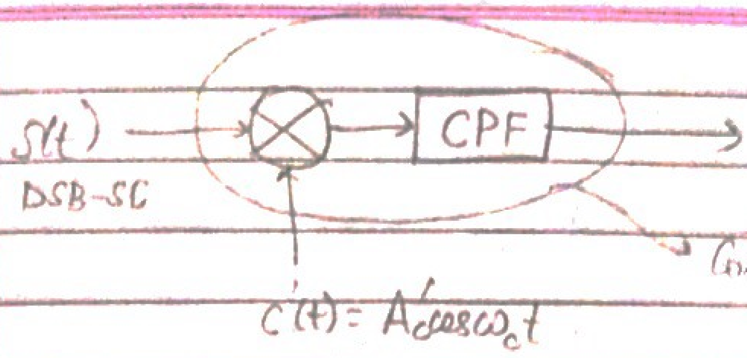


SS → signifies that its far away from origin

* When $\phi = \frac{\pi}{2}$, o/p = 0. ∴ "QUADRATURE-NULL EFFECT"
(QUADRATURE $\equiv \frac{\pi}{2}$, NULL $\equiv 0$)

* Crystal oscillator advantage : high ↑ endurance

Consider  → Envelop of signal



→ Coherent / Synchronous detector

[not req^d in conventional AM, but can be used]

Consider: $\equiv$ freq.

Single-tone modul^n

$$m_{(t)} = A_m \cos\omega_m t \; ; \; \omega_c >>> \omega_m$$

So, $$S_{AM}(t) = A_c\left[1 + k_a A_m \cos\omega_m t\right]\cos\omega_c t$$

→ Defining $\boxed{\mu \triangleq k_a A_m}$ Modul^n index or Depth of modul^n

→ extent to which c(t) has been influenced by m(t)

$$\Rightarrow S_{AM}(t) = \underbrace{A_c\left[1 + \mu\cos\omega_m t\right]}_{A(t)}\cos\omega_c t$$

★ Note :-

$$\boxed{\mu \triangleq \frac{A(t)\big|_{max} - A(t)\big|_{min}}{A(t)\big|_{max} + A(t)\big|_{min}}}$$

→ $A(t)\big|_{max} = A_c\left[1 + \mu\right]$

→ $A(t)\big|_{min} = A_c\left[1 - \mu\right]$

Plotting waveforms

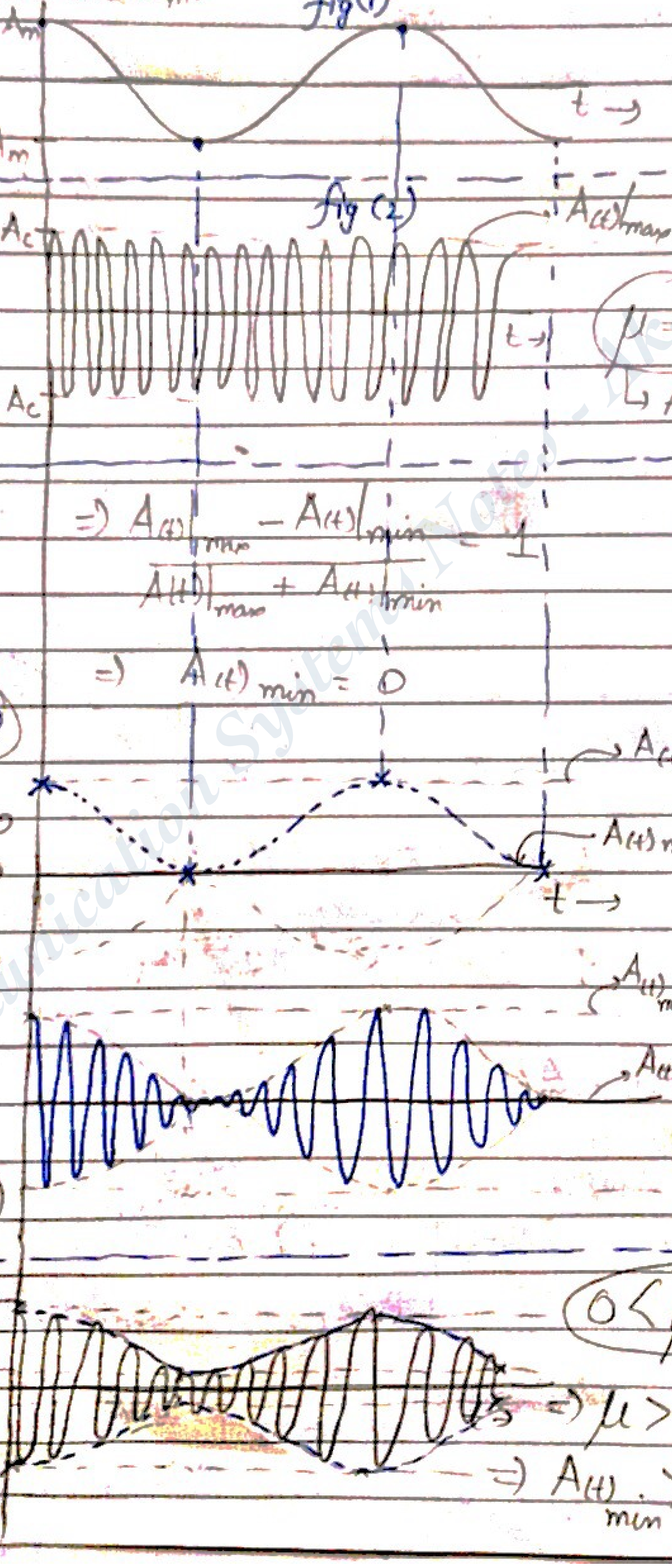
$m(t) = A_m \cos \omega_m t$

fig(1)

$C(t) = A_c \cos \omega_c t$

$\boxed{= S_{AM}(t)\Big|_{\mu=0}}$

fig(2)



$A(t)|_{max}$ & $A(t)|_{min}$

$\mu = 0$

$\hookrightarrow A(t)_{max} = A(t)_{min}$

$\Rightarrow \dfrac{A(t)|_{max} - A(t)|_{min}}{A(t)|_{max} + A(t)|_{min}} = 1$

$\mu = 1$

$\Rightarrow 100\%$ modulⁿ

wrt fig(1)

fig(3)

$\Rightarrow A(t)_{min} = 0$

only envelope

$A(t)_{max}$

$A(t)_{min}$

envelope × carrier $= fig(1) \times fig(2)$

$A(t)_{max}$

$A(t)_{min}$

Envelope changes wrt message signal.

$A(t)_{max}$

$A(t)_{min}$.

$0 < \mu < 1$

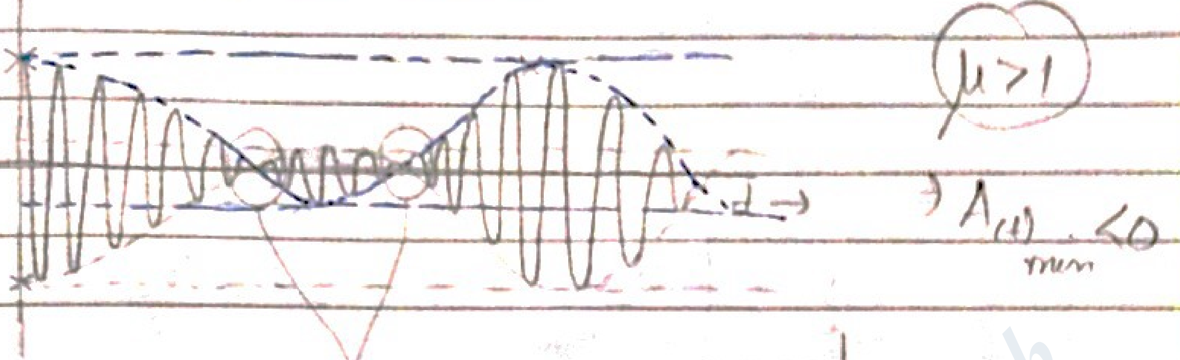
$\Rightarrow \mu > 0$ & $\mu < 1$

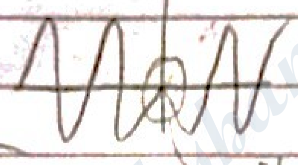
$\Rightarrow A(t)_{min} > 0$ & $A(t)_{max} > A(t)_{min}$

(from formula)

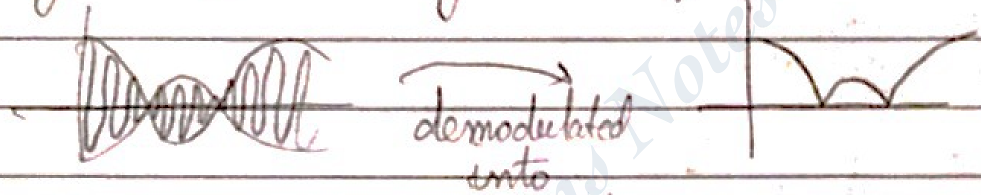
* We are able to identify the modulated carrier (the info. carrying signal) as long as

$$0 < \mu \text{ or } m < 1$$ → modul^n index



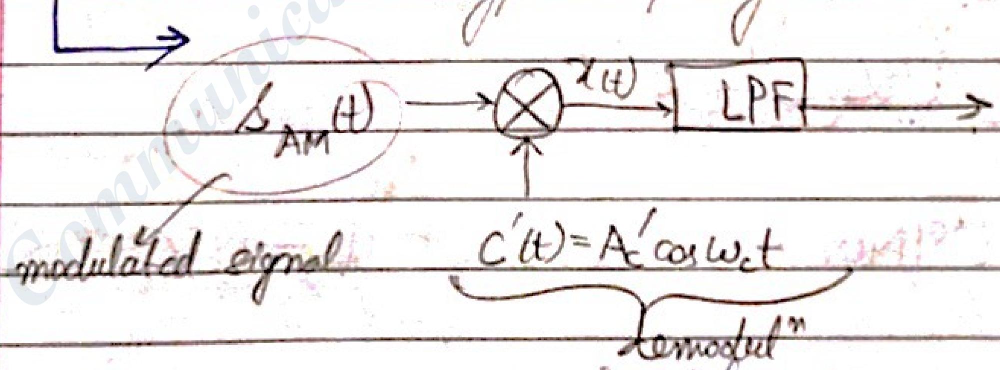


* **Envelope detector** : Detects only +ve wave
eg :- for a modulated signal with ($\mu > 1$)



* The −ve part shows the phase change. That is detected by **Phase detector**

* **Coherent detector** :- A high cost receivers, which can detect all types of signals



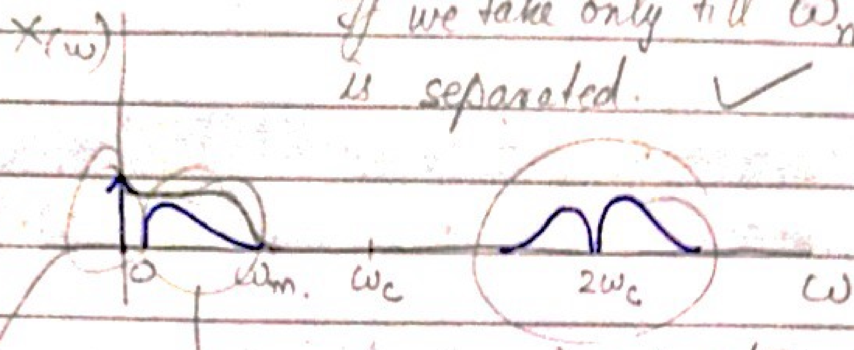
Mathematically

$$x(t) = \underbrace{A_c[1 + k_a m(t)] \cos \omega_c t}_{s_{AM}(t)} \times \underbrace{A_c' \cos \omega_c t}_{c'(t)}$$

$$= \frac{A_c A_c'}{2}[1 + k_a m(t)] + \frac{A_c A_c'}{2}[1 + k_a m(t)] \cos 2\omega_c t$$

Graphically

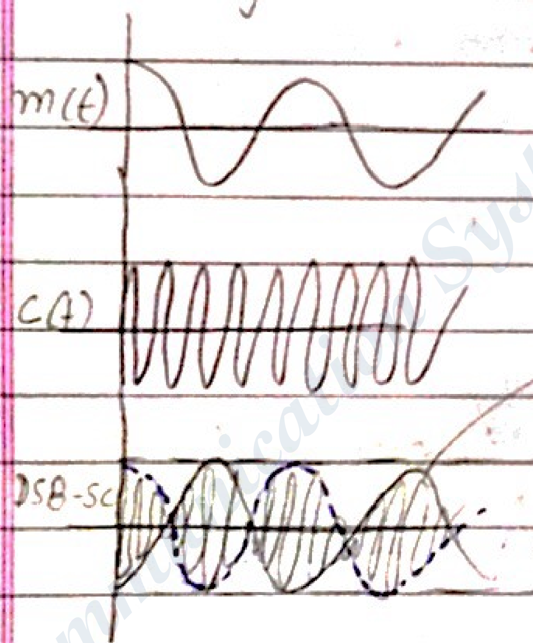
If we take only till ω_m, $m(t)$ is separated ✓



$\frac{A_c A_c'}{2}$ $\frac{A_c A_c'}{2} k_a m(t)$ $\frac{A_c A_c'}{2}[1+k_a m(t)]\cos 2\omega_c t$

—o—

☆ Generating DSB-SC



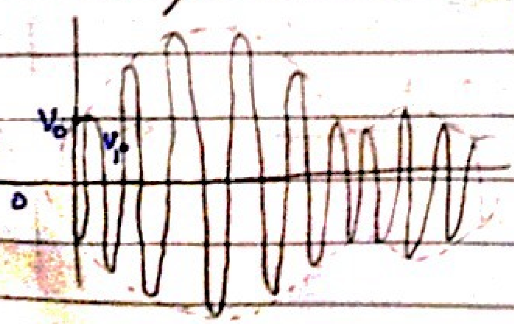
coherent detector works (but costly)

μ or $m \rightarrow \infty$

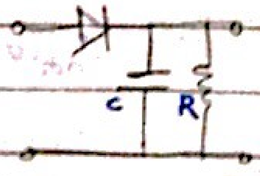
$\rightarrow A_{t_{min}} = -A_{t_{max}}$

☆ USING AN ENVELOP DETECTOR

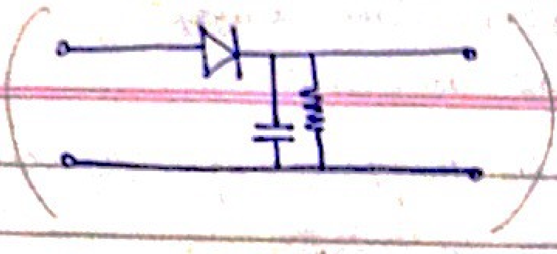
eg Consider a general modulated signal $0<\mu<1$ that is given to envelope detector



→



☆ Envelope detector: R, C, Diode



At $t=0$ → initially in my signal

i/p V is ↑ & C is initially not charged

So, forward biasing happens in diode (nearly ideal). So, at

$$V \simeq V_0$$

V across R is V_0.

Now,

When V ↓ (as in signal), V at R (V_0) wont ↓ immediately, So,

$$V \text{ (at i/p)} < V \text{(across R)}$$

So, diode is in RB → OC.

Now,

as V is ↓ further (V),

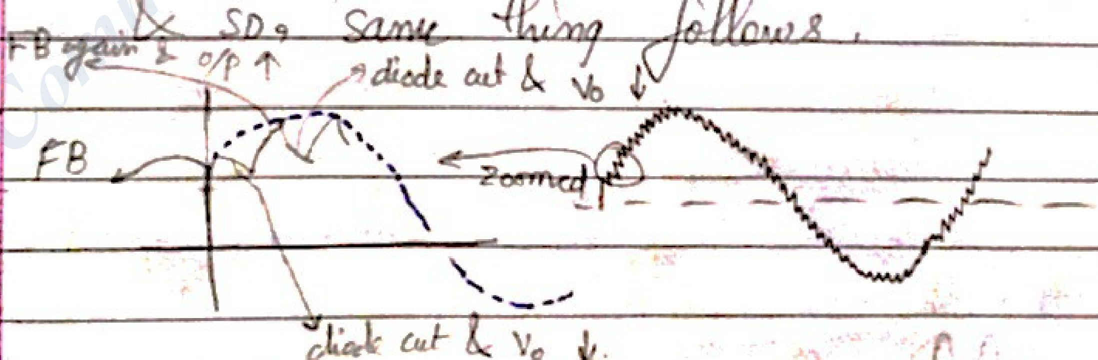
o/p is slightly ↓.

Now, when i/p V starts ↑ (at V_1) by that time o/p has decreased & i/p V has now increased. So, diode comes back (FB) & so, same thing follows.

Note (concept):

i/p V > o/p V
high V   low V
+ ⊳| −
FB

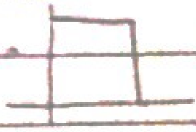
i/p V < o/p V
low V   high V
− ⊳| +
RB

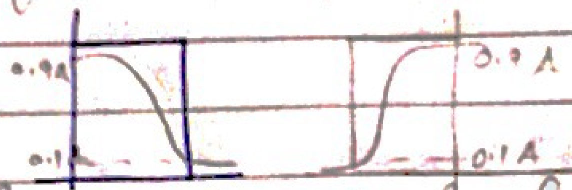


* So, with envelope detector, we are nearly getting our m(t).

* Understanding concept of Time Constant etc

* Charging / discharging of capacitor

Consider RC network & i/p  applied to it. it takes 5τ (time constt - τ) to discharge from 90% of final value to 10%.

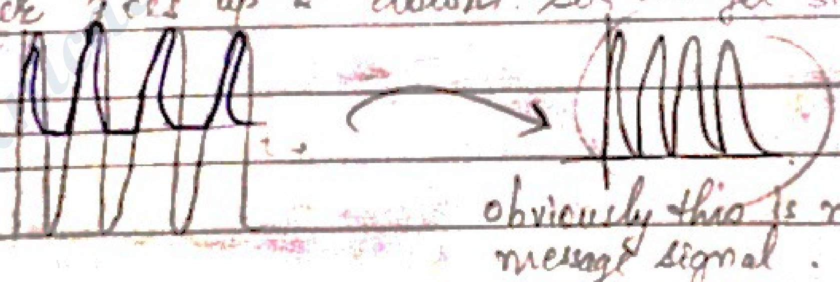


Illy, in charging, RC network charges from 10% to 90% of final value in 5τ sec

* For a carrier signal ($\nu = f_c$)

If $\tau_{RC} <<< \frac{1}{f_c}$

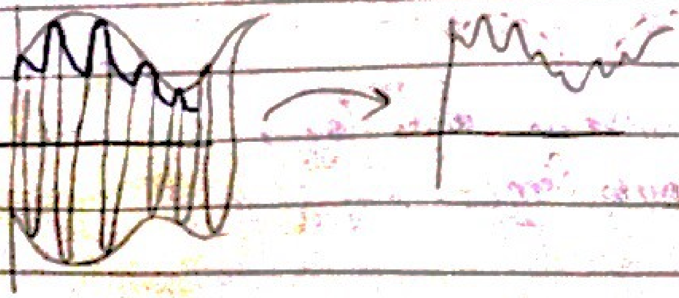
So, that means, capacitor network has enough time to discharge & charge (before carrier goes up & down) So, we get sth like →



obviously this is not our message signal.

Trying for $\tau_{RC} >>> \frac{1}{f_c}$

(i.e. RC network takes more time to charge / discharge than the carrier wave)



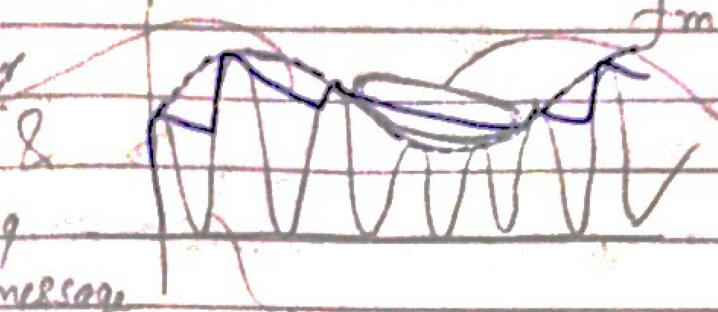
→ Seems like we nearly got m(t). But, how much τ_{RC} can we ↑ ?

Try $\tau_{RC} >> \frac{1}{f_m}$

Contd

In case of $\tau_{RC} \gg \frac{1}{f_m}$

how capacitor is charging & discharging with message



diagonal clipping happened (capacitor not able to discharge with same pace as that of msg)

* Diagonal clipping not desired.

So, ideally, we should take—

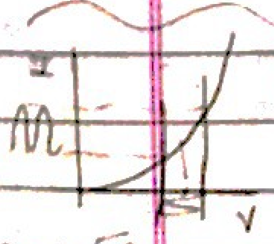
$$\boxed{\frac{1}{f_m} \gg \tau_{RC} \gg \frac{1}{f_c}}$$

↳ under this cond^n, we nearly get the same message signal using an envelop detector (cheap) as compared to coherent detector (costly)

☆ GENERATION OF AM WAVES :-

M2 SQUARE LAW MODULATOR

↳ makes use of a device which satisfies the sq. law cond^n.



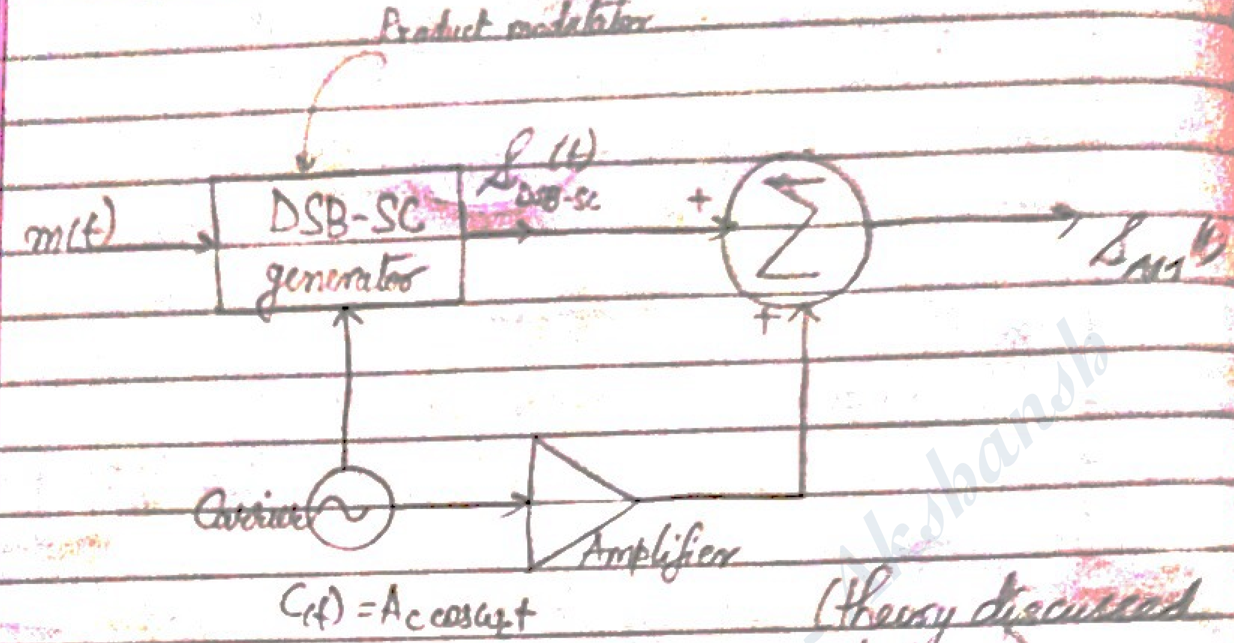
$v_i(t) \longrightarrow$ Non-linear device $\longrightarrow v_o(t) = k[v_i(t)^2]$

$v_o(t) = [k_0 + k_1 v_i(t) + k_2 v_i(t)^2]$ → General form for sq. law cond^n

$(o/p \propto (i/p)^2)$

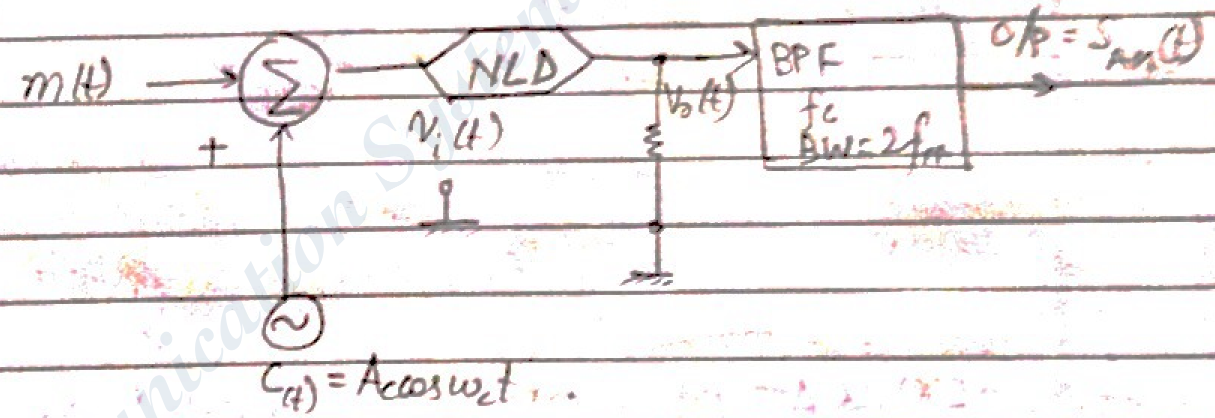
(detail later)

M1



(theory discussed before)

M2 Continued



Now,

$$V_0(t) = k_0 + k_1 [V_i(t)] + k_2 [V_i(t)]^2$$

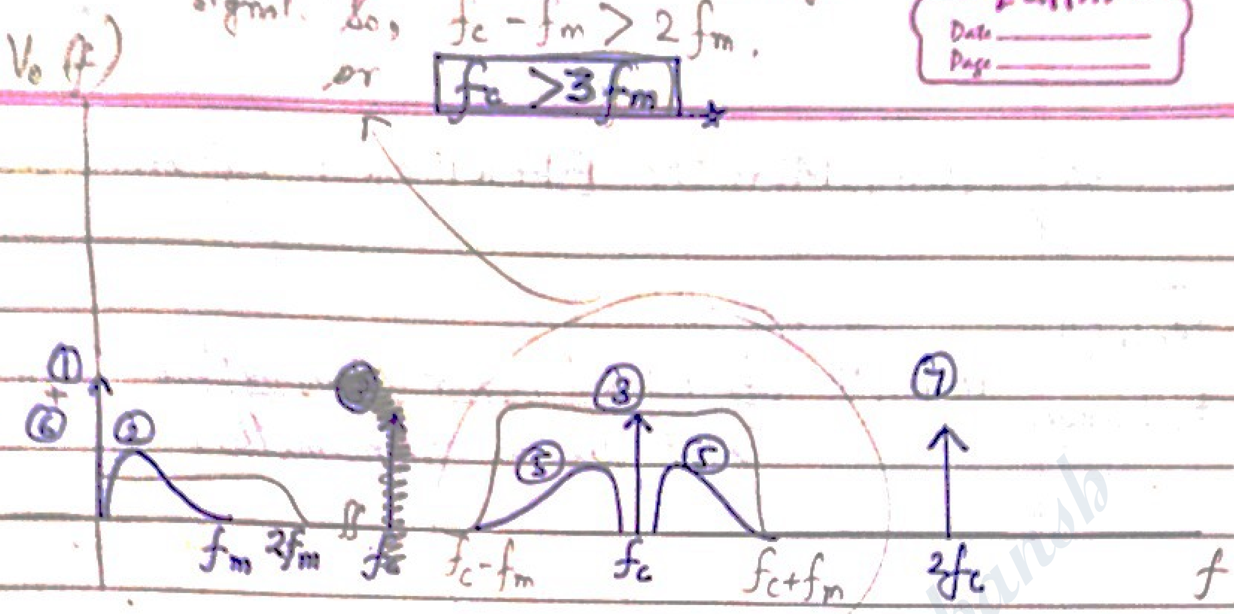
$$= k_0 + k_1 [m(t) + A_c \cos \omega_c t] + k_2 [m(t) + A_c \cos \omega_c t]^2$$

$$V_0(t) = \underbrace{k_0}_{1} + \underbrace{k_1 m(t)}_{2} + \underbrace{A_c k_1 \cos \omega_c t}_{3} + \underbrace{k_2 m^2(t)}_{4} + \underbrace{2 A_c k_2 m(t) \cos \omega_c t}_{5} + \underbrace{k_2 A_c^2 \cos^2 \omega_c t}_{6}$$

$$\rightarrow \frac{1}{2}[1 + \underbrace{\cos 2\omega_c t}_{7}]$$

Plotting this f^n in $V_0(f)$ vs f graph →

$f_c - f_m$ should not intersect message signal. So, $f_c - f_m > 2f_m$
or $\boxed{f_c > 3f_m}$



Note :-

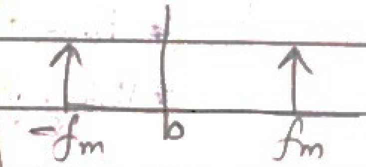
Say, $m(t) \overset{FT}{\rightleftharpoons} M(\omega)$

So, $m^2(t) = \underbrace{m(t) \cdot m(t)}_{\text{Multiplic}^n \text{ in time domain}} \rightleftharpoons \frac{1}{2\pi}\left[M(\omega) * M(\omega)\right]$

Multiplic^n in time domain $\} = \{$ Convolution in freq. domain

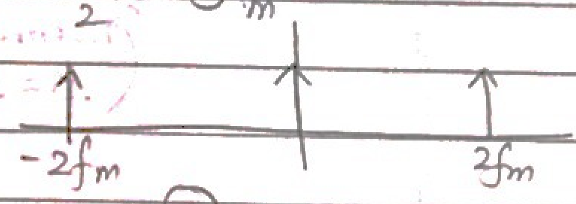
In amp Sys { ✱ When you convolve 2 signals, its width equals the sum of widths of individual signals

eg :- if $m(t) = A_m \cos \omega_m t$



& $m^2(t) = A_m^2 \cos^2 \omega_m t$

$= \frac{A_m^2}{2} + \frac{A_m^2}{2} \cos 2\omega_m t$



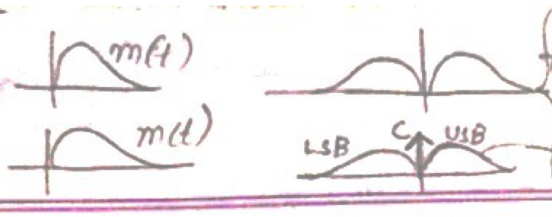
$o/p := A_c k_1 \cos \omega_c t + 2A_c k_2 m(t) \cos \omega_c t$

$= A_c k_1 \cos \omega_c t \left[1 + \frac{2k_2}{k_1} m(t)\right]$

$o/p = A_c k_1 \left(1 + \frac{2k_2}{k_1} m(t)\right) \cos \omega_c t$

Std. AM

Conventional AM



transmitted signal

due to C, 100% efficiency ✗ (but with C, demodulⁿ easy)

☆ TRANSMISSION EFFICIENCY OF AM.

(what %age of transmitted signal contains useful message)

or Modulⁿ Efficiency

Definⁿ :- $\dfrac{\text{The useful power transmitted}}{\text{Total power transmitted}} \times 100\%$

Now, we had

$$s_{AM}(t) = A_c[1 + k_a m(t)]\cos\omega_c t$$

$$= \underbrace{A_c \cos\omega_c t}_{\text{Carrier (C)}} + \underbrace{A_c k_a\, m(t) \cos\omega_c t}_{SB = LSB + USB.}$$

Power $= P_C \qquad\qquad P_{SB}$

So,

$$\eta = \frac{P_{SB}}{P_{SB} + P_c} \times 100\%$$

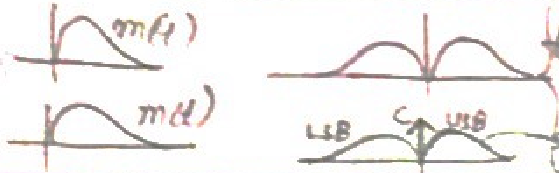
* Power = Mean square ~~val~~ value
* Voltage/Current = Root mean square value.

$$P_c = \overline{[A_c \cos\omega_c t]^2} = (A_c^2)\left(\frac{\pi}{2}\right)$$

(normalised $R = 1\,\Omega$)

Std. AM

Conventional AM



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☆ TRANSMISSION EFFICIENCY OF AM

or Modulⁿ Efficiency

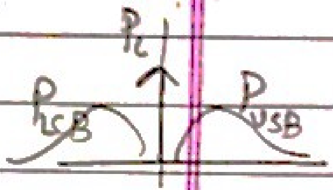
(what %age of transmitted signal contains useful message)

Definⁿ:- $\dfrac{\text{The useful power transmitted}}{\text{Total power transmitted}} \times 100\%$

Now, we had, $s_{AM}(t) = \overbrace{(m(t) A_c k_a \cos w_c t)}^{SB} + \overbrace{A_c \cos w_c t}^{C}$

$s_{AM}(t) = A_c[1 + k_a m(t)] \cos w_c t$

$= \underbrace{A_c \cos w_c t}_{\text{Carrier }(c)} + \underbrace{A_c k_a m(t) \cos w_c t}_{SB = LSB + USB}$



Power $= P_c \qquad P_{SB}$

So,

(A) $\eta = \dfrac{P_{SB}}{\underbrace{P_{SB} + P_c}_{= P_T}} \times 100\%$

* Power = Mean square ~~vol~~ value
* Voltage/Current = Root mean square value.

(1) $P_c = \overline{[A_c \cos w_c t]^2} = (A_c^2)\left(\dfrac{1}{\sqrt{2}}\right)^2$

(normalised $R = 1\,\Omega$)

mean

Mean sq. value of $\cos w_c t$.

(2) Now $P_{SB} = \overline{[A_c k_a m(t) \cos w_c t]^2} = A_c^2 k_a^2 [\overline{m^2(t)}]\left(\dfrac{1}{2}\right)$

Mean sq. Value of m(t)	$\overline{m^2(t)} = \dfrac{1}{T}\displaystyle\int_0^T m^2(t)\, dt$

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Using (1) & (2) in (A)

$\Rightarrow \eta = \dfrac{\frac{1}{2} A_c^2 k_a^2 \overline{m^2(t)}}{\frac{1}{2} A_c^2 k_a^2 \overline{m^2(t)} + \frac{1}{2} A_c^2} \times 100 \%$

Imp ★ $\Rightarrow \boxed{\eta = \dfrac{k_a^2 \overline{m^2(t)}}{k_a^2 \overline{m^2(t)} + 1} \times 100 \%}$

→ for transmission efficiency or modul^n efficiency of AM

→ k_a : circuit constt.

★ SPECIAL CASE

Case : Single Tone Modul^n

→ $m(t) = A_m \cos \omega_m t$; $\omega_m \ll \omega_c$.

$\eta = \dfrac{k_a^2 \left[A_m^2 \left(\frac{1}{\sqrt{2}}\right)^2 \right]}{k_a^2 \left[A_m^2 \left(\frac{1}{\sqrt{2}}\right)^2 \right] + 1}$

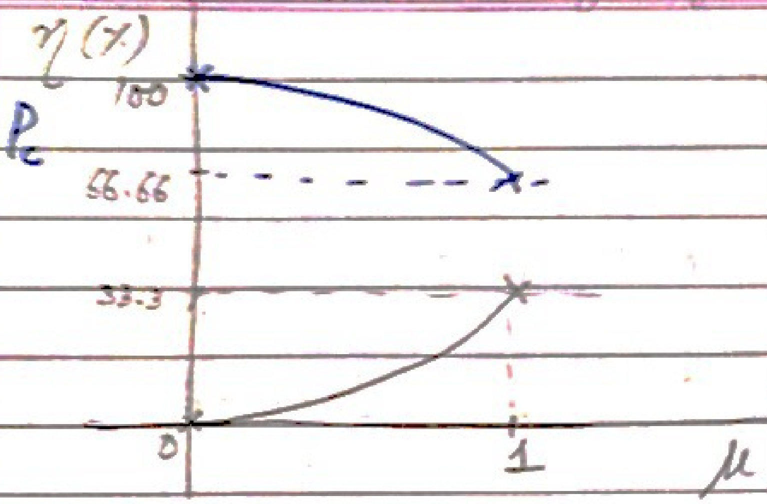
$\Rightarrow \eta = \dfrac{\frac{k_a A_m^2}{2}}{\frac{k_a^2 A_m^2}{2} + 1} = \dfrac{\mu^2}{\mu^2 + 2} \times 100 \%$

$\Rightarrow \boxed{\eta = \dfrac{\mu^2}{\mu^2 + 2} \times 100 \%}$

→ Transm^n efficiency for a single tone modulated wave

→ $\mu \triangleq k_a A_m$ (assumed before)

☆ Note:- $\eta_{max} = 33.3\%$ for single tone
$\eta_{max} = 50\%$ for signals like speech

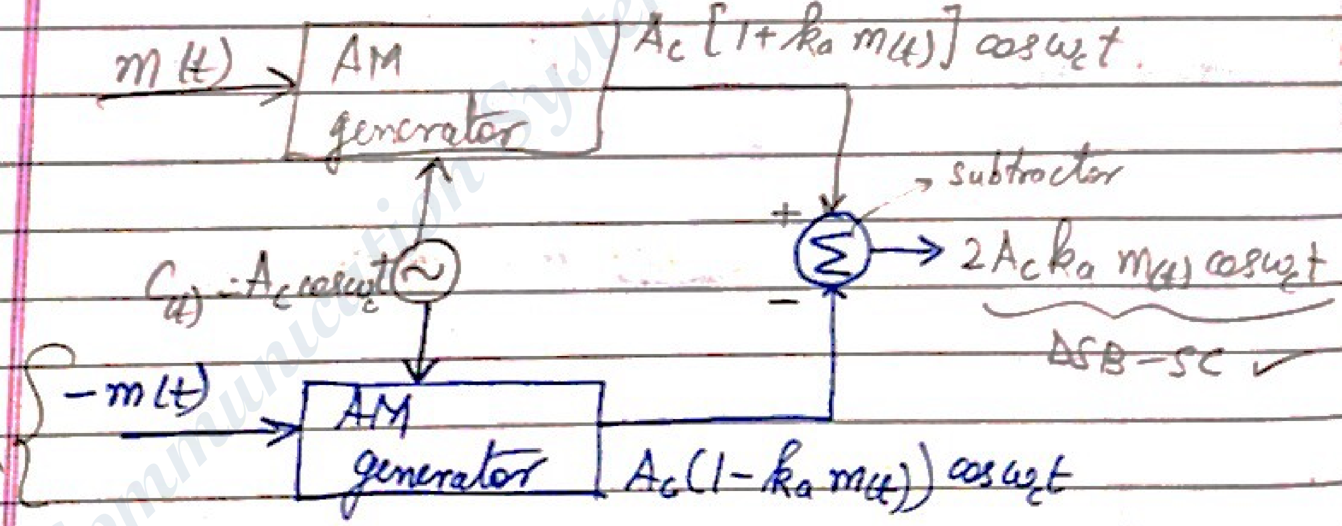


μ	0	1
η	0	33.33

μ	0	1
P_c	100%	66.66

§ DSB-SC

↳ Produced by product modulator or Balanced modulator



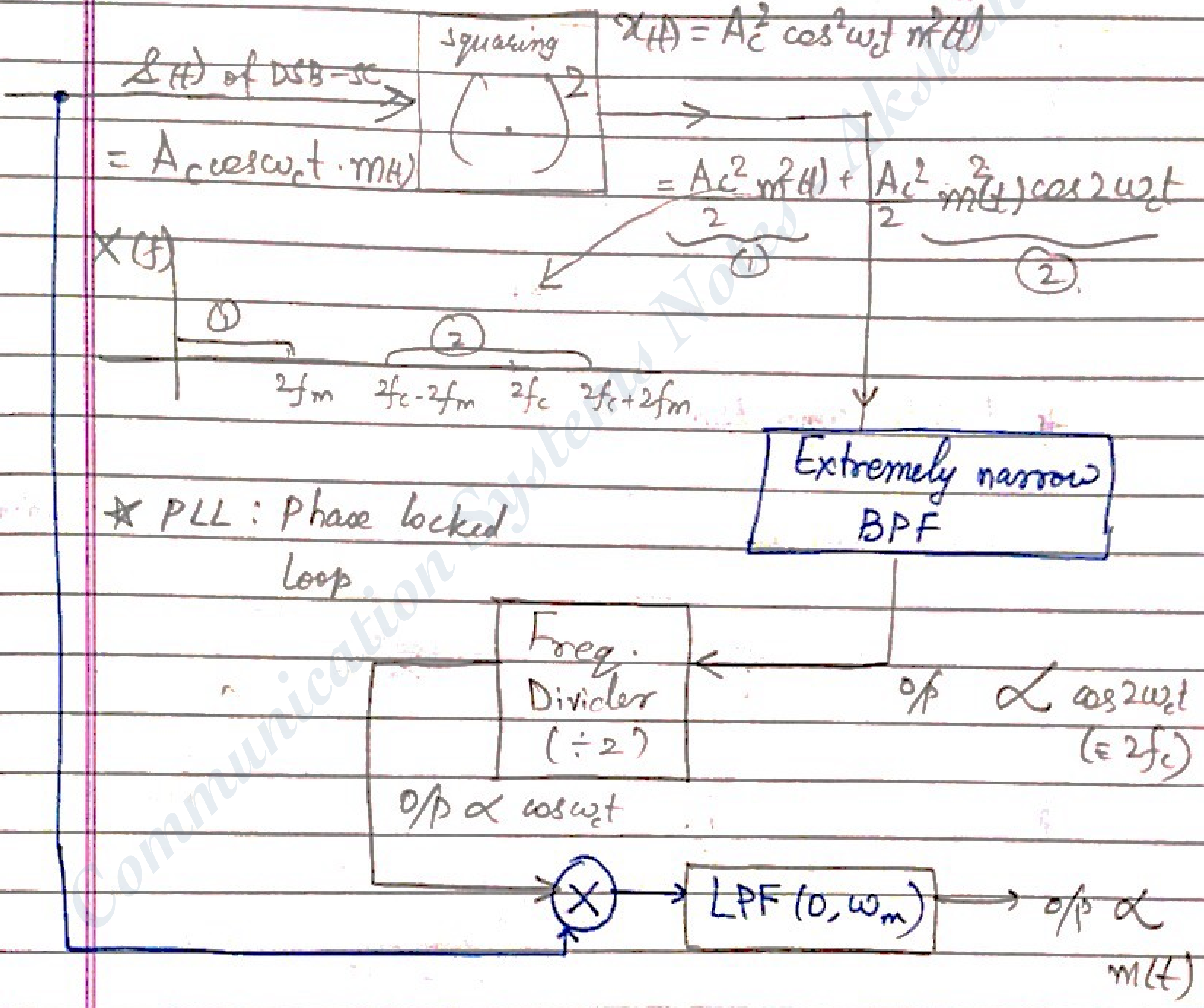
using another identical AM generator with a modified m(t).

☆ DSB-SC Demodulⁿ :-

Idea :- Remove carrier signal from incoming DSB-SC locally & use it to multiply with the signal & passed through a LPF to get m(t).

★ Extremely narrow BPF : allows only one freq. to go through.

SQUARE - LAW DEMODULATION



★ PLL : Phase locked Loop

Advantages:
saves power, BW saved, as compared to DSB-SC

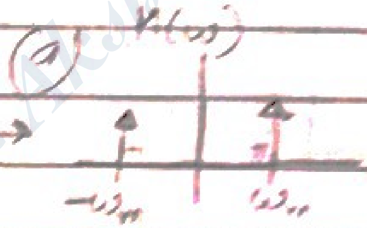
③ SINGLE SIDEBAND SUPPRESSED CARRIER MODULATION (SSB-SC)

Recall :- General expressions

DSB-SC : $s_{DSB\text{-}SC}(t) = A_c m(t) \cos \omega_c t$

Conventional AM : $s_{AM}(t) = A_c [1 + k_a m(t)] \cos \omega_c t$

SSB-SC : ?



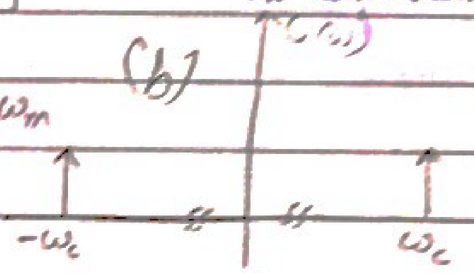
*** Single tone DSB-SC**

$m(t) = A_m \cos \omega_m t$

$M(\omega) = \underbrace{\pi [\delta(\omega - \omega_m) + \delta(\omega + \omega_m)]}$

An impulse at ω_m & $-\omega_m$ of area π.

* convolution



$(\omega_c >> \omega_m)$

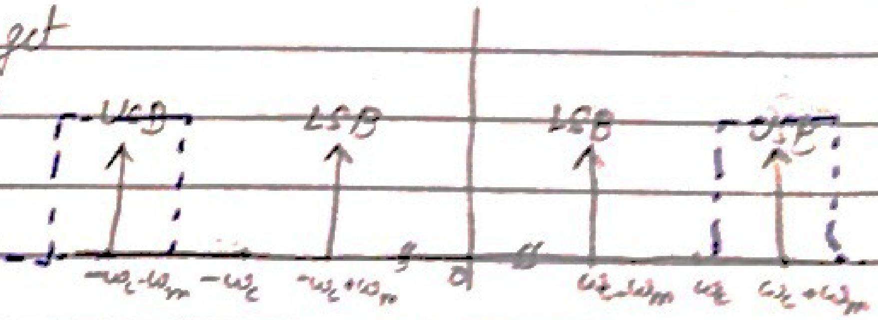
$s_{DSB\text{-}SC}(t) = m(t) \times c(t)$

$= A_c A_m \cos \omega_c t \cos \omega_m t$

Multiplicⁿ in time domain = Convolⁿ in freq domain

(a) * (b) =

Remove any sideband LSB or USB to get the signal; this is SSB-SC.



i.e., both +ve and -ve side of m(t) were pulled towards ω_c & $-\omega_c$

* Fourier transform $\subset$ Laplace transform
$\to$ subset

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Now, Which one to remove (extract)?

$$SSB \begin{cases} \to USB\text{-}SSB & (\text{called } USSB) \\ \quad or. & \\ \to LSB\text{-}SSB & (\text{called } LSSB) \end{cases}$$

We had

$$s_{DSB\text{-}SC}(t) = A_c A_m \cos\omega_c t \cos\omega_m t$$
$$= \frac{A_c A_m}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

Suppose we choose LSB

$$\Rightarrow s_{LSSB}(t) = k\left[\underbrace{\cos\omega_m t}_{m(t)\ \text{part}} \cos\omega_c t + \sin\omega_m t \sin\omega_c t\right]$$

choose USB

$$\Rightarrow s_{USSB}(t) = k\left[\cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t\right]$$

Generalising $\to \forall\ m(t)$

$$s_{LSSB}(t) = k\left[m(t)\cos\omega_c t + \tilde{m}(t)\sin\omega_c t\right]$$

PTO

Generalising further $\to$ LSB or USB

$$\boxed{s_{SSB}(t) = k\left[m(t)\cos\omega_c t \overset{LSSB}{\underset{USSB}{\pm}} \tilde{m}(t)\sin\omega_c t\right]}$$

* Understanding relⁿ b/w $m(t)$ & $\tilde{m}(t)$

Hilbert Transform:

- ✓ Retains magnitude.
- ✓ Changes phase by 90°.
- ✓ Has TF = $H(\omega)$

Fourier Transform of TF of Hilbert's Transform

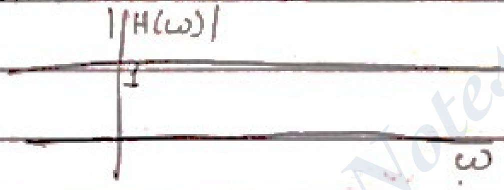
Consider

$$m(t) \rightarrow \boxed{\text{HILBERT TRANSFORM} \; H(\omega)} \rightarrow \tilde{m}(t)$$

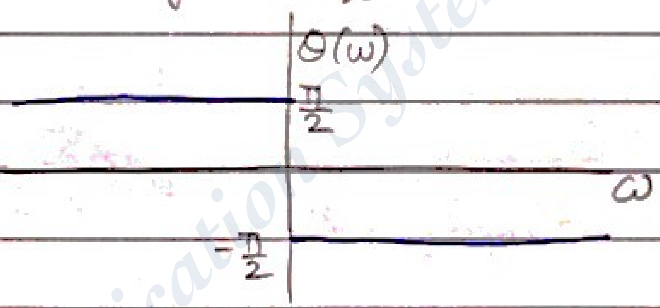
$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

- → Magnitude response $|H(\omega)|$ vs ω — always even f^n of ω
- → Phase response : $\theta(\omega)$ vs ω. — always odd f^n of ω.

Magnitude remains same
⇒



Phase changes by $\frac{\pi}{2}$



So,

$$H(\omega) = \begin{cases} 1\, e^{-j\frac{\pi}{2}} & ; \ \omega > 0 \\ 1 . e^{+j\frac{\pi}{2}} & ; \ \omega < 0 \end{cases}$$

$$= \begin{cases} \cos(-\frac{\pi}{2}) + j\sin(-\frac{\pi}{2}) & , \ \omega > 0 \\ \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) & ; \ \omega < 0 \end{cases}$$

$$= \begin{cases} -j & ; \ \omega > 0 \\ j & ; \ \omega < 0 \end{cases}$$

$$= (-j) \begin{cases} 1 & ; \ \omega > 0 \\ -1 & ; \ \omega < 0 \end{cases} \rightarrow \text{sgn (Signum } f^n)$$

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$\Rightarrow H(\omega) = (-j)\, sgn(\omega)$

↳ freq. domain representⁿ of Hilbert Transform.

Now, what is $h(t)$?

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$

$Y(\omega) = X(\omega) \cdot H(\omega)$

Now,

if $x(t) = \delta(t)$

$X(\omega) = 1$

$Y(\omega) = H(\omega)$

$\Rightarrow y(t) = h(t)$ → Response to unit impulse

So,

$m(t) \xrightarrow[\text{Trans.}]{\text{Hilbert}} \tilde{m}(t)$

$= \cos\omega_m t \qquad\qquad = \sin\omega_m t$

---

* Understanding Hilbert Transform:-

We know:- $H(\omega) = -j\, sgn(\omega)$ & $h(t) = \dfrac{1}{\pi t}$.

$m(t) \rightarrow \boxed{\text{Hilbert Transform, } h(t)} \xrightarrow{} \boxed{\text{Hilbert Transform, } h(t)} \xrightarrow{x(t)} ?$

90° phase shift — more 90° phase shift

o/p $= m(t) * h(t) * h(t)$

$X(\omega) = M(\omega) \cdot H(\omega)\, H(\omega)$

$= -M(\omega)[-j\,sgn(\omega)]^2$

$= -M(\omega)$

Now,

$X(\omega) = -M(\omega) \longrightarrow$

$\Rightarrow x(t) = -m(t)$.

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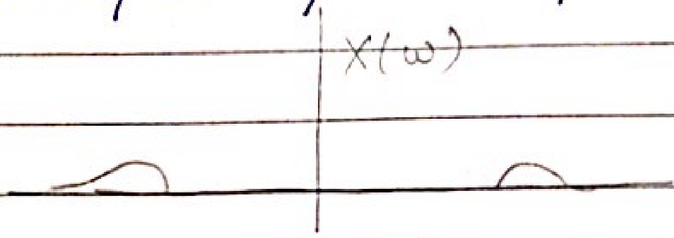
* Bandpass Represent^n of a Signal.



$$x(t) = x(t)\cos\omega_c t \pm \tilde{x}_{(t)}\sin\omega_c t$$

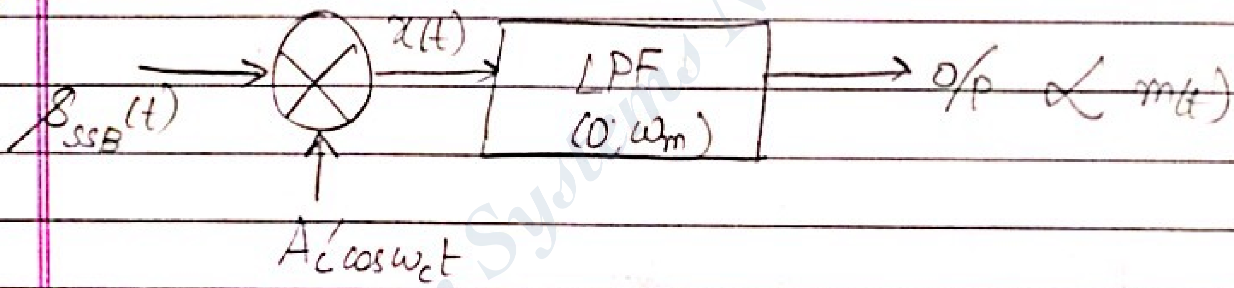
$x(t)$ → Inphase component; $\tilde{x}_{(t)}$ → Quadrature component

→ In DSB-SC

$x(t) = x(t)\cos\omega_c t$

i.e $\tilde{x}_{(t)}\sin\omega_c t = 0$

☆ DEMODULATION OF SSB-SC



Ideally, $x(t) = s_{SSB-SC}(t)\, A_c' \cos\omega_c t$

But, signal can have phase & freq. changes during transmission (errors)

So, $x_{(t)} = s_{SSB}(t)\cos[(\omega_c + \Delta\omega)t + \phi]$

$\Delta\omega$ → freq error; ϕ → phase error

$$= k\, m(t)\cos\omega_c t \cos[(\omega_c+\Delta\omega)t+\phi]$$
$$\pm\, k\tilde{m}_{(t)}\sin\omega_c t \cdot \cos[(\omega_c+\Delta\omega)t+\phi]$$

$$\Rightarrow x(t) = \frac{k\, m(t)}{2}\left\{\cos[(2\omega_c+\Delta\omega)t+\phi] + \cos[\Delta\omega t+\phi]\right\}$$
$$\mp \frac{k\, \tilde{m}(t)}{2}\left\{\sin[(2\omega_c+\Delta\omega)t+\phi] - \sin(\Delta\omega t+\phi)\right\}$$

Considering an ideal LPF, all freq out of the range $[-\omega_m, \omega_m]$ are cut

$2\omega_c > \omega_m$. So, $\cos(2\omega_c - \Delta\omega)t$ freq or $\sin(2\omega_c - \Delta\omega)t$ are cut

$$\Rightarrow \left(o/p\right)_{LSSB} = \frac{k}{2} m(t) \cos(\Delta\omega t + \delta) - \frac{k}{2} \tilde{m}(t) \sin(\Delta\omega t + \delta)$$

(Demodulated o/p should have had m(t) alone)

Considering ideal case, $\nexists$ no freq & phase error ($\Delta\omega = 0$, $\delta = 0$)

$$\text{So, } \left(o/p\right)_{LSSB} = \frac{k}{2} m(t) (1) - 0.$$

So we get m(t) under ideal case.
So, Coherent Detector gives o/p

★ Analysing freq & phase errors in SSB-SC demodulation:-

CASE I : Only FREQ error

$$\Rightarrow o/p \propto m(t) \cos(\Delta\omega t) - \tilde{m}(t) \sin(\Delta\omega t)$$

Difficult to see together. So, seeing demodulated o/p — SINGLE-TONE CASE

$$\text{So, } m(t) \propto \cos\omega_m t$$

$$o/p \propto \cos\omega_m t \cos(\Delta\omega t) - \sin\omega_m t \sin(\Delta\omega t)$$

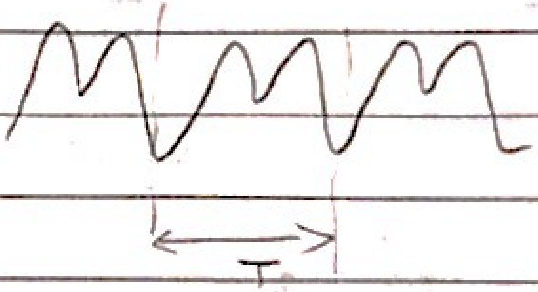
$$\Rightarrow o/p \propto \cos[(\omega_m + \Delta\omega)t]$$

(but we needed $o/p \propto \cos[\omega_m t]$)

So, it gets a little distorted

So, error occurs (corresponding to $\omega_m = 100\,Hz$ at i/p, we get o/p at $\omega_m + \Delta\omega = 110\,Hz$ (for $\Delta\omega = 10$)
or, $\omega_m - \Delta\omega = 90\,Hz$ (for $\Delta\omega = -10$)

Consider a music note with freq 300 Hz



So, at different harmonics, we have

300 Hz, 600 Hz, 900 Hz

So, $\underbrace{1 : 2 : 3}_{\text{a ratio continued}}$

So, good music

Now, with errors of say, $\Delta\omega = 10$ Hz

o/p → 310 Hz, 610 Hz, 910 Hz

& o/p wave is a combinⁿ of all these waves

It's no longer in harmonic (ratios)

So, good music lost

Case II : Only PHASE error

$$o/p \propto m(t)\cos(\phi) - \hat{m}(t)\sin\phi$$

Taking FT

$$\Rightarrow \mathcal{F}(o/p) \propto M(\omega)\cos\phi - \hat{M}(\omega)\sin\phi$$

(assuming $\cos\phi$, $\sin\phi$: constt instantaneously)

$$\text{So, } \mathcal{F}(o/p) \propto M(\omega)\cos\phi + j\,sgn(\omega)M(\omega)\sin\phi$$

($\because \hat{M}(\omega) = -j\,sgn(\omega)M(\omega)$ by Hilbert Transform)

So, $\mathcal{F}(o/p) \propto \begin{cases} M(\omega)\,[\cos\phi + j\sin\phi] & \omega > 0 \\ M(\omega)\,[\cos\phi - j\sin\phi], & \omega < 0 \end{cases}$

$\Rightarrow \mathcal{F}(o/p) \propto \begin{cases} M(\omega)e^{j\phi} & ;\ \omega > 0 \\ M(\omega)e^{-j\phi} & ;\ \omega < 0 \end{cases}$

So,

$\mathcal{F}(o/p) = \begin{cases} |M(\omega)|\, e^{j[\theta(\omega)+\phi]} & \omega > 0 \\ |M(\omega)|\, e^{j[\theta(\omega)-\phi]} & ;\ \omega < 0 \end{cases}$

uniform phase shift

$\left(\because M(\omega) = \underbrace{|M(\omega)|}_{\text{Magnitude}}\, \underbrace{e^{j\theta(\omega)}}_{\text{phase}} \right)$

Ideally, we needed

$M(\omega) = |M(\omega)|\, e^{j\theta(\omega)}$.

we got :

$|M(\omega)|\, e^{j(\theta(\omega) \pm \phi)}$ → uniform Phase shift (not desirable)

* Uniform phase shift : Constt phase shift irrespective of freq of message

* linear phase shift : proportional phase shift.

Linear phase shift { for
1 kHz → 10 × 1 k = 10° phase shift (say)
2 kHz → 20° phase shift.
4 kHz → 40° phase shift

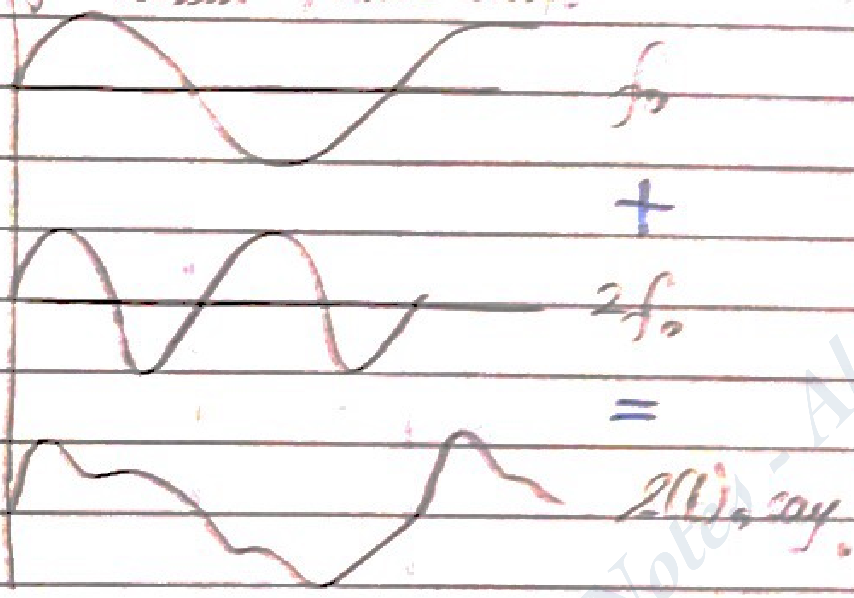
Uniform phase shift { for
1 kHz → 10° phase shift
5 kHz → 10° phase shift
500 kHz → 10° phase shift

Extra :

What is desirable ?

Uniform phase shift OR Linear Phase shift.

→ Seeing Linear Phase shift



Passing $s(t)$ through a channel

$$s(t) \longrightarrow \boxed{\begin{array}{c}\text{Channel}\\ H(\omega)\end{array}} \longrightarrow y(t) = \underbrace{k}_{\text{attenuation}}\, s(t - \overbrace{t_0}^{\text{delay}})$$

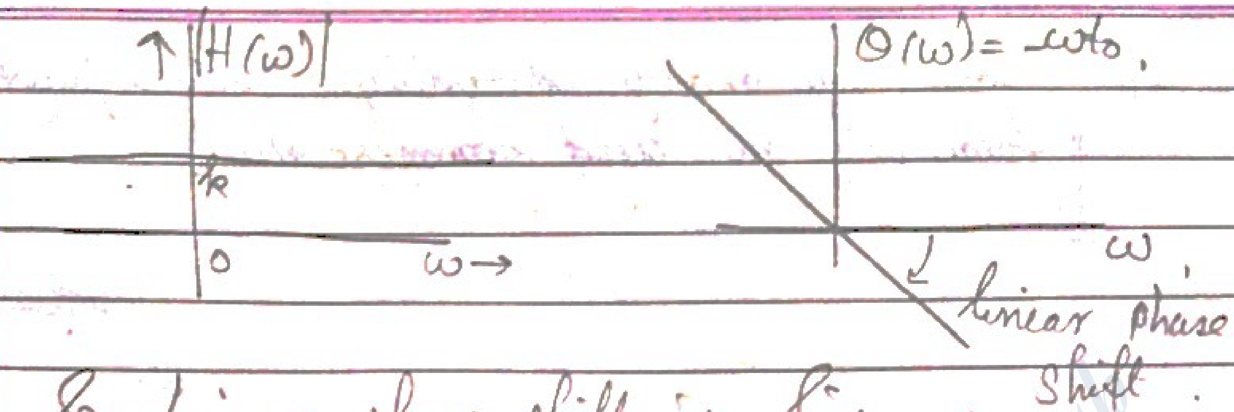
Basically,

$y(t) = k\,s(t - t_0)$ is not considered as signal distortion because we can manage communicating.

Taking FT

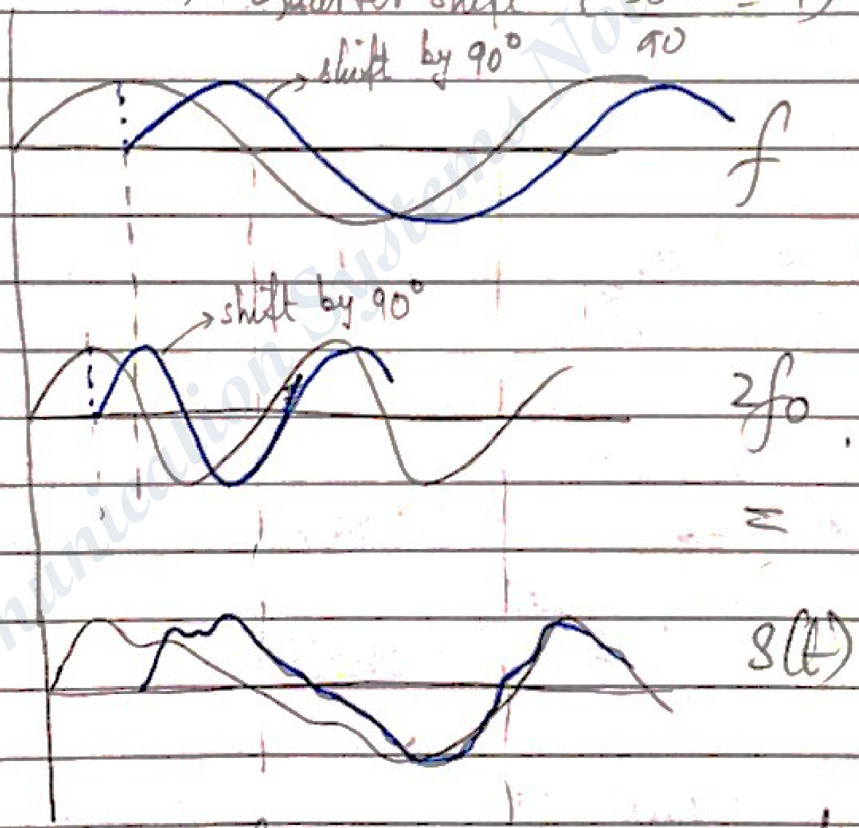
$\Rightarrow Y(\omega) = k\,S(\omega)e^{-j\omega t_0}$ (Time shifting)

$\Rightarrow TF = H(\omega) = \dfrac{Y(\omega)}{S(\omega)} = k\,e^{-j\omega t_0}$



So, linear phase shift is fine.

→ Seeing Uniform phase shift
= 90°, say.
A full wave = 360°
⇒ Quarter shift $\left(\frac{360}{90} = 4\right)$



As visible, fo & 2fo dont start from same point. So, the resultant is a distortion (s(t)). This is what happens for a uniform phase shift & is obviously not desirable.

Q. SSBSC has many advantages (saves power & BW). Then, why not used commercially?

Reason 1 :- It requires coherent detection. Doesn't work with envelope detection &, coherent detection is costly.

envelope detection is basically adding carrier, shifting signal & taking it out. So, lets try envelope detection for SSB-SC

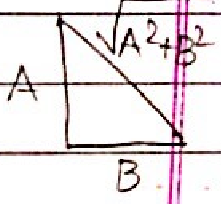
i.e

$$s_{SSB-SC}(t) + c(t) = \left[m(t)\cos\omega_c t \pm \tilde{m}(t)\sin\omega_c t\right] + A_c\cos\omega_c t$$

&

Takes only magnitude

$A\cos\theta + B\sin\theta \rightarrow$ Envelope detector



$$= \sqrt{A^2+B^2}\cos(\theta+\phi)$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right)$$

envelope, $A(t)$

Now

$$s_{SSB-SC}(t) + c(t) = (m(t) + A_c)\cos\omega_c t \pm \tilde{m}(t)\sin\omega_c t$$

$$\left[A\cos\omega_c t \pm B\sin\omega_c t\right]$$

So,

$$A(t) = \sqrt{(m(t)+A_c)^2 + [\tilde{m}(t)]^2}$$

$$= \left\{m^2(t) + 2A_c m(t) + A_c^2 + (\tilde{m}(t))^2\right\}^{1/2}$$

Now,

If $A_c >> |m(t)|, |\tilde{m}(t)|$

So, ignoring $m(t)$ & $\tilde{m}(t)$

$$\Rightarrow A(t) = [A_c^2 + 2A_c m(t)]^{1/2}$$

$$\Rightarrow A(t) = A_c\left[1 + \frac{2m(t)}{A_c}\right]^{1/2}$$

$\Rightarrow 0$

We know

$$(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \ldots$$

$$\approx 1 + nx, \text{ if } |x| << 1$$

So, $A(t) \approx A_c$.

So,

$$A(t) = A_c\left[1 + \frac{1}{2}\left[\frac{2m(t)}{A_c}\right]\right]$$

$$= A_c\left[1 + \frac{m(t)}{A_c}\right]$$

$$A(t) = A_c + m(t)$$

↳ valid when $\underbrace{A_c >>> |m(t)|, |\tilde{m}(t)|}$

⇒ ≃95 of power is $\underbrace{\text{pushed to carrier}}_{\text{not desired}}$.

⇓

DSB-SC ✓

* Single-tone AM

↳ general expression (derived before)

$$s_{AM}(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t$$

should be 1 (normalised)

* Finding μ :-

eg: Consider :- $s_{AM}(t) = A_c[2 + 0.9 \cos \omega_m t] \cos \omega_c t$

(M1) $\mu = \dfrac{A_{max}(t) - A_{min}(t)}{A_{max}(t) + A_{min}(t)}$

$$= \frac{2.9 A_c - 1.1 A_c}{2.9 A_c + 1.1 A_c} = \frac{1.8}{4} = \frac{0.9}{2}.$$

(M2) Comparing with std. form, $\mu = \dfrac{0.9}{2}$*.

* Relⁿ b/w P_T, P_c, μ

Total power

carrier power

We know $P_{SB} = P_{LSB} + P_{USB}$.

$$s_{AM}(t) = A_c[1 + \mu \cos \omega_m t] \cos \omega_c t$$

$$= \underbrace{A_c \cos \omega_c t}_{\text{Carrier}} + \underbrace{A_c \mu \cos \omega_m t \cos \omega_c t}_{SB = LSB + USB}.$$

Now,

$$P_T = P_c + P_{SB}$$

Mean sq value of carrier

Mean sq. value of SB (= LSB + USB)

$$\Rightarrow P_T = [A_c \cos\omega_c t]^2 + [A_c \mu \cos\omega_m t \cos\omega_c t]^2$$

$$= (A_c)^2 \left(\frac{1}{\sqrt{2}}\right)^2 + (A_c)^2 (\mu)^2 \left(\frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2.$$

$$= \underbrace{\frac{A_c^2}{2}}_{P_C} + \underbrace{\frac{A_c^2 \mu^2}{4}}_{P_{SB}}$$

$$\Rightarrow P_T = \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)$$

$$\Rightarrow \boxed{P_T = P_C \left[1 + \frac{\mu^2}{2}\right]}$$

↳ for a signal made of many freq. components, i.e, various values of μ, Take the mean as $\mu = \sqrt{\mu_1^2 + \mu_2^2 + \ldots}$ & use in this eq^n.

* Modul^n ⇒ making signal, $s(t)$ a fn of $m(t)$.

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§ Angle Modulation

↳ While studying AM, we saw 3 components :-

Revising AM

$m(t)$: message signal.

$c(t) = A_c \cos[\omega_c t + \phi]$ — constants

adding →

$s(t) = A_c(t) \cos[\omega_c t + \phi]$ — remains same

→ doesn't remain const anymore. It gets amplified for amplitude modul^n. It depends on $m(t)$

another form

$s_{AM}(t) = \underbrace{A_c[1 + k_a m(t)]}_{A(t) = f\{m(t)\}} \cos(\omega_c t + \phi)$

$A(t) = f\{m(t)\}$ ⇒ Ac got changed.

★ What change comes for Angle Modul^n?
here, ω_c & ϕ get changed on adding (instead of Ac).

★ Angle Modul^n
↳ Frequency modulation.
↳ Phase modulation.

General expression: $\boxed{s(t) = A_c \cos\theta(t)}$ → Generalised angle

where generalized angle $\theta(t)$ is a f^n of $m(t)$.

In the unmodulated carrier,

$c(t) = A_c \cos(\omega_c t + \phi) = A_c \cos\theta(t)$.

→ generalized angle $\theta(t) = \omega_c t + \phi$ where ω_c & ϕ are constants

$\triangleq$: means : "is defined as"

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- Instantaneous frequency of a signal. ($\omega_i(t)$)

$$\omega_i(t) \triangleq \frac{d}{dt}\theta(t)$$

So, what is $\omega_i(t)$ of $c(t) = A_c \cos(\underbrace{\omega_c t + \phi}_{\theta(t)})$

$$= \frac{d}{dt}(\omega_c t + \phi) = \omega_c$$

&, given inst. freq., find $\theta(t)$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\lambda)\, d\lambda \quad \text{or} \quad \overbrace{\int \omega_i(t)\,dt}^{\text{no need for limit}}$$

$-\infty \rightarrow = 0$ for sys. starting from $t=0$ (causal systems)

—x—

⟨1⟩ PHASE MODULATION (PM)

for unmodulated carrier, $\theta(t) = \omega_c t + \phi$

for PM, $\theta(t) = \omega_c t + k_p m(t)$

$k_p \rightarrow$ proportionality constt

(i.e., modulated carrier has a phase component proportional to $m(t)$)

★ So, $s_{PM}(t) = A_c \cos[\underbrace{\omega_c t + k_p m(t)}_{\theta(t)}]$

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(2) FREQUENCY MODULATION (FM)

↳ Instt. freq. (ω_i) of carrier varies about carrier freq. in proportion to $m(t)$

mathematically,

$$\omega_i(t) = \omega_c + k_f\, m(t)$$

for this, $\theta(t) = \int_{-\infty}^{t} \omega_i(\lambda)\, d\lambda$ or $\int \omega_i(t)\, dt$

$$= \int \left[\omega_c + k_f\, m(t)\right] dt$$

$$\Rightarrow \theta(t) = \omega_c t + k_f \int m(t)\, dt$$

Now

* $s_{FM}(t) = A_c \cos \underbrace{\left[\omega_c t + k_f \int m(t)\, dt\right]}_{\theta(t)}$

Note ☆

In PM, $\phi(t) \propto m(t)$

In FM, $\phi(t) \propto \int m(t)$

—x—

☆ Note : For a linear $m(t) - kt$, we find PM & FM have diff^t waveforms (See →)

But, if $m(t)$ = sinusoidal waveform, we find that PM & FM have same waveform (only delayed by some amount)

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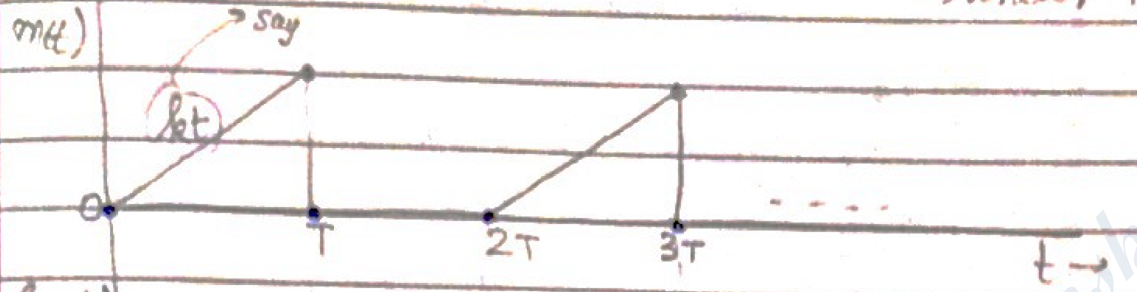
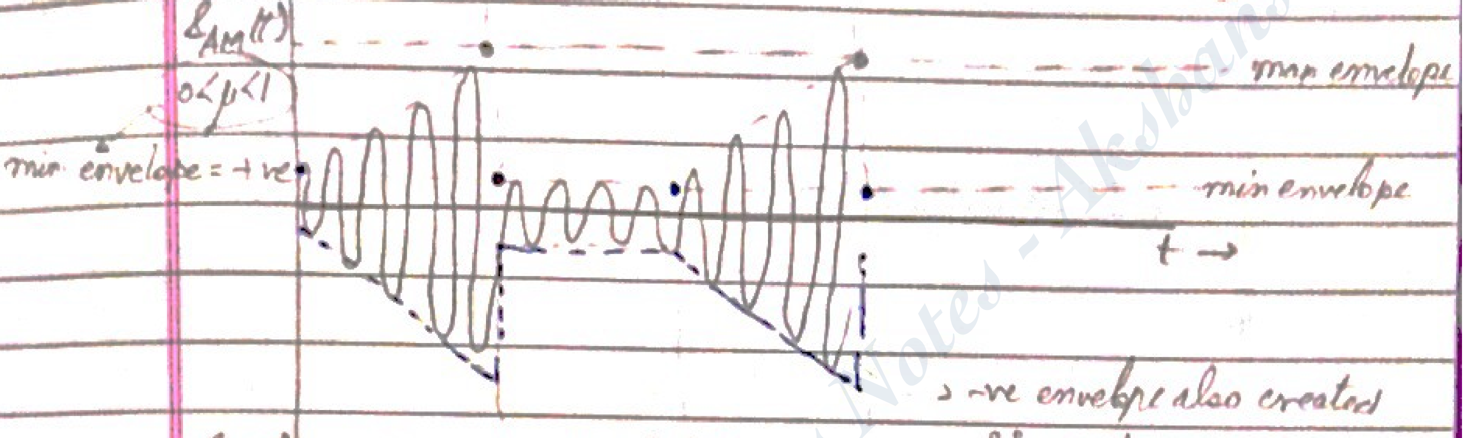
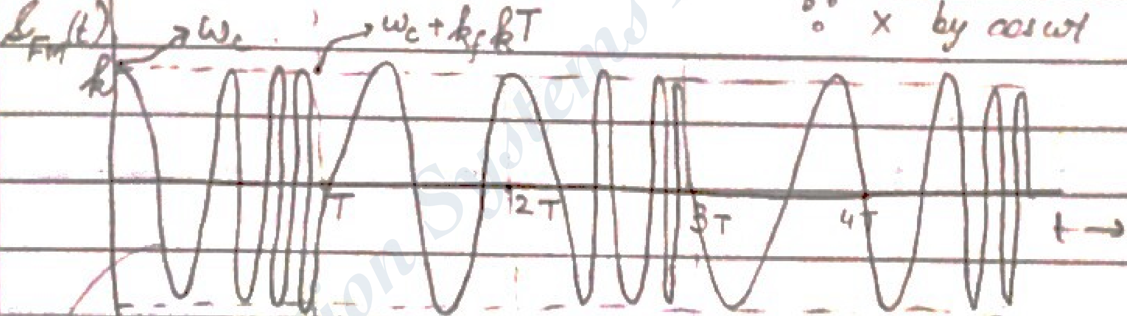
* FM waveform for Triangular wave

☆ Time waveforms (for $m(t) = kt$)

linear.

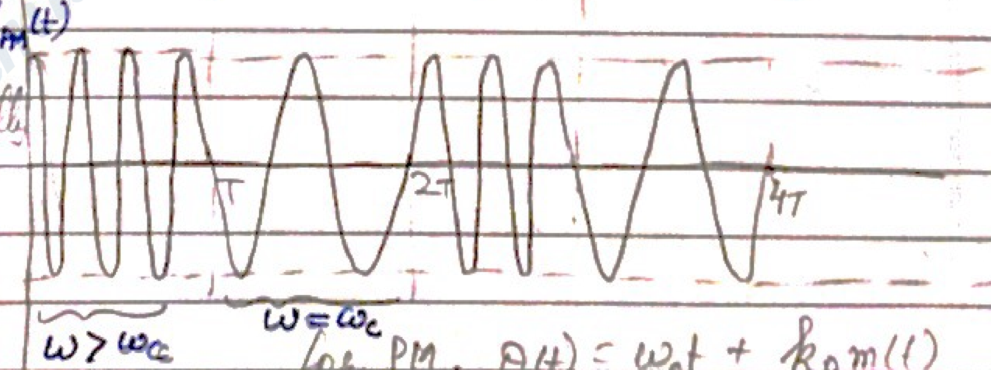






for FM: for $0<t<T$, $\omega_i = \omega_c + k_f(kt)$

as visible from formula, instt freq = ω_c initially & keeps ↑ till T. From T to 2T, amplitude is constt & $m(t)=0$. So, we have only $\omega_i = \omega_c$, i.e. carrier freq.



for PM, $\theta(t) = \omega_c t + k_p m(t)$

$= \omega_c t + k_p(kt)$

$\Rightarrow \omega_i(t) = \dfrac{d\,\theta(t)}{dt}$

$\Rightarrow \omega_i(t) = \underbrace{\omega_c + k_p k}_{\text{constt}}$

* **Single tone FM**

In single tone, $m(t) = A_m \cos\omega_m t$

In FM,

$$\omega_i(t) = \omega_c + k_f\, m(t)$$

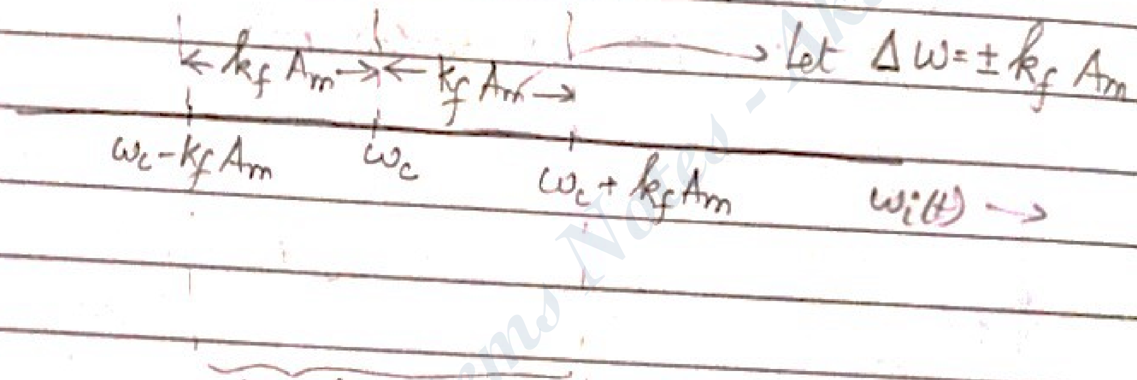
instt freq. of carrier

carrier freq

$$\Rightarrow \omega_i(t) = \omega_c + k_f (A_m \cos\omega_m t)$$

max :- $\omega_c + k_f A_m (1)$

min :- $\omega_c + k_f A_m (-1)$



Instt freq. $\omega_i(t)$ is within this region

$\Delta\omega$ = Max. freq. ~~variation~~ deviation (of instt. freq. from carrier freq).

$= \pm k_f A_m$

$$\Rightarrow \omega_i = \omega_c + \Delta\omega \cos\omega_m t$$

We know

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

$$\Rightarrow \theta(t) = \int_0^t \omega_i(t)\, dt$$

$$= \int_0^t \left(\omega_c(t) + \Delta\omega \cos\omega_m t\right) dt$$

$$= \omega_c t + \Delta\omega \left[\frac{\sin\omega_m t}{\omega_m}\right]$$

* Modul^n^ index of FM $= k_f \dfrac{A_m}{\omega_m} \propto \left(\dfrac{\text{Amplitude}}{\text{Frequency}}\right)$

On changing β of FM, always change A_m, NEVER ω_m.

$$\Rightarrow \theta(t) = \omega_c t + \left[\frac{\Delta\omega}{\omega_m}\right]\sin\omega_m t$$

$\rightarrow \dfrac{\Delta\omega}{\omega_m}$ = Modul^n^ index of FM $\triangleq \beta$.

$k_f A_m$

$\rightarrow = \dfrac{2\pi\Delta f}{2\pi f_m} = \dfrac{\Delta f}{f_m} = m_f$

Using β or m_f is same

$\rightarrow$ for single tone.

for general m(t), with bandwidth W, is defined with Deviation ratio, D ⇒

$$\boxed{D = \frac{\Delta f}{W}}$$

So, $\theta(t) = \omega_c t + \beta\sin\omega_m t$.

So, $S_{FM}(t) = A_c\cos[\omega_c t + \beta\sin\omega_m t]$

$\rightarrow$ for single tone FM signal.

* Single tone - PM

for single tone, let $m(t) = A_m\cos\omega_m t$.

for PM,

$$\theta(t) = \omega_c t + k_p m(t)$$

$$\Rightarrow \theta(t) = \omega_c t + k_p A_m\cos\omega_m t.$$

So,

$$S_{PM}(t) = A_c\cos[\omega_c t + k_p A_m\cos\omega_m t]$$

Now, $\omega_i(t) \triangleq \frac{d}{dt}\theta(t) = \omega_c - k_p A_m \omega_m \sin\omega_m t$

$$(\equiv \omega_c \pm \Delta\omega)$$

So, $|\Delta\omega| = k_p A_m \omega_m$.

modul^n index for PM $= \beta_p = \frac{\Delta\omega}{\omega_m} = \frac{k_p A_m}{\omega_m}$.

↳ changing β can be done for

FM: By changing A_m or ω_m (But A_m should be changed). We have 2 choices.

PM: By changing A_m only. We just have one option

☆ The waveforms for FM & PM for single tone will be the same, but, only for a certain TIME SHIFT.

☆ SPECTRUM OF FM Signal

↳ considering single tone case for simplicity

for $s_{FM}(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$

[≡ trignometric fn of trigonometric fn]

for conventional AM, single tone:

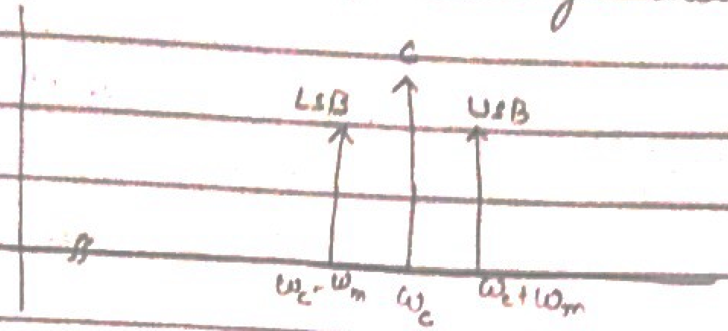
$$s_{AM}(t) = A_c[1 + \mu\cos\omega_m t]\cos\omega_c t$$

$$= \underbrace{A_c\cos\omega_c t}_{\text{carrier}} + \frac{A_c\mu}{2}\cos(\omega_c+\omega_m)t + \frac{A_c\mu}{2}\cos(\omega_c-\omega_m)t$$

* $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

Representing conventional AM to get idea :-



Proceeding similarly for single tone FM;

$$S_{FM}(t) = A_c \cos[\underbrace{\omega_c t}_{A} + \underbrace{\beta \sin \omega_m t}_{B}]$$

$$= \boxed{A_c \cos(A+B)}$$

$$S_{FM}(t) = A_c \cos \omega_c t \cos(\beta \sin \omega_m t) - A_c \sin \omega_c t \sin(\beta \sin \omega_m t) \longrightarrow (1)$$

For simplifying eq^n (1), we define 2 types of FM

Types: FM

Narrow Band FM (NB FM)	Wideband FM (WB FM)
$\beta \ll \frac{\pi}{2}$ or 1	$\beta \gg \frac{\pi}{2}$ or 1

So, let $x = \beta \sin \omega_m t$

$\rightarrow 1 - \frac{x^2}{2!} + \dots$

$$\Rightarrow S_{NBFM}(t) = A_c \cos \omega_c t \left[1 - \frac{\beta^2 \sin^2 \omega_m t}{2!} + \dots\right]$$

$$- A_c \sin \omega_c t \left[\beta \sin \omega_m t - \frac{\beta^3 \sin^3 \omega_m t}{3!} + \dots\right]$$

$\rightarrow x - \frac{x^3}{3!} + \dots$

* Power depends on Amplitude of any wave, not its angular component

If β is very small,

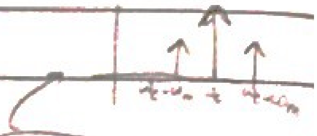
$\beta^2 \simeq 0$ & $\beta^3 \simeq 0$ & so on.

$$\Rightarrow s_{NBFM}(t) = A_c \cos \omega_c t - A_c \sin \omega_c t (\beta \sin \omega_m t)$$

$$= A_c \cos \omega_c t - \frac{A_c \beta}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

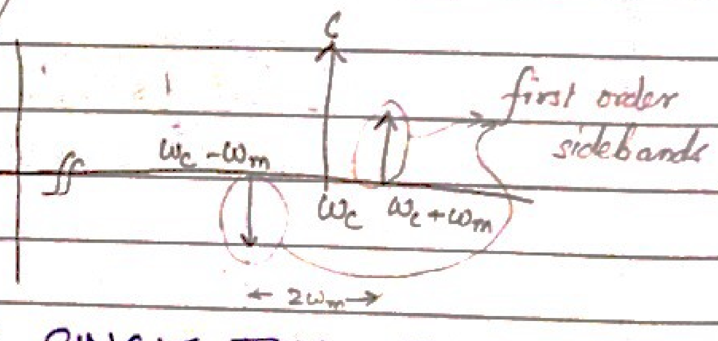
$$\Rightarrow s_{NBFM}(t) = A_c \cos \omega_c t - \frac{A_c \beta}{2} \cos(\omega_c - \omega_m)t + \frac{A_c \beta}{2} \cos(\omega_c + \omega_m)t$$

$\longrightarrow \beta <<< (\frac{\pi}{2}$ or $1)$



(this expression is similar to AM signal, just a phase difference is there $(+\frac{A_c \beta}{2}$ & $-\frac{A_c \beta}{2}))$

So, fig:



☆ SPECTRUM OF A SINGLE TONE FM SIGNAL

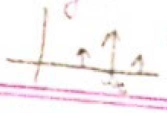
① TAYLOR SERIES METHOD (continued)

Now, in previous case, $\beta << \frac{\pi}{2}$ or 1

So, β^2 & β^3, ... terms were neglected

Now, let us increase β a little, s.t. β^2 cannot be neglected; $\beta^3, \beta^4 \ldots$ can still be neglected

So, going from NBFS $\rightarrow$ WBFM

... μ in AM Signal, BW remains same
But amplitude changes
So, s.th. like  (initially)  (can happen)

Remember, $\beta = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m} = \frac{k_f A_m}{\omega_m}$.

So, now,

$$S(t) = A_c \cos\omega_c t \left[1 - \frac{\beta^2 \sin^2\omega_m t}{2}\right] - A_c \sin\omega_c t \, \beta \sin\omega_m t$$

$$= A_c \cos\omega_c t - \frac{A_c\beta}{2}\left[\cos(\omega_c - \omega_m)t\right] + \frac{A_c\beta}{2}\left[\cos(\omega_c + \omega_m)t\right]$$

$$- \frac{A_c\beta^2}{2} \cos\omega_c t \, \sin^2\omega_m t$$

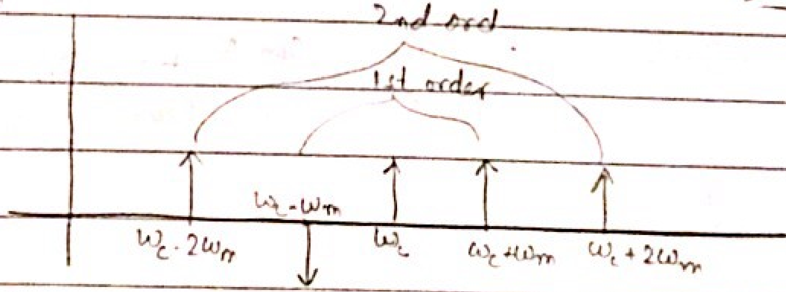
$$\sin^2\omega_m t \rightarrow \frac{1 - \cos 2\omega_m t}{2}$$

$$= A_c \cos\omega_c t - \frac{A_c\beta}{2}\cos(\omega_c - \omega_m)t + \frac{A_c\beta}{2}\cos(\omega_c + \omega_m)t$$

$$- \frac{A_c\beta^2}{4}\cos\omega_c t + \frac{A_c\beta^2}{4}\underbrace{\cos\omega_c t \cos 2\omega_c t}_{(=\cos A \cos B)}$$

$$= A_c\left[1 - \frac{\beta^2}{4}\right]\cos\omega_c t - \frac{A_c\beta}{2}\cos(\omega_c - \omega_m)t + \frac{A_c\beta}{2}\cos(\omega_c + \omega_m)t$$

$$+ \frac{A_c\beta^2}{4}\left[\frac{1}{2}\left(\cos(\omega_c + 2\omega_m)t + \cos(\omega_c - 2\omega_m)t\right)\right]$$



Observation: As $\beta \uparrow$, more sideband components come,
& all odd order sidebands are out of phase ($\downarrow \uparrow$)
& all even order sidebands are in phase ($\uparrow \uparrow$)

Also, initially, BW was $(\omega_c + \omega_m) - (\omega_c - \omega_m) = 2\omega_m$.
Now, it has increased to $(\omega_c + 2\omega_m) - (\omega_c - 2\omega_m) = 4\omega_m$.

As we keep ↑ value of β, ∴ BW $\rightarrow \infty$.
Infinite BW is not practical (i.e., 100% power)
So, for practical purposes, we take 98% power
i.e., for an FM signal as: $A_c \cos[\omega_c t + \beta \sin \omega_m t]$
we focus on achieving $(0.98)\frac{A_c^2}{2}$.

Max. Power = $\frac{A_c^2}{2}$. As new components come (on ↑ β), it comes out of $\frac{A_c^2}{2}$. Power doesn't increase above $\frac{A_c^2}{2}$ (rms value)

(2) FOURIER-SERIES APPROACH

Same spectrum will be maintained — no change in β value in this approach

Restricting ourselves to single-tone FM:

General FM wave: $s_{FM}(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t]$

$$= A_c \, Re\left[e^{j\theta}\right]$$

$\hookrightarrow \theta = \omega_c t + \beta \sin \omega_m t$

$\left(\because e^{j\theta} = \cos\theta + j\sin\theta \right.$
$\left. \Rightarrow Re(e^{j\theta}) = \cos\theta \ \& \ Im(e^{j\theta}) = \sin\theta \right)$

$$= A_c \, Re\left[e^{j(\omega_c t + \beta \sin \omega_m t)}\right]$$

$$= A_c \, Re\left[e^{j\omega_c t} \cdot e^{j\beta \sin \omega_m t}\right] \rightarrow (1)$$

Let $v(t) = e^{j\beta \sin \omega_m t}$

(If $v(t)$ is periodic, it can be expressed as Fourier series)

Sol) Checking for periodicity: If $v(t) = v(t+T)$, $T = \frac{2\pi}{\omega_m}$

Now, as its periodic signal, Fourier series is applicable (why are we doing it? $\longrightarrow$ To convert an unknown fn in a linear combinⁿ of simple f's.)

So, $v(t) = e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} V_n e^{jn\omega_m t} \longrightarrow (2)$

$\hookrightarrow$ V_n: Fourier coeff corresponding fn.

; $V_n \triangleq \frac{1}{T}\int_{-T/2}^{T/2} v(t)\, e^{-jn\omega_m t}\, dt$

$\Rightarrow V_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\beta \sin \omega_m t} \cdot e^{-jn\omega_m t}\, dt$

let $\omega_m t = x$

$\Rightarrow \omega_m dt = dx$

$\Rightarrow$ at $t = -\frac{T}{2} \Rightarrow x = \omega_m\left(-\frac{T}{2}\right)$

$= \left(\frac{2\pi}{T}\right)\left(-\frac{T}{2}\right)$

$= -\pi$

at $t = +\frac{T}{2}$, $x = +\pi$

$\Rightarrow V_n = \frac{1}{T}\int_{-\pi}^{\pi} e^{j\beta(\sin x - nx)} \frac{1}{\omega_m}\, dx$

V. Imp ★★★

$\Rightarrow V_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx = f(n, \beta)$

$\hookrightarrow$ variables : n, β. $\rightarrow$ sideband no.

$\hookrightarrow$ x: dummy variable

$= J_n(\beta)$

$=$ Bessel's fⁿ of first kind, first order

From ②,

$$v(t) = e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}$$

Alternate expression:

$$s_{FM}(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t]$$
$$= A_c \, Re\left[e^{j\omega_c t} \cdot \sum_{n=-\infty}^{\infty} \underbrace{J_n(\beta)}_{\text{Bessel's } f^n} e^{jn\omega_m t}\right]$$

* Properties of Bessel's f^n.

P1) $J_n(\beta)$ is a real valued f^n.

* P2) $J_{-n}(\beta) = (-1)^n J_n(\beta)$ (tells if terms are in phase/out of phase)

P3) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$ ($\because$ total probability $= 1$)

Now,

$$s_{FM}(t) = \left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\right) \cdot Re\left\{e^{j(\omega_c + n\omega_m)t}\right\}$$

$$\Rightarrow s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

Taking $N = 0$

$\Rightarrow \quad = A_c \, J_0(\beta) \cos \omega_c t$

$N = 1$

$A_c \, J_1 \beta \, [\cos(\omega_c + \omega_m)t]$

$N = -1$

$-A_c \, J_1 \beta \cos[(\omega_c + \omega_m)t]$

$N = 2, -2$

$= A_c J_2(\beta)[\cos(\omega_c + 2\omega_m)t + \cos(\omega_c - 2\omega_m)t]$

Observation: Even order sidebands in phase
Odd order sideband out of phase

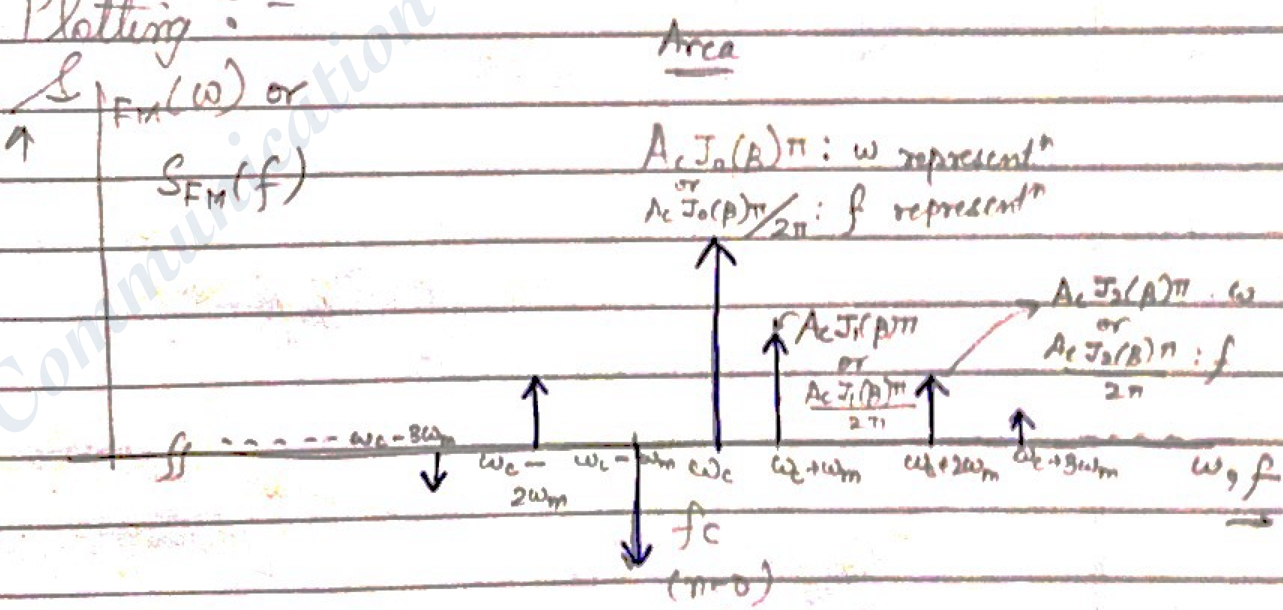
So, $S_{FM}(t) = N = 0\ (+)\ N = \pm 1\ (+)\ N = \pm 2$

$= A_c J_0(\beta)\cos\omega_c t +$

$\{A_c J_1(\beta)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t]\}$

$+ A_c[J_2(\beta)\cos(\omega_c + 2\omega_m)t + J_2(\beta)\cos(\omega_c - 2\omega_m)t]$

Plotting :-



← Bandwidth (= BW) → ∞ →

(Theoretically)

Now, $P_T = \frac{A_c^2}{2}$ (Normalised, $R = 1\,\Omega$).

Seeing if this power is shown by this new Bessel's expression.

So,

$$P_T = \left[\frac{A_c J_0(\beta)}{\sqrt{2}}\right]^2 + 2\left\{\left[\left(\frac{A_c J_1(\beta)}{\sqrt{2}}\right)^2 + \left(\frac{A_c J_2(\beta)}{\sqrt{2}}\right)^2 + \dots\right]\right\}$$

$$= \frac{A_c^2}{2}\underbrace{\left[J_0(\beta)^2 + 2\left\{J_1^2(\beta) + J_2^2(\beta) + \dots\right\}\right]}_{\to (A)}$$

from Bessel's property 3,

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

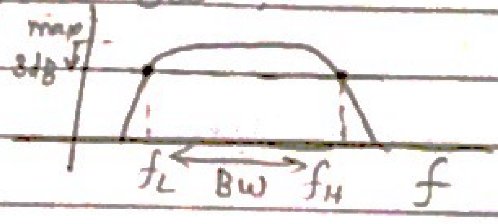
$\Rightarrow$ $P_T = \frac{A_c^2}{2}$ (same)

Considering practical BW (as, practically, BW can't be ∞, as seen before)

Std. defin^n: One way of defining BW:

The points where $\exists$ 3 dB lower from max (half of power). That will give cut off freq. The diff of that gives BW.

So, for any signal, say



As discussed before, we don't take 100% power in practical cases ($\because BW \to \infty$). So, decreasing little accuracy, we take 98% power. This concept is given by **CARSON'S FORMULA**

from (A) ($\leftarrow$), we went from values of $J^2(\beta)$ from $-\infty$ to ∞. $\left(\sum_{n=-\infty}^{\infty} J_n^2(\beta)\right)$

Instead,

make it $\displaystyle\sum_{n=-k}^{k} J_n^2(\beta)$

Now, $P_{T_{new}} = 98\% \, P_T$.

$$\Rightarrow \frac{P_{T_{new}}}{P_T} \geq 0.98 \Rightarrow \frac{\cancel{\left(\frac{A_c^2}{2}\right)} \sum_{n=-k}^{k} J_n^2(\beta)}{\cancel{\left(\frac{A_c^2}{2}\right)} \underbrace{\left(\sum_{n=-\infty}^{\infty} J_n^2(\beta)\right)}_{\to 1}}$$

$$\Rightarrow \sum_{n=-k}^{k} J_n^2(\beta) \geq 0.98$$

$$\Rightarrow J_0^2(\beta) + 2\left[J_1^2(\beta) + J_2^2(\beta) + \dots + J_k^2(\beta)\right] \geq 0.98$$

Start taking values for $n = 0, 1, 2, \dots$ of $J_n^2(\beta)$

At some point, there will be a point when the values of $J_n^2(\beta)$ becomes ≥ 0.98.

That value of k is the value which gives P_T as 98%.

From the previous frequency spectrum, the practical bandwidth changes to $2k\omega_m$ $\left(\omega_c - k\omega_m \quad \omega_c \quad \omega_c + k\omega_m ;\ \longleftarrow 2k\omega_m \longrightarrow\right)$

$J_0(0.2)$ ($J_n(\beta)$)

* Partial table of Bessel function values $J_n(\beta)$

can have any value

$\beta \left(= \frac{\Delta f}{f_m} = \frac{\Delta\omega}{\omega_m}\right)$

n	0.2	0.5	1.0	2.0	5.0	8.0
0	0.990	0.938	0.765	0.224	-0.178	0.172
1	0.100	0.242	0.440	0.577	-0.328	0.235
2	0.005	0.031	0.115	0.353	0.047	-0.113
3		0.003	0.020	0.129	0.365	-0.291
4			0.002	0.034	0.391	-0.105
5				0.007	0.261	0.186
6				0.001	0.131	0.338
7					0.053	0.321
8					0.018	0.223
9					0.006	0.126
10					0.001	0.026
⋮						

Underline indicates the value of n we need to take for ONE SPECIFIC VALUE of β

eg :- for $\beta = 0.2$

$n = 0$ gives $(0.990)^2 \geq 0.98$. Don't take other values of n.

for $\beta = 0.5$

$n = 0$ gives $(0.938)^2 \not\geq 0.98$

So, take $n = 1$ added to it

i.e., $(0.938)^2 + 2(0.242)^2 \geq 0.98$

So, stop here.

Illy,

for $\beta = 1$,

take $n = 0, 1, 2$ . . . & so on

So, for $\beta = 2$,

take $n = 0, 1, 2, 3$

Generalised,

$\beta = a$

$n = 0, 1, 2, ---, a$

So,

$$(k = \beta + 1) \star$$

hence, BW of FM can be given as:-

$$BW_{FM} = 2k\omega_m$$
$$= 2(\beta+1)\omega_m \text{ or } 2(\beta+1)f_m$$
$$= 2\left(\frac{\Delta\omega}{\omega_m}+1\right)\omega_m \text{ or } 2\left(\frac{\Delta f}{f_m}+1\right)f_m$$
$$= 2\Delta\omega + 2\omega_m \text{ or } 2\Delta f + 2f_m$$
$$= 2(\Delta\omega + \omega_m) \text{ or } 2(\Delta f + f_m)$$

So,

for a general m(t) of BW = W,

$$B_{FM} = 2(D+1)W$$

→ $D = \frac{\text{max. freq. deviation } (\Delta f)}{W}$

→ In commercial radio broadcasting,

$W = 15\ kHz$ & $\Delta f = 75\ kHz$.

$\Rightarrow D = \frac{75}{15} = 5$

$\Rightarrow B_{FM} = 2(5+1)15\ kHz$
$= 180\ kHz$.

Note: had it been conventional AM or DSB-SC, i.e., for commercial AM broadcasting with freq of m(t) = 15 KHz
So, BW of modulated signal = $2 \times 15\ kHz = 30\ kHz$.

Deductions:

① BW for same message freq.

More : FM (commercial = 180 kHz)

less : AM (commercial = 30 kHz)

⇒ FM requires more BW.

② Commercial FM BW for any message freq is 180 kHz

So, suppose ∃ a station 101.2 MHz

180 kHz ⇒ 0.18 MHz

So, there cannot be any station b/w

101.2 + 0.18 &

101.2 − 0.18 MHz.

for safety, we take ± .2 MHz

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Q. Given the single tone FM signal :

$$s_{FM}(t) = 20\cos[(2\pi\times10^6 t) + 2\sin(2\pi\times10^4 t)]$$

① Sketch the FM spectrum for the carrier & the first three sideband terms.

② What is the BW using Carson's rule ?

③ What is the BW within which 95% of Total FM power is contained

Given :-

$J_0(2) = 0.224$ $\quad$ $J_3(2) = 0.129$

$J_1(2) = 0.577$

$J_2(2) = 0.353$

From formula of $s_{FM}(t)$, we find,

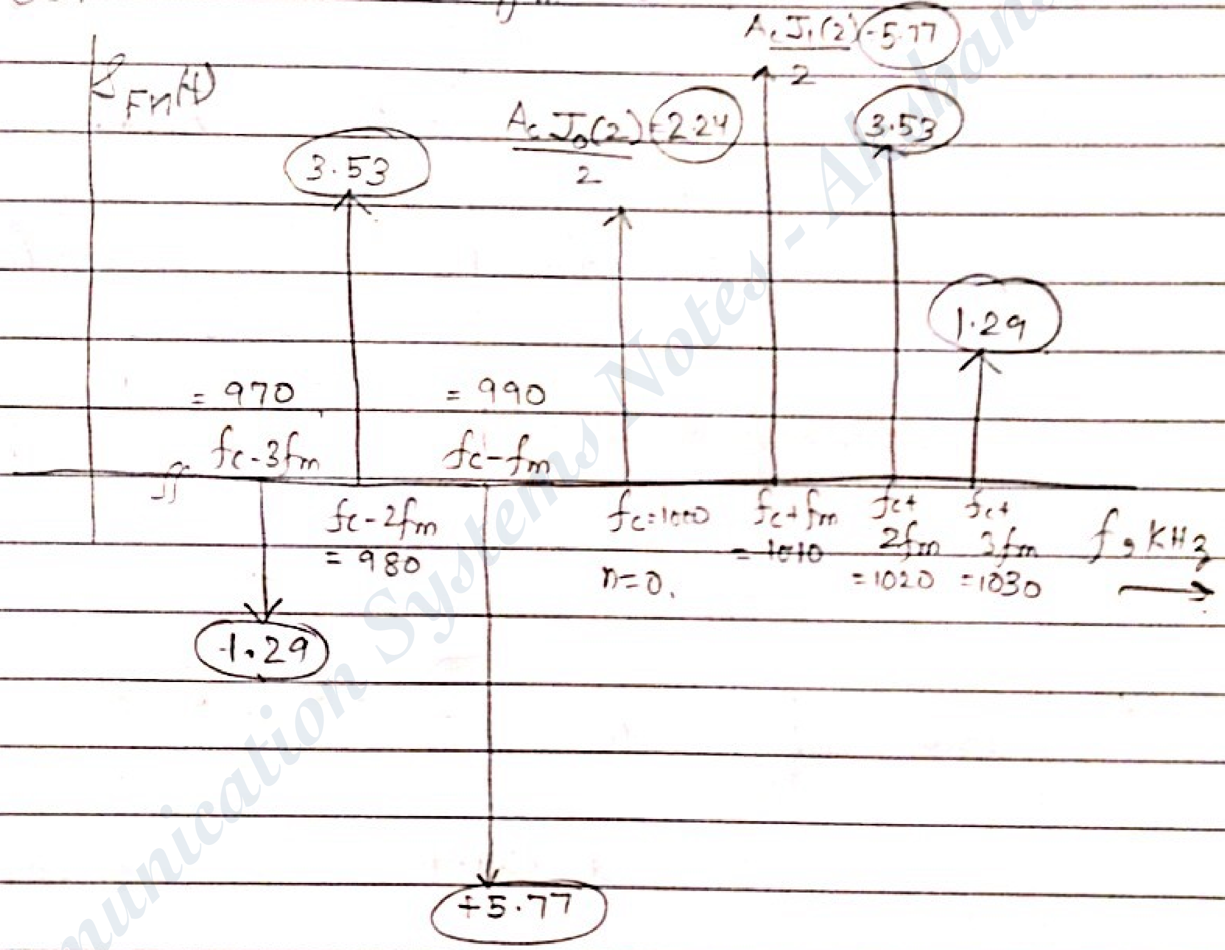
$A_c = 20\,V$

$\beta = 2$.

$\omega_c = 2\pi \times 10^6 \Rightarrow f_c = 10^6$.

$\omega_m = 2\pi \times 10^4 \Rightarrow f_m = 10^4$

①



② $BW = 970$ to 1030 kHz $= 60$ kHz

③ $J_0^2(2) + 2[J_1^2(2) + J_2^2(2) + \ldots + J_k^2(2)] \geq 0.95$

$= \underbrace{0.0502 + 0.666}_{\simeq 0.7} + 2[J_2^2(2) + \ldots] \geq 0.95$

$\simeq 0.7$, so take next term also,

$= \underbrace{0.0502 + 0.666 + 0.249}_{0.965} + 2[J_3^2(2) + \ldots] \geq 0.95$

$0.965 \rightarrow \geq 0.95$ ✓

So, $k = 2$.

So, BW within 95% of total FM power

$= 2kf_m$

$= 2(2)\times10^4 = 40\,kHz$

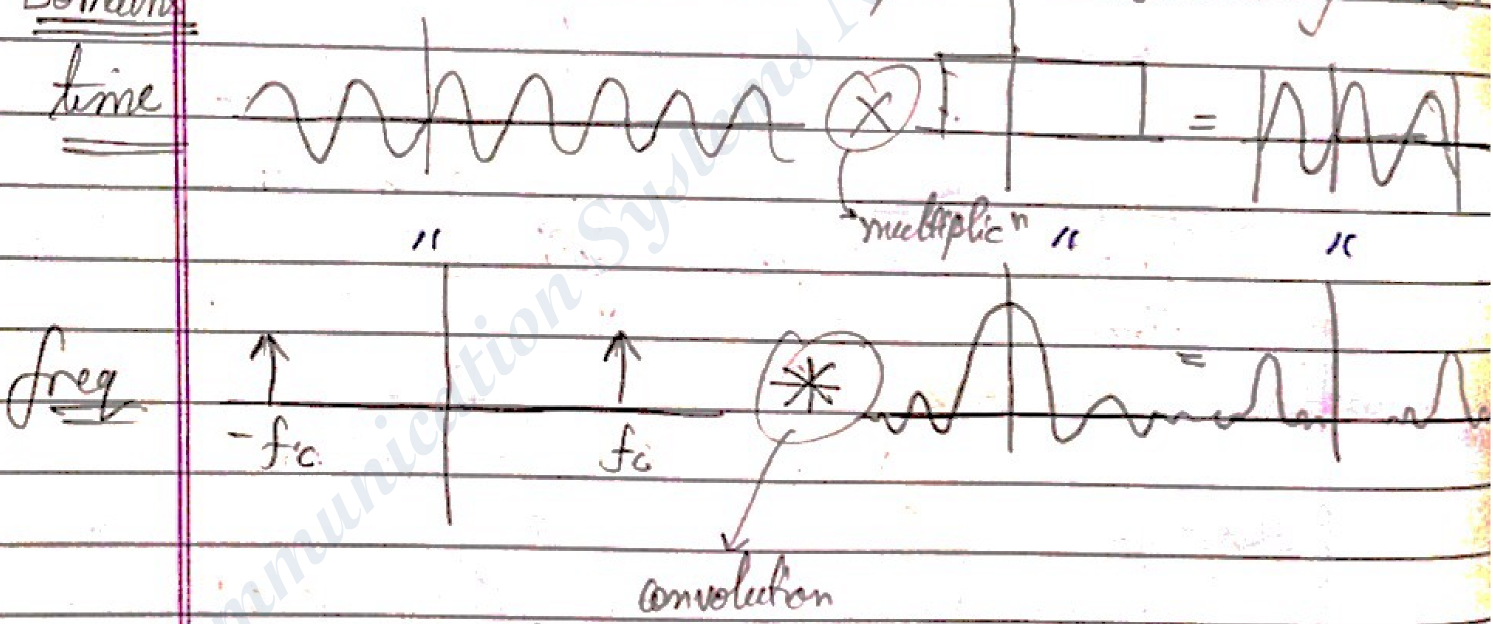
hence,

By Carson's (98% of Power) : BW = 80 kHz

(95% of Power) : BW = 40 kHz

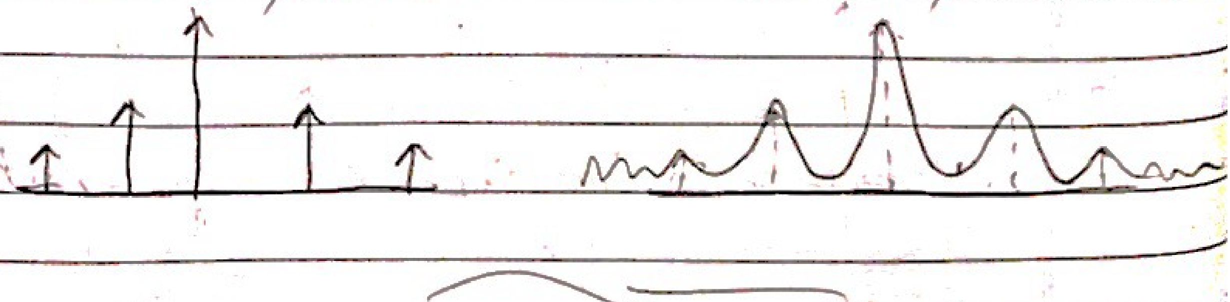
Note :- Mathematically, sine/cosine waves are infinite. So, practically, in lab, we use a window f^n so as to reduce the spectrum. Something like :

Domain

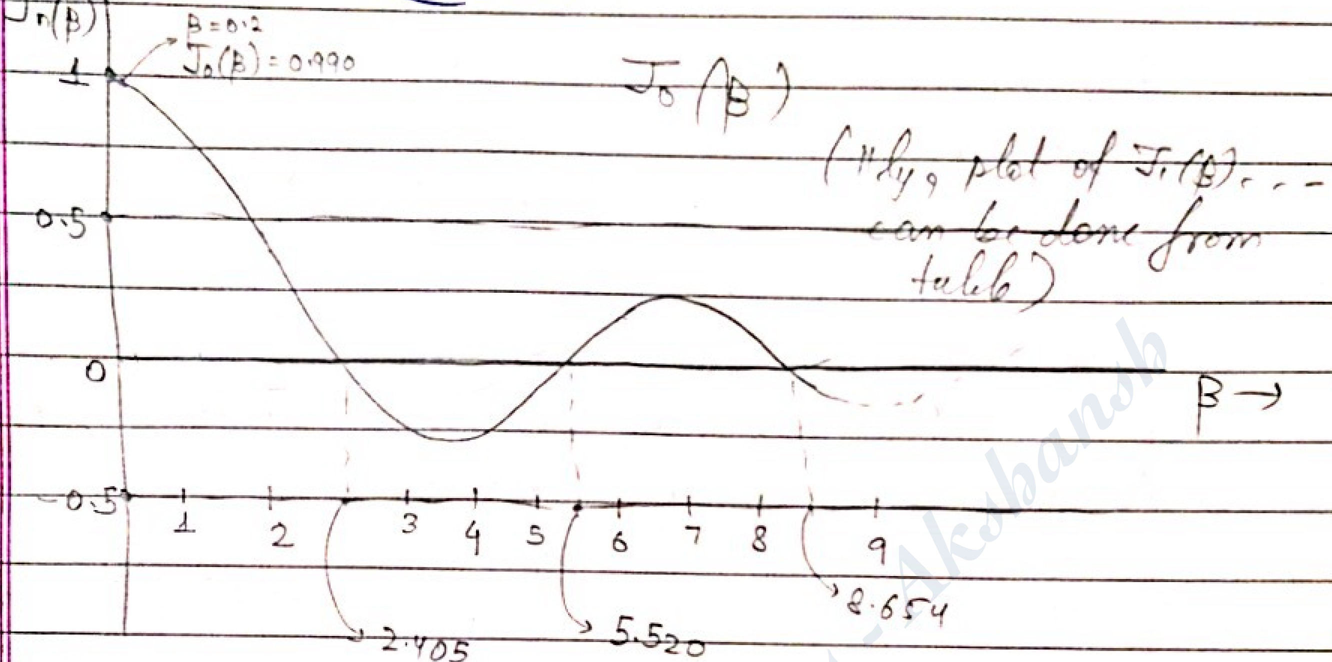


Basically, for any FM signal,

theoretical spectrum   actual spectrum



PLOTTING BESSEL'S f^n



Idea: We want carrier freq. to be lowest & other sidebands with high freq. to improve Transmission efficiency.

That can be done :-

$\exists$ certain β for which $\exists$ no carrier component. So, as in plot of $J_0(\beta)$ above, $\beta = 2.405$, 5.520 are values with zero carrier component.

Q In order to obtain the k_f (freq. deviation constt) of an FM generator, the amplitude (A_m) of a single tone modulating signal of freq. 1 kHz was slowly increased. It was observed by using a spectrum analyser, that the carrier component of the FM signal became zero for the first time, when $A_m = 1V$. What is the value of k_f. If we continue to increase the modulating signal amplitude, for what value of AM will the carrier component reduce to zero for second time?

We have :-

$$\underset{rad/s}{\omega_i(t)} = \omega_c + \underbrace{k_f}_{rad/sec\text{-}volt} \underbrace{m(t)}_{volt}$$

or $f_i(t) = f_c + \underbrace{k_f}_{Hz/volt} m(t)$.

In above problem, we are controlling A_m. we know

$$\beta = \frac{\Delta\omega}{\omega_m} = \frac{k_f A_m}{\omega_m}$$

, hence, we are controlling β.

Initially, $A_m = 0 \Rightarrow \beta = 0$,

Now, carrier component became zero for first time,

$\Rightarrow \beta = 2.405$ → taken from prev. graph for $J_0(\beta)$

$k_f = 2.405 \times 2\pi \times 1000$.

$\Rightarrow k_f = 4810\pi$ rad/sec-V.

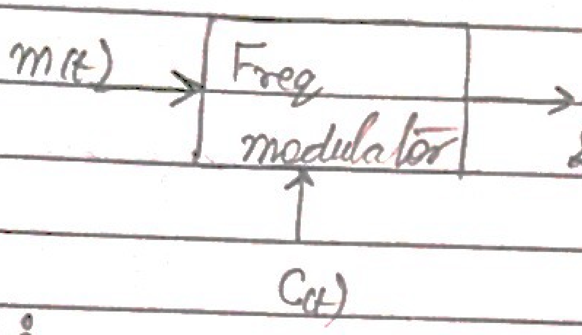
Now, for carrier component to become zero for second time, $\beta = 5.520$.

$\Rightarrow$

$$5.520 = \frac{k_f A_m}{\omega_m}$$

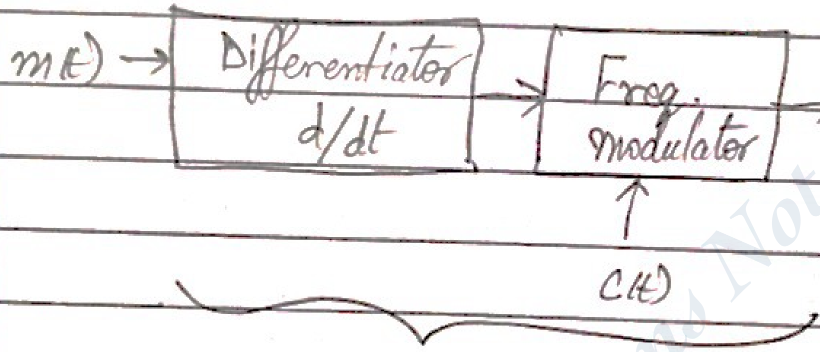
$$\Rightarrow A_m = \frac{5.520 \times \omega_m}{4810\pi} = \frac{5.520 \times 2\pi \times 1\,KHz}{4810\pi}.$$

* General note: how FM & PM can be derived from one another.



$$s_{FM}(t) = A_c \cos[\omega_c t + k_f \int m(t)\,dt]$$

Suppose:



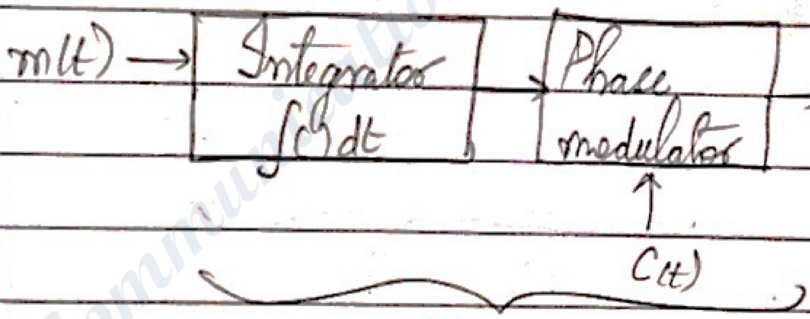
$$s(t) = A_c \cos[\omega_c t + k_p \int \frac{d}{dt} m(t)\,dt]$$

$$= A_c \cos[\omega_c t + k_p m(t)]$$

$$= s_{PM}(t)$$

PM generator.

||ly,



$$s(t) = A_c \cos[\omega_c t + k_f \int m(t)\,dt]$$

$$= s_{FM}(t)$$

FM generator.

Date
Page

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So, suppose ∃ a station 101.2 MHz

180 kHz ⇒ 0.18 MHz

So, there cannot be any station b/w

$101.2 + 0.18$ &

$101.2 - 0.18$ MHz

for safety, we take $\pm .2$ MHz

Q. Given the single tone FM signal :

$$s_{FM}(t) = 20\cos[(2\pi\times10^6 t) + 2\sin(2\pi\times10^4 t)]$$

① Sketch the FM spectrum for the carrier & the first three sideband terms.

② What is the BW using Carson's rule ?

③ What is the BW within which 95% of Total FM power is contained

Given :-

$J_0(2) = 0.224$ $\quad$ $J_3(2) = 0.129$

$J_1(2) = 0.577$

$J_2(2) = 0.353$

From formula of $s_{FM}(t)$, we find,

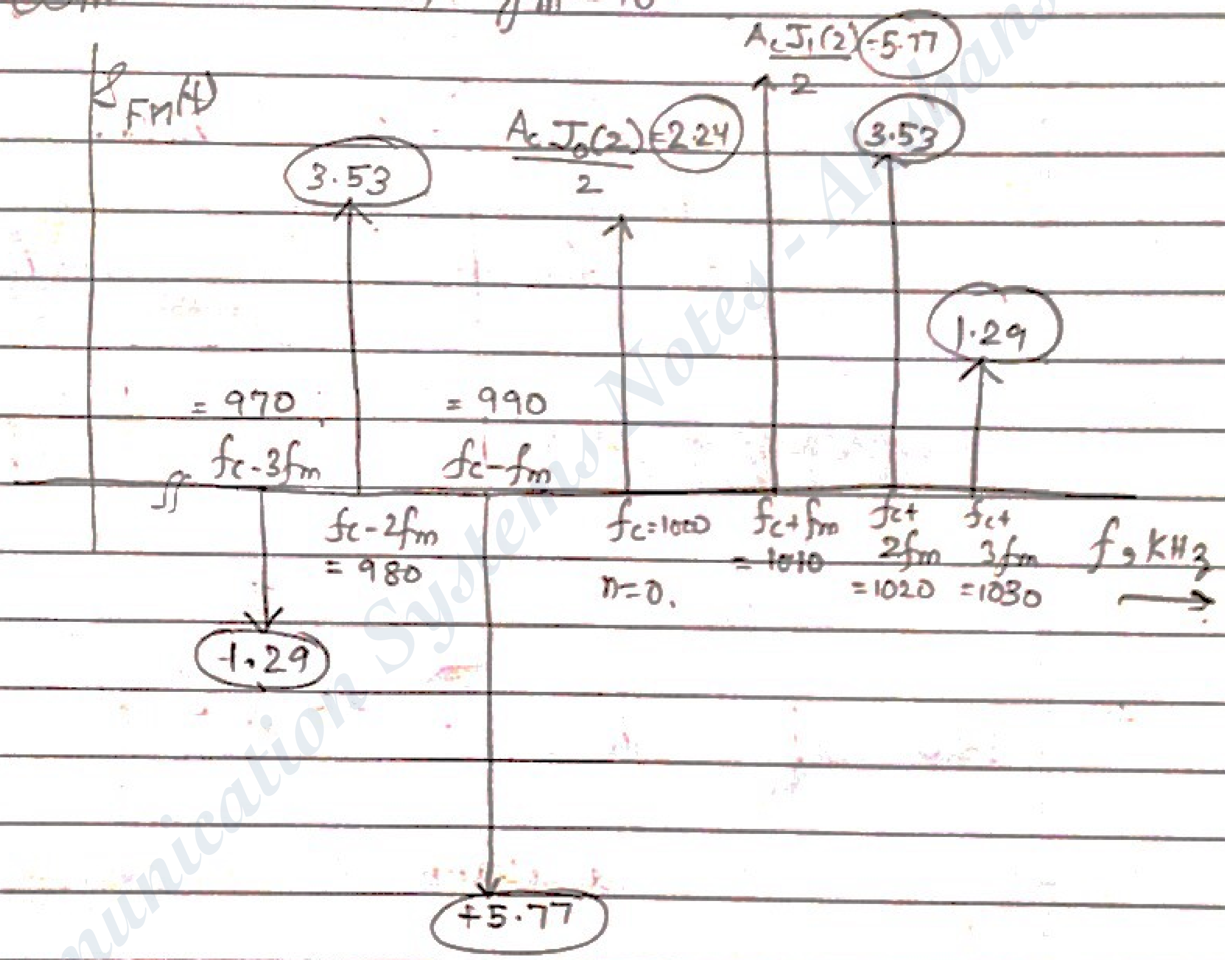
$A_c = 20\,V$

$\beta = 2.$

$\omega_c = 2\pi \times 10^6 \Rightarrow f_c = 10^6.$

$\omega_m = 2\pi \times 10^4 \Rightarrow f_m = 10^4$

(1)



(2) $BW = 970$ to 1030 kHz $= 60$ kHz

(3) $J_0^2(2) + 2[J_1^2(2) + J_2^2(2) + \dots + J_k^2(2)] \geq 0.95$

$= \underbrace{0.0502 + 0.666}_{\simeq 0.7} + 2[J_2^2(2) + \dots] \geq 0.95$

$\simeq 0.7$, so take next term also,

$= \underbrace{0.0502 + 0.666 + 0.249}_{0.965} + 2[J_3^2(2) + \dots] \geq 0.95$

$0.965 \rightarrow \geq 0.95$ ✓

So, $k = 2$.

So, BW within 95% of total FM power

$= 2 k f_m$

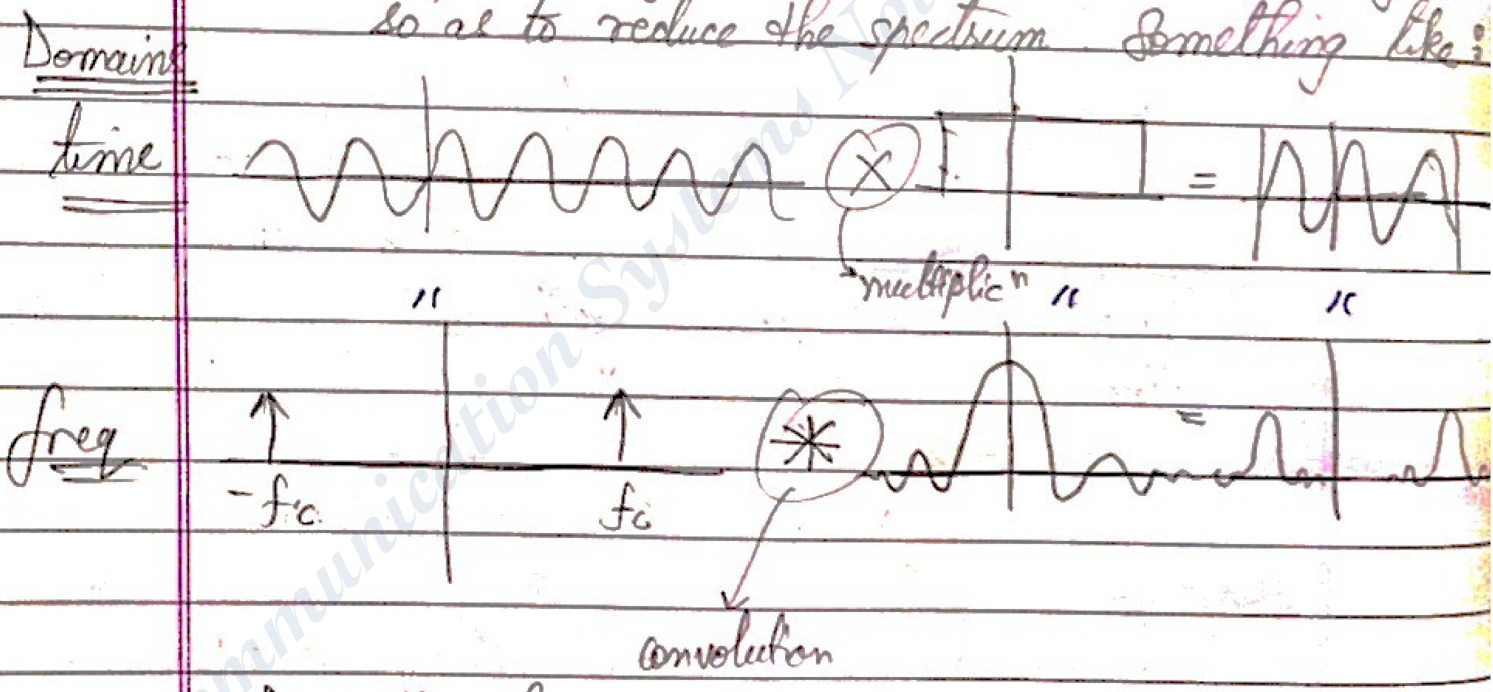
$= 2(2)\times10^4 = 40\,kHz$

hence,

By Carson's (98% of Power) : BW = 80 kHz

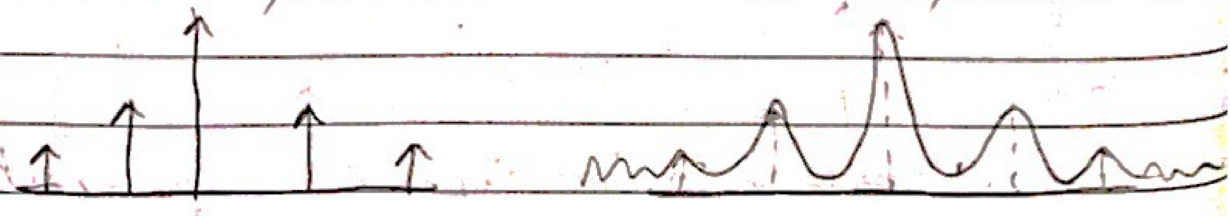
(95% of Power) : BW = 40 kHz

Note :- Mathematically, sine/cosine waves are infinite. So, practically, in lab, we use a window fnⁿ so as to reduce the spectrum. Something like :

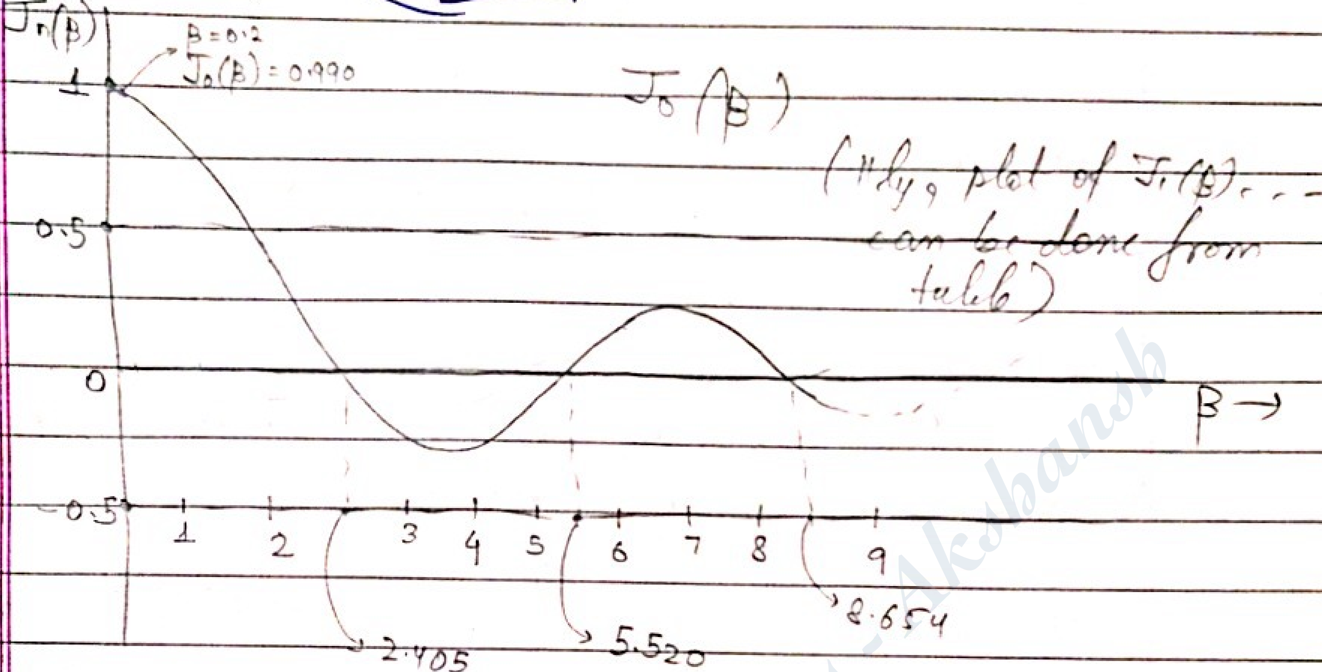


Basically, for any FM signal,

theoretical spectrum   actual spectrum



PLOTTING BESSEL'S f^n



(Similarly, plot of $J_1(\beta)$.... can be done from table)

Idea: We want carrier freq. to be lowest & other sidebands with high freq. to improve Transmission efficiency.
That can be done :-
$\exists$ certain β for which $\exists$ no carrier component. So, as in plot of $J_0(\beta)$ above, $\beta = 2.405, 5.520$ are values with zero carrier component.

Q. In order to obtain the k_f (freq. deviation constt) of an FM generator, the amplitude (A_m) of a single tone modulating signal of freq. 1 kHz was slowly increased. It was observed by using a spectrum analyser, that the carrier component of the FM signal became zero for the first time, when $A_m = 1\,V$. What is the value of k_f. If we continue to increase the modulating signal amplitude, for what value of AM will the carrier component reduce to zero for second time?

We have :-

$$\underset{rad/s}{\omega_i(t)} = \omega_c + \underbrace{k_f}_{rad/sec\text{-}volt} \underbrace{m(t)}_{volt}$$

or $f_i(t) = f_c + \underbrace{k_f}_{Hz/volt} m(t)$.

In above problem, we are controlling A_m.
we know

$$\beta = \frac{\Delta\omega}{\omega_m} = \frac{k_f A_m}{\omega_m}$$, hence, we are controlling β.

Initially, $A_m = 0 \Rightarrow \beta = 0$,

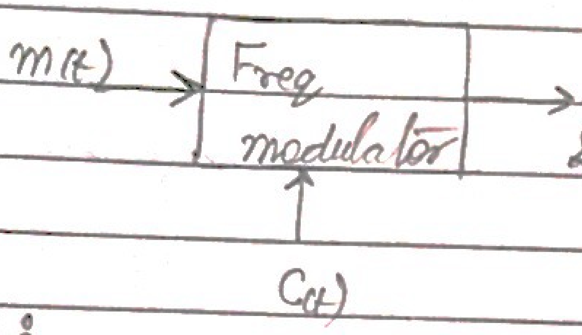
Now, carrier component became zero for first time,
$\Rightarrow \beta = 2.405$ → taken from prev. graph for $J_0(\beta)$
$k_f = 2.405 \times 2\pi \times 1000$.
$\Rightarrow k_f = 4810\pi$ rad/sec-V.

Now, for carrier component to become zero for second time, $\beta = 5.520$.

$$\Rightarrow 5.520 = \frac{k_f A_m}{\omega_m}$$

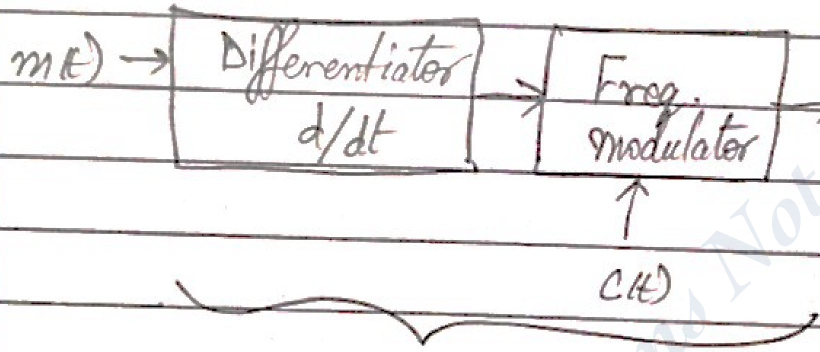
$$\Rightarrow A_m = \frac{5.520 \times \omega_m}{4810\pi} = \frac{5.520 \times 2\pi \times 1\,KHz}{4810\pi}$$

✲ General note: how FM & PM can be derived from one another.



$$s_{FM}(t) = A_c \cos[\omega_c t + k_f \int m(t)\,dt]$$

Suppose:



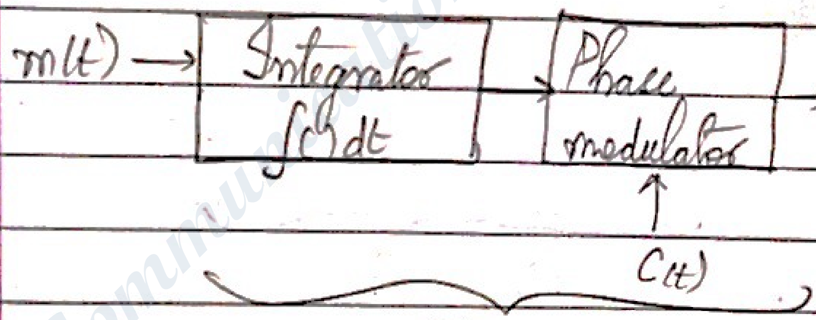
PM generator.

$$s(t) = A_c \cos\left[\omega_c t + k_p \int \frac{d}{dt} m(t)\,dt\right]$$

$$= A_c \cos[\omega_c t + k_p m(t)]$$

$$= s_{PM}(t)$$

||ly,



FM generator.

$$s(t) = A_c \cos[\omega_c t + k_f \int m(t)\,dt]$$

$$= s_{FM}(t)$$

GENERATION OF ANGLE-MODULATED SIGNALS

(1) Narrow Band FM (NBFM)

$$s_{FM}(t) = A_c \cos[\underbrace{\omega_c t}_{A} + \underbrace{k_f \int m(t)\,dt}_{B}]$$

$$= A_c(\cos\omega_c t)(\cos k_f \int m(t)\,dt) - A_c \sin(\omega_c t)\sin(k_f \int m(t)\,dt)$$

For an FM to be NBFM or WBFM, it depends on $k_f \int m(t)\,dt$.

If $\underbrace{k_f \int m(t)\,dt}_{\theta,\ say} \ll \frac{\pi}{2}$, its NBFM.

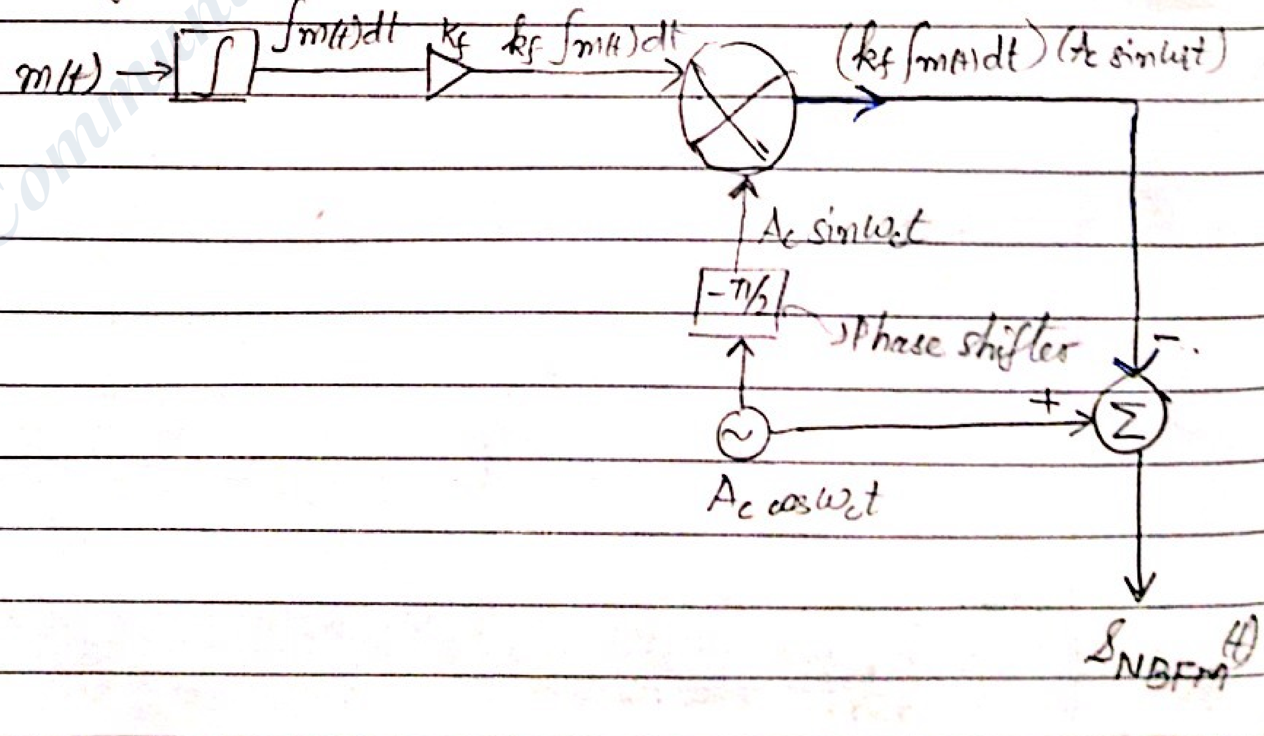
So, $\cos\theta \simeq 1$ & $\sin\theta \simeq \theta$

$$\Rightarrow s_{NBFM}(t) = A_c \cos\omega_c t\,(1) - A_c \sin\omega_c t\,(\theta)$$

$$\Rightarrow s_{NBFM}(t) = A_c \cos\omega_c t - (A_c k_f \int m(t)\,dt)\sin\omega_c t$$

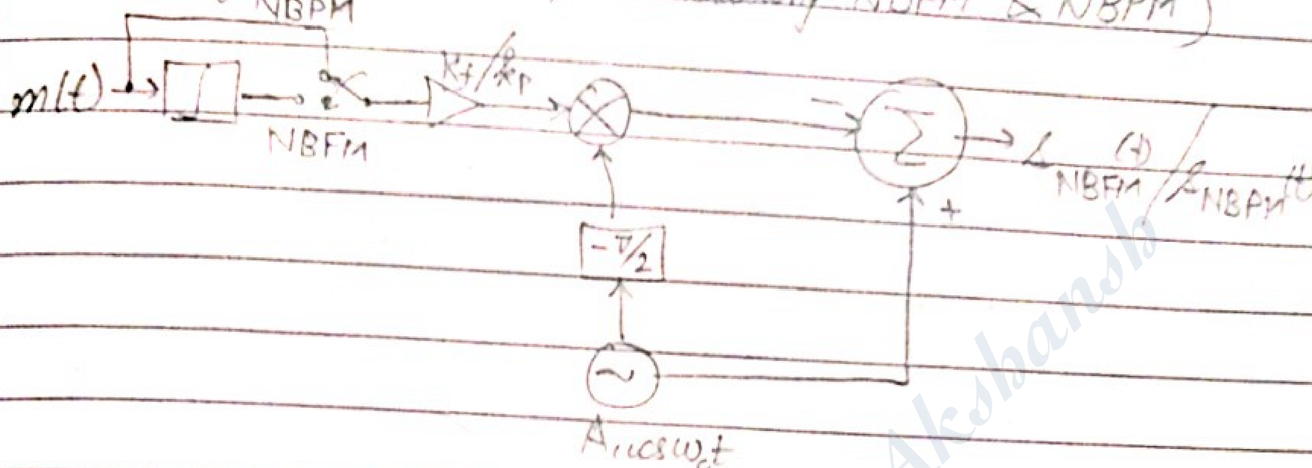
$$= A_c \cos\omega_c t - (k_f \int m(t)\,dt)(A_c \sin\omega_c t)$$

Now, generating this :

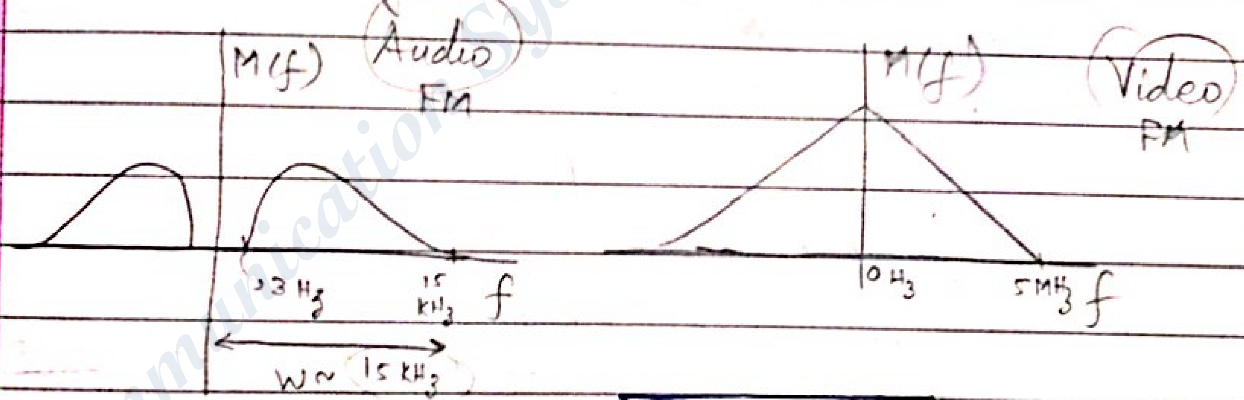


Ily, for generating NBPM, remove the integrator, or bypass it.

Something like : (after including NBFM & NBPM)

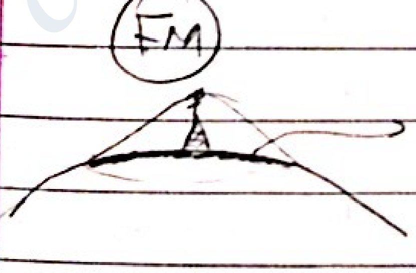


Note : Any voice/music signal starts from 30 Hz (never from 0) (for commercial FM broadcasting)
Picture signal starts from 0.



$\boxed{\Delta f = 75\ kHz}$ → Std. value.

FM



→ for an antenna, by line of sight, only freq. with one particular range are seen in any area.

Usually, f_c is b/w 88 – 108 MHz.

Now, By Carson's rule (98% power),

$BW \simeq 2(\Delta f + W)$

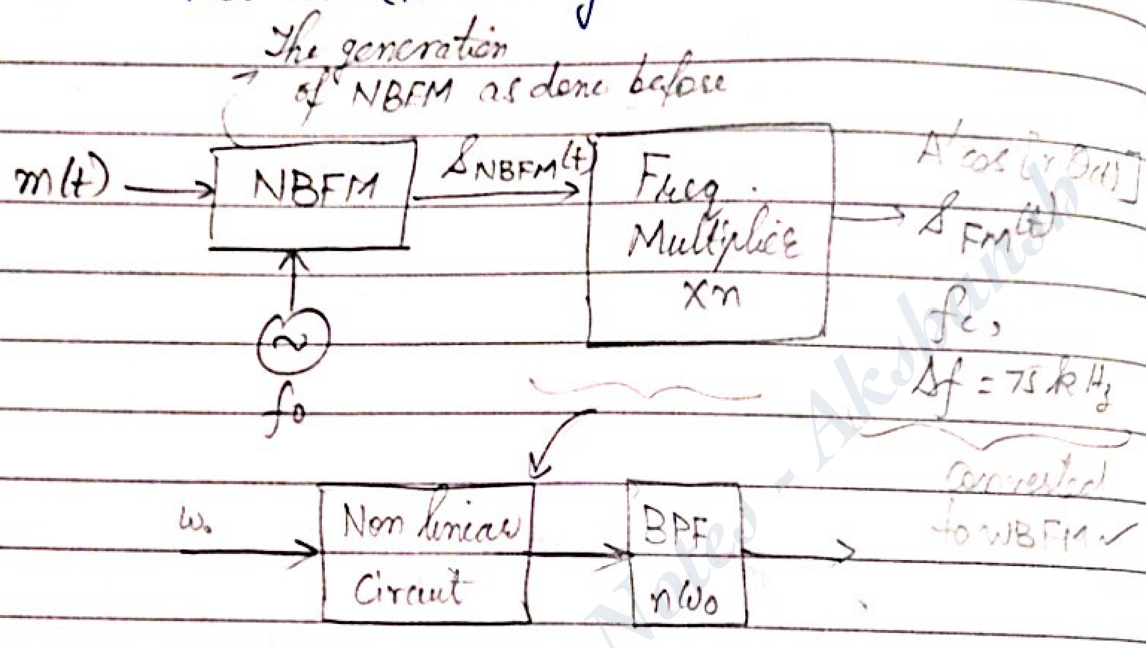
$\simeq 2(75 + 15)\ kHz = 180\ kHz \simeq 200\ kHz$

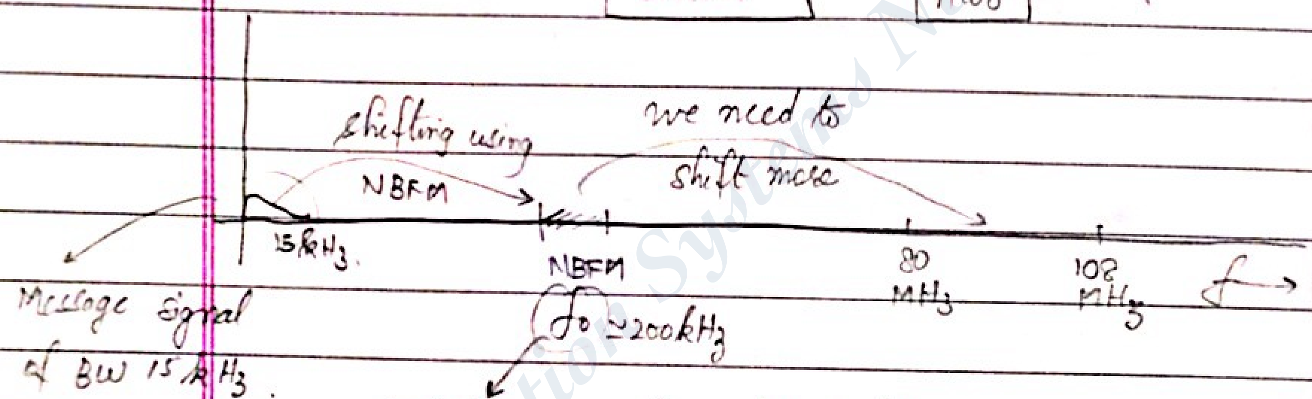
88 MHz — 108 MHz.

$\frac{20\ MHz}{200\ kHz} = 100$ stations

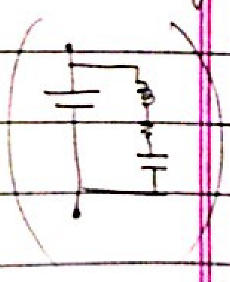
✱ FM Generation

(M1) Indirect method (Armstrong's method)





initially generating signal at a very stable freq (f_0)
Stable: using Crystal Oscillators — using Pizo electric crystal



So, freq $\simeq 200\,kHz$ is stable & signal can be oscillated in this freq.

Its $BW = 2 \times (\Delta M)$
$= 2 \times (15\,kHz)$
$= 30\,kHz$

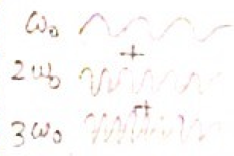
We want:
→ Convert to WBFM with $\beta >> 1$ & $BW \simeq 200\,kHz$

how? : Use Frequency multiplier

In amplitude model^n,
$A\cos\theta(t) \rightarrow kA\cos\theta(t)$
$k >> 1$

How will Freq multiplier work?

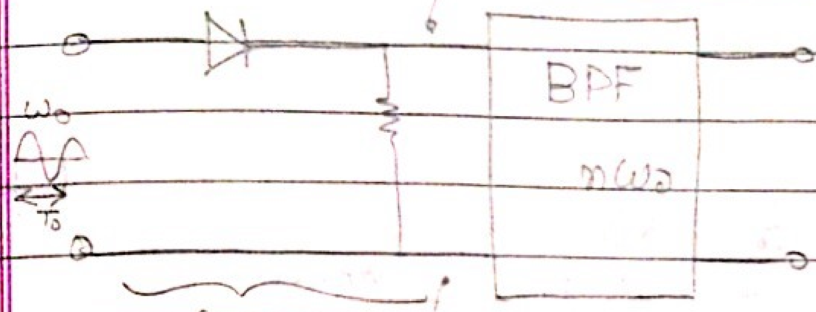
$A\cos\theta(t) \rightarrow A'\cos[n\theta(t)]$



A periodic signal
Combinⁿ of diff. sine/cosine waves

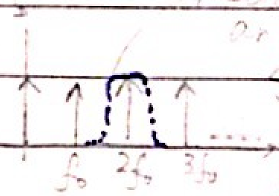
Puffin
Date
Page

eg



half wave rectifier

✓ only one sine/cosine wave.
✓ has only one freq component



→ Selecting on o/p: BPF
If $2f_0$ is selected: It's frequency doubler.
If $3f_0$ is selected: It's freq tripler
(only freq. doubler & tripler are possible, not higher)
So higher freq. multiplicⁿ is reqd,
CASCADE.

$2f_0 \longrightarrow \boxed{\times 2} \longrightarrow 2^2 \longrightarrow \boxed{\times 2} \longrightarrow 2^3 \ldots$

* Effects of freq. multiplicⁿ on an FM signal.

Consider single tone FM :-

Initially — $s(t) = A_c' \cos[\omega_0 t + \beta' \sin \omega_m t] \longrightarrow (1)$

let $\beta' < 1 \Rightarrow$ NBFM

$\downarrow$

$\boxed{\text{Freq} \times n}$

$s_{FM}(t) = A_c \cos[n\omega_0 t + n\beta' \sin \omega_m t]$

after freq. multiplicⁿ — $s_{FM}(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$

Observations from (1) & (2)

$\beta' \rightarrow \beta = n\beta'$

$\omega_0 \rightarrow \omega_c = n\omega_0$

$\omega_m \rightarrow \omega_m$ (unaffected)

$\Delta\omega' \rightarrow \Delta\omega = n\Delta\omega'$ (let $\beta' = \dfrac{\Delta\omega'}{\omega_m}$. So, $n\beta' = \dfrac{\Delta\omega}{\omega_m} \Rightarrow \Delta\omega = n\Delta\omega'$)

Consider a band of freq

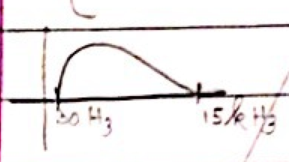
let $m(t)$ be i/p signal with BW = 15 kHz

→ using highly stable freq. of carrier generated using crystal oscillator

$f_0 = 200\,kHz$

$\longrightarrow$ NBFM $\longrightarrow$ $\boxed{\text{Freq.} \times n}$ $\xrightarrow{\text{WBFM}}$

$\boxed{\beta'_{max} = 0.5}$



we decide that for keeping in NBFM, we won't exceed $\beta = 0.5$.

(We know

$\beta = \dfrac{\Delta f}{f_m}$

So, $\beta \propto \dfrac{1}{f_m}$

So, β_{max} depends on $f_{m_{min}}$)

If $\beta'_{max} = 0.5$,

$f_{m_{min}} = 30\,Hz$

So, $\Delta f' = (0.5)(30)$

$\Delta f' = 15\,Hz$

$\Delta \omega = n \Delta \omega'$

$\Rightarrow n = \dfrac{\Delta \omega}{\Delta \omega'}$ or $\dfrac{\Delta f}{\Delta f'}$

$= \dfrac{75\,kHz}{15\,Hz}$ (for commercial FM broadcasting)

$n = 5000$.

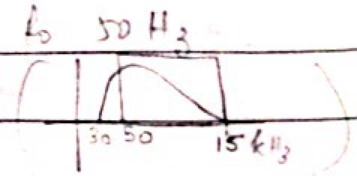
(So, a multiplic^r factor of 5000 is req^d)

Now, how to reduce the value of n?

Value of β_{max} can be increased (has limit)

So, increase f_m. (⇒ some degradation in quality)

So, instead of $f_{min} = 30\,Hz$, change it to $50\,Hz$



Now, $\Delta f' = (0.5)(50) = 25$

$\Rightarrow n = \dfrac{75000}{25} = 3000$

So, reduced ✓

So, now $\Delta f = n \Delta f' = 75\,kHz$ (as desired)

$\beta_{max} = n \beta'_{max} = 1500 \Rightarrow$ WBFM

$f_c = n f_0 = 600\,MHz$ ✗
↳ (very large value)

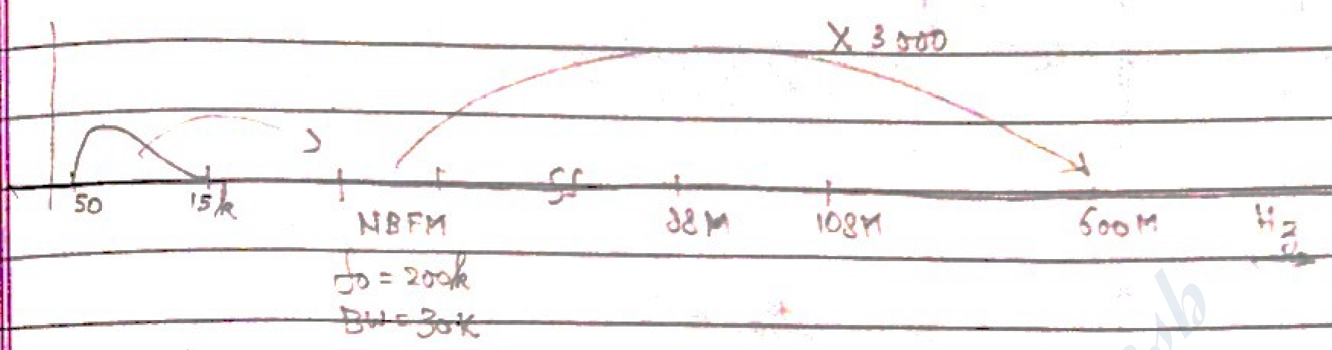
It should have been b/w 88 – 108 MHz

FM : 100% modulation ⇒ $\Delta f = 75\ kHz$

80% modul^n ⇒ $\Delta f = 80\% \times 75k = 50k$ Hz

50% " ⇒ $\Delta f = 50\% \times 75k = 37.5\ kHz$

So, what we did :-



So, idea: Instead of ×3000 (×n), we divide n into n_1 & n_2 say $n_1 = 50$, $n_2 = 60$, s.t

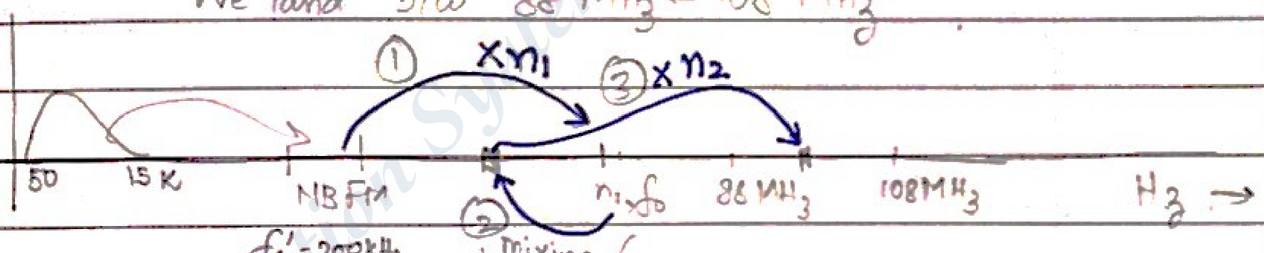
$n = n_1 \times n_2$ ✓

Now, 1. use $\times n_1$

2. go back (MIXING): for freq translation

3. use $\times n_2$.

We land b/w 88 MHz – 108 MHz



$f_c' = 200\ kHz$

$\beta'_{max} = 0.5$

$\Delta f' = 25\ Hz$

BW = 30 kHz

$f_c'' = n_1 f_c'$

$\beta''_{max} = n_1 \beta'_{max}$

$\Delta f'' = n_1 \Delta f'$

WBFM

BW

diff

f_c'''

β''_{max}

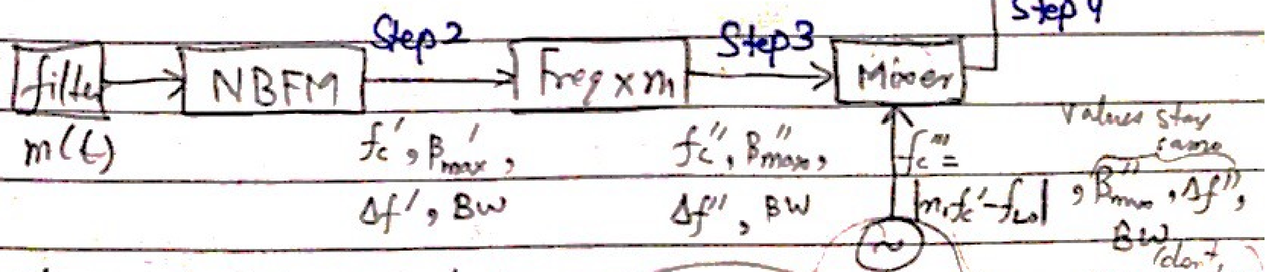
$\Delta f''$

WBFM

BW

same

So,



For practical purposes, dont use this. Choose another multiplier ($\times n_3$) s.t $n_3 \times 45$ gives of req.

✲ Choosing apt. multipliers.

Suppose we want ×3000 freq. multiplicⁿ

Idea: Implementⁿ can only be done in doublers & triplers (i.e. multiples of 2 & 3 only)

So, $n = n_1 \times n_2$.

to start, let $n_1 = n_2$

So, $n_1 = n_2 = \sqrt{3000} \simeq 57$

This is not a multiple of 2 or 3.

So, let $n_1 \uparrow$ & $n_2 \downarrow$.

$n_1 \uparrow$. So, $57 \longrightarrow$ next 64 is a multiple

So, $n_2 = \frac{3000}{64} \simeq 46.87$

$46.87 \longrightarrow$ changing up/down $\simeq 48$ — multiple ✓

So, $n_1 \times n_2 = 3072$ (cant always get exact value)
(64×48)

Now,

In step 2

$B''_{max} = 64 \times (0.5) = 32$

$f_c'' = n_1 f_c' = 12.8\ MHz$

$\Delta f'' = n_1 (25) \simeq 1.6\ kHz$

Step 3

$B'''_{max} = 32$

$\Delta f''' = 1.6\ kHz$.

$f_c''' = 2\ MHz$

(b) to get this, we should have $|12.8 - f_L| = 2$.

So, $f_{LO} = 10.8 / 14.8$

~~Take 10.8 MHz~~

Using $f_c' = 2\ kHz$, So, $n_3 = \frac{108}{2} = 54$

Choosing f_c b/w 88-108 = 96, say (in multiples)

So, $96/48 = 2\ MHz$ for b3)

(a)

Step 4.

$B_{max} = 32 \times 48$

$f_c = 96\ MHz$
(2×48)

$\Delta f = 1.6 \times 48\ kHz$

working back

* PLL detection : mostly used both for AM & FM detection in modern receivers

* We got $\Delta f = 1.6 \times 48$ in step 4 $\rightarrow BW = 200\ kHz$

For 100% modulⁿ in FM, $\Delta f = 75\ kHz$

Now, if we take 80% modulⁿ,

$$\Delta f = 0.8 \times 75 = 60\ kHz$$

Hence, $BW = 2(\Delta f + W)$

$= 2(60 + 15)$

$= 150\ kHz$

hence, BW reduced from 200 to 150 kHz

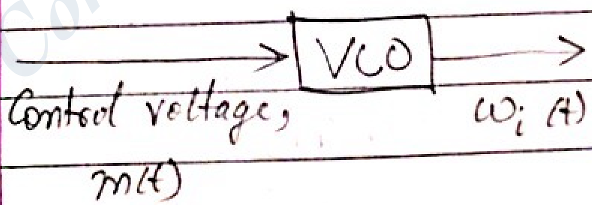
We got

$\Delta f = 1.6 \times 48$

If this is $> 75\ kHz$, adjust values.

(M2) Direct Method

↳ use of VCO (Voltage Controlled Oscillator)



$\omega_i(t) = \omega_c + k_f\, m(t)$

✱ DETECTION of ~~AM~~ Angle Modulated Signal.

$$s_{FM}(t) = A_c \cos[\underbrace{\omega_c t + k_f \int m(t)dt}_{\theta(t)}]$$

$$s_{PM}(t) = A_c \cos[\omega_c t + k_p m(t)]$$

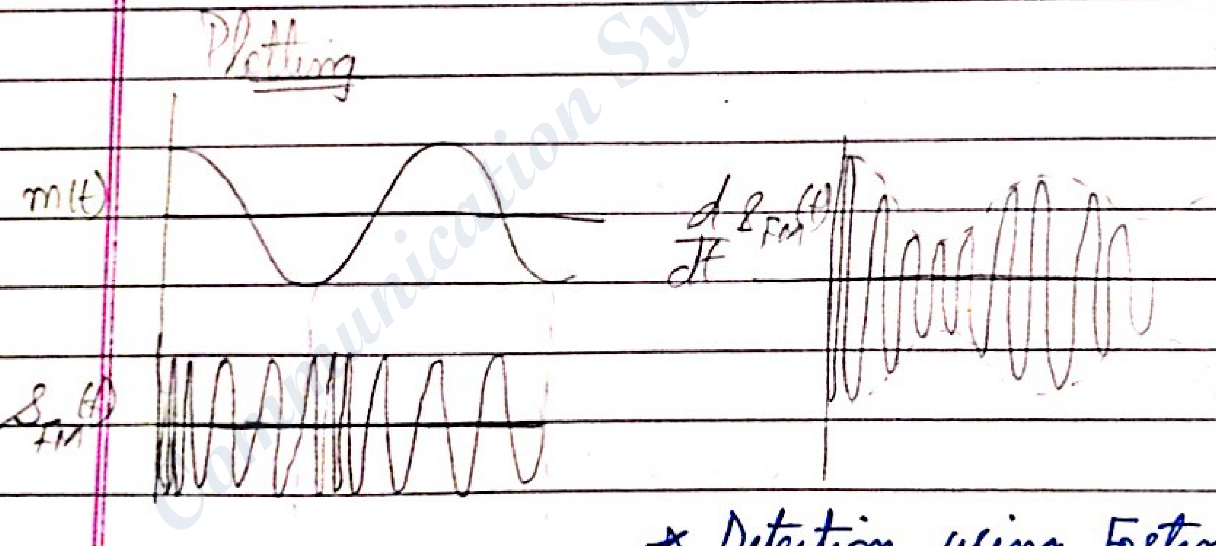
Now, $\frac{d}{dt} s_{FM}(t) = -A_c \sin\theta(t) \cdot \frac{d\theta(t)}{dt}$

$$= -A_c \sin(\omega_c t + k_f \int m(t)dt)[\omega_c + k_f m(t)]$$

$$= \underbrace{-A_c [\omega_c + k_f m(t)]}_{\text{Amplitude} \propto m(t) \Rightarrow AM} \underbrace{\sin[\omega_c t + k_f \int m(t)dt]}_{\text{Angle} \propto \int m(t) \Rightarrow FM}$$

AM — FM

Plotting



pt. for max freq

pt. for min. value of m(t). f_0, min. freq

✱ Detection using Foster Seeley detector, Ratio discriminator.

Imp Reading Assignment (can come in Quiz)

→ PLL (Phase Locked Loop) for FM/PM/AM detection.

Ch- Sampling of Signals

* MULTIPLEXING.

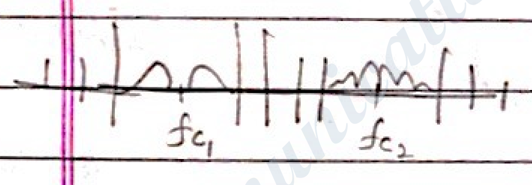
A process where 2 or more signals are sent through common channel without mutual interference & ability to separate them at receiver

FDM

Freq. Division Multiplexing

(for analog signals)

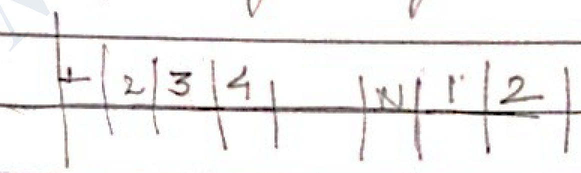
Dividing freq. axis into diff^t slots with gap b/w slots & allotting diff^t freq^s to each slot. They co-exist.



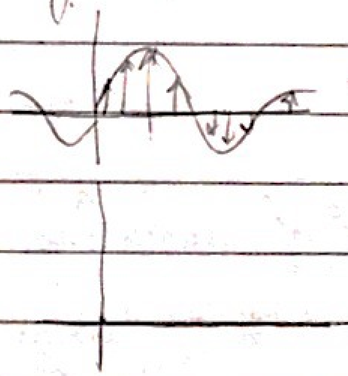
TDM

Time Division Multiplexing

(for digital signals)



Taking samples of analog signal



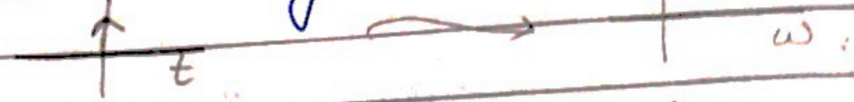
→ discrete values.

* SAMPLING Theorem:

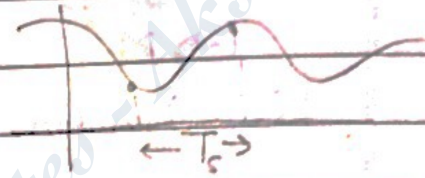
↳ for LPF

↳ given a cts time signal $m(t)$, band limited to $|f| \leq f_m$, it is possible to reconstruct the signal fully from its samples, provided that sampling rate $f_s \geq 2f_m$, or, alternatively, if sampling period, $T_s = \frac{1}{f_s} \leq \frac{1}{2f_m}$ sec.

- Any time limited signal is of infinite BW.



∀ practical signals, all signals are "essentially band limited"
(∵ All signals are time limited. So, BW → ∞. But, practically not possible. So, they have finite BW i.e., are band limited)



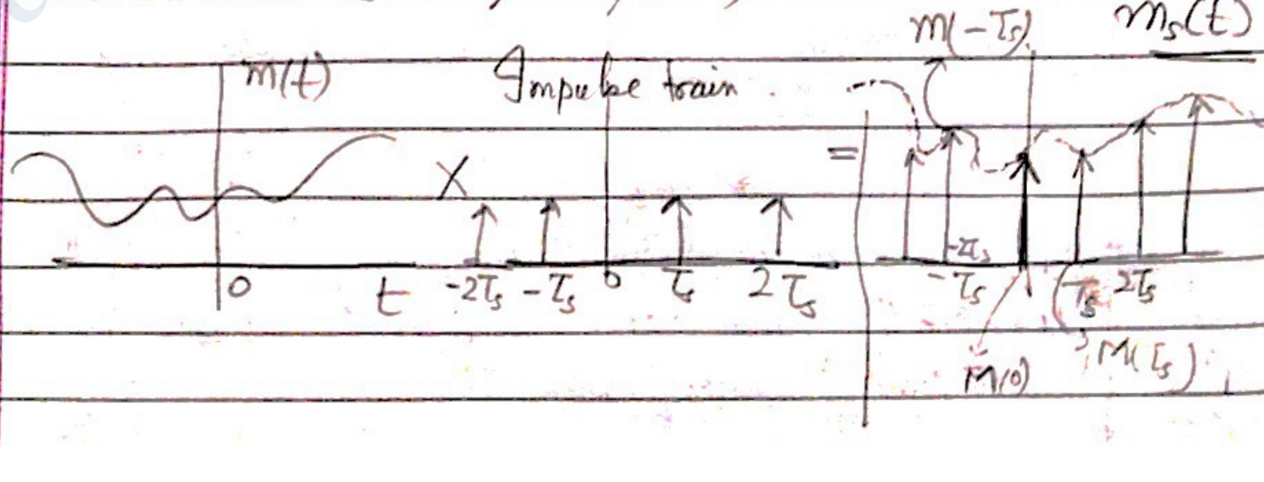
★ $T_s \left(= \frac{1}{f_s}\right) \ll \frac{1}{2 f_m}$ sec.

↳ for any signal of say $f_m = 5$ kHz

So, $f_s \geq 2 \times 5 = 10000$ samples/sec.
These should be there so that ALIASING doesn't take place.
(Another signal can be used in the free space b/w samples).

★ **proof of sampling theorem:**

Consider idealised sampling / Instt. sampling
(Ideal / Impulse)



* So, $m_\delta(t) = m(t) \times \delta_{T_s}(t) = m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$m_\delta(t)$ → train of impulses

* Periodic Signals ⇒ $FS = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_s t}$ (exponential FS)

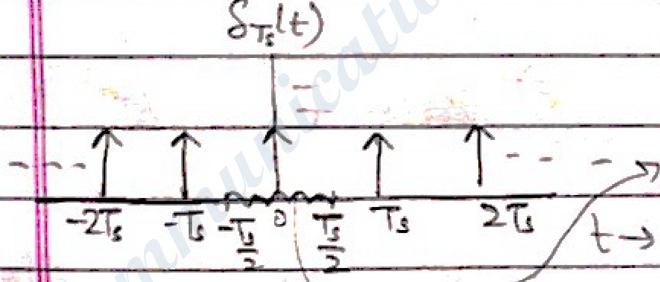
- *Instantaneous sampling*

$$m_\delta(t) = m(t) \cdot \delta_{T_s}(t)$$

$$= m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Let $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_s t}$

$$\hookrightarrow F_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta_{T_s}(t)\, e^{-jn\omega_s t}\, dt$$



$$F_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t)\, e^{-jn\omega_s t}\, dt$$

$$\int_{-\infty}^{\infty} \delta(t - 0)\, f(t)\, dt$$

$$= f(t)\Big|_{t=t_0}$$

$$\Rightarrow F_n = \frac{1}{T_s} \left[e^{-jn\omega_s t} \right]_{t=0} = \frac{1}{T_s}$$

V. Imp. Property (only these 2 fns) {
* Impulse train signal has same shape in time & freq domain
* Gaussian fn. has same shape in time & freq domain
  ↳ bell shaped curve

$$\Rightarrow m_s(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} m(t) e^{jn\omega_s t} \longrightarrow (1)$$

We are trying to prove :- we need not give complete signal, only samples. But samples should be sufficient

for $f_m = 5000$ Hz, samples should be $\geq 2f_m$

i.e ≥ 10000 samples.

Taking FT of (1)

$$\Rightarrow M_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \mathcal{F}[m(t) e^{jn\omega_s t}]$$

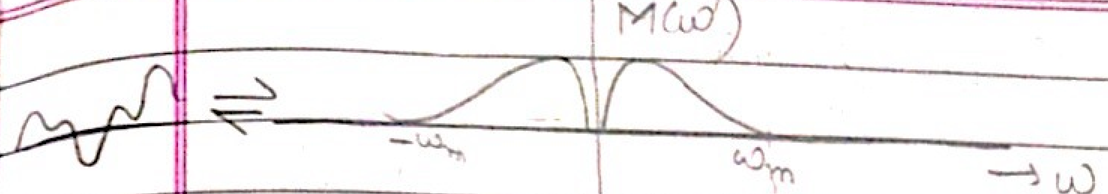
(By Freq. shift property of FT,

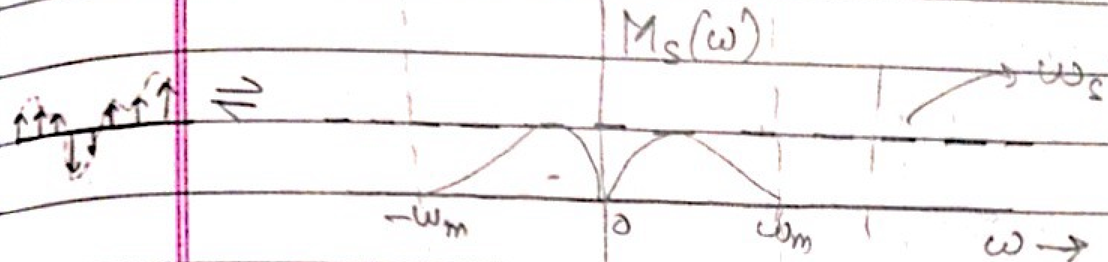
If $m(t) \underset{F^{-1}}{\overset{FT}{\rightleftharpoons}} M(\omega)$,

then, $m(t) e^{\pm j\omega_0 t} \rightleftharpoons M(\omega \mp \omega_0)$)

So, $$\boxed{M_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)}$$

↳ spectrum of a sampled signal.



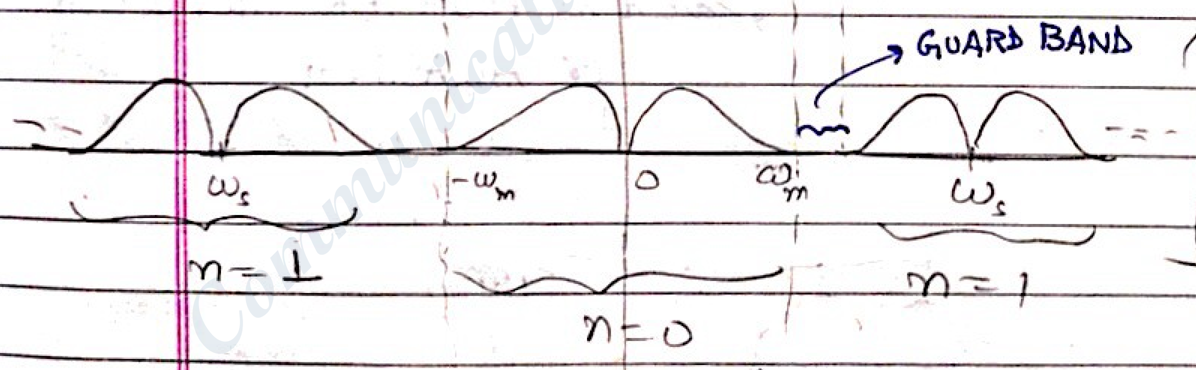


Oversampling → $\omega_s > 2\omega_m$ or $f_s > 2f_m$

$n = 0$ in $M_s(\omega)$

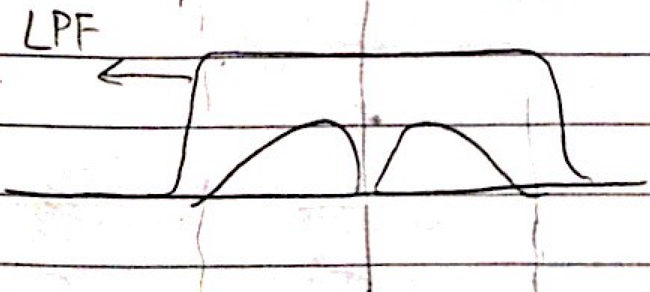
IIly $\forall$ n.

So, combining these,



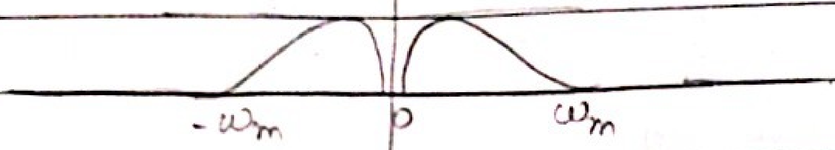
So, its a mix of infinite spectral components.

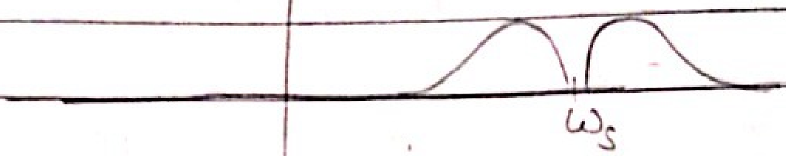
Now, put a LPF to remove single freq.



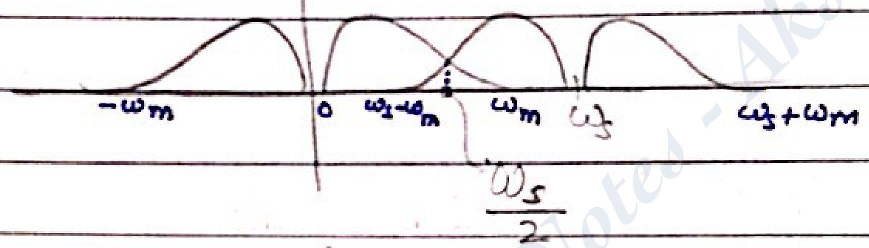
If guard band = 0, i.e, the spectrums just touch each other. So, thats when $f_s = 2f_m$.

for $f_s < 2f_m$: Undersampling

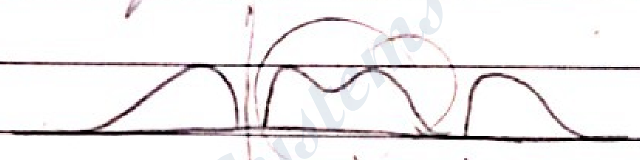




combining

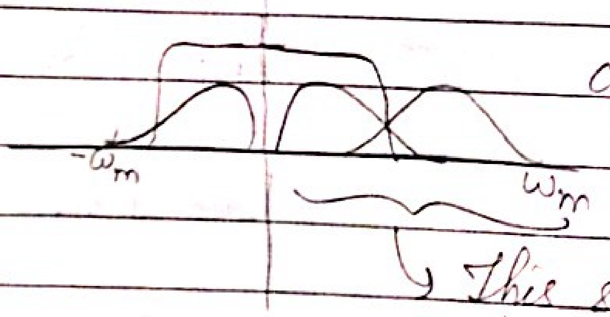


equivalent



distorted, called as FOLDOVER DISTORTION or ALIASING.

If sampling is done at $\frac{\omega_s}{2}$, NYQUIST RATE

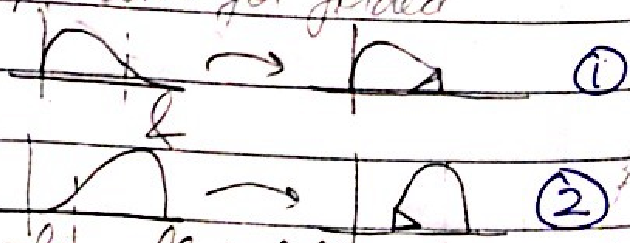


cutting done at Nyquist freq.

This seems as if the spectrums got folded

So, seeing overall, some high freq part of ① behaves/goes lower freq. ||ly for ②. This is termed as ALIASING.

like :



So, it's called foldover distortion.

* RF Digital Schemes (Radio Freq. Digital Schemes) } ASK, FSK, PSK, DPSK, QPSK

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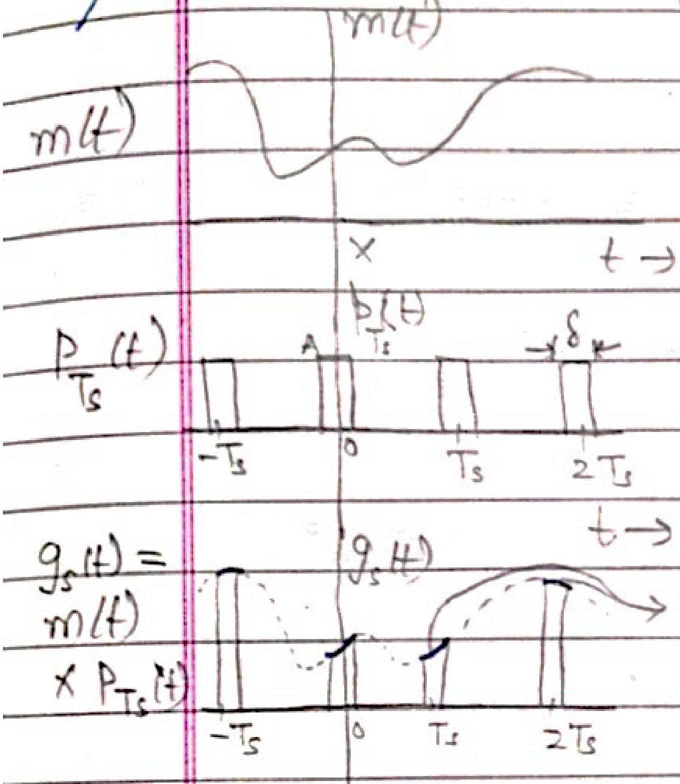
* NON-IDEAL SAMPLING {PULSE MODULATION}

(i.e, sampling using pulses (⊓ ⊓ ⊓) instead of impulses (↑ ↑ ↑))



(1) Pulse Amplitude Model^n (PAM)

$g_s(t)$

$= m(t) \cdot p_{T_s}(t)$

→ Natural Sampled
(i.e, how m(t) changes, similar change is seen at top of $g_s(t)$)

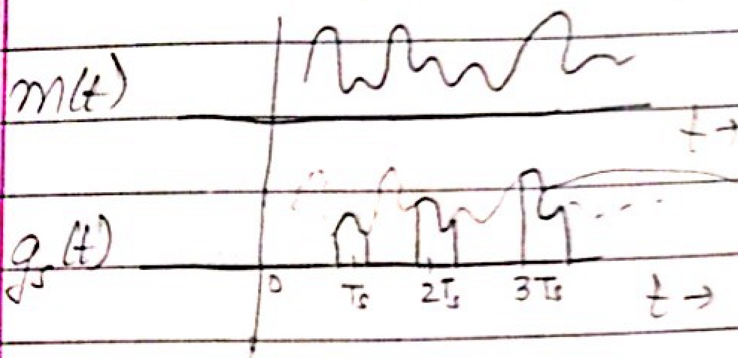
* PSK, FSK can be of diff. sizes — M-ary
 → binary (2^1)
 → quarternary (2^2)
 ⋮ (2^n)

Modulation

→ Analog (~~Continuous~~)
 → Continuous-time Model^n.
 (Carrier, $c(t) = A_c \cos(\omega_c t + \phi)$)
 • we can change its Amplitude, freq and phase
 → AM: Amplitude Model^n — Changing amplitude
 → FM, PM: Angle Model^n — Changing freq &/or phase
 → Pulse Model^n
 It has Pulse trains (for carrier) (⊓ ⊓ ⊓ ⊓)
 • we can change its amplitude, width, position
 → PAM: Pulse Amplitude Model^n
 → PWM or PDM: Pulse Width Model^n or Pulse Delay Model^n.
 → PPM: Pulse Pos^n Model^n

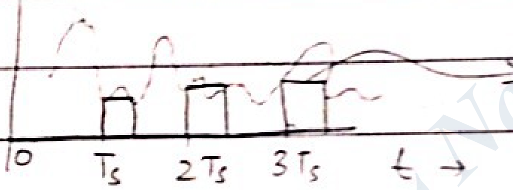
→ Digital
 → ✓ Pulse Code Model^n (PCM) & its variants (DPCM)
 → ✓ Delta Model^n.
 → low pass Spectrum.

* Natural sampled PAM signal.



→ Amplitude changes according to m(t)

Ideally, we won't want that. So, we use sampling & holding process, & we get

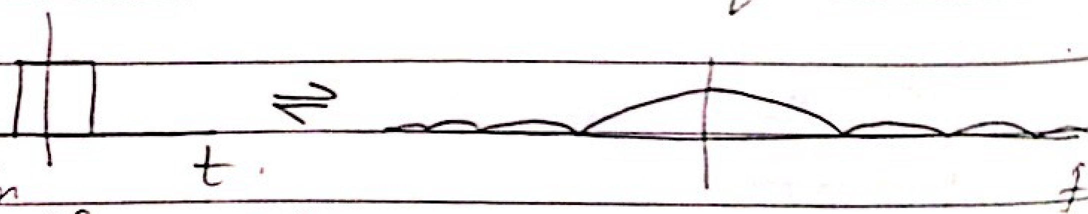


→ This gives us flat at top. This is called **Flat-top PAM**.

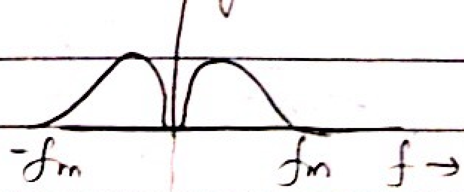
hence, PAM → Natural sampled PAM, Flat top PAM

V. Imp ★ Time Domain ⇌ Freq Domain

for a pulse in time domain

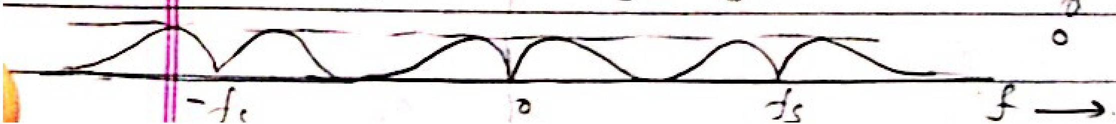


★ Consider the signal:

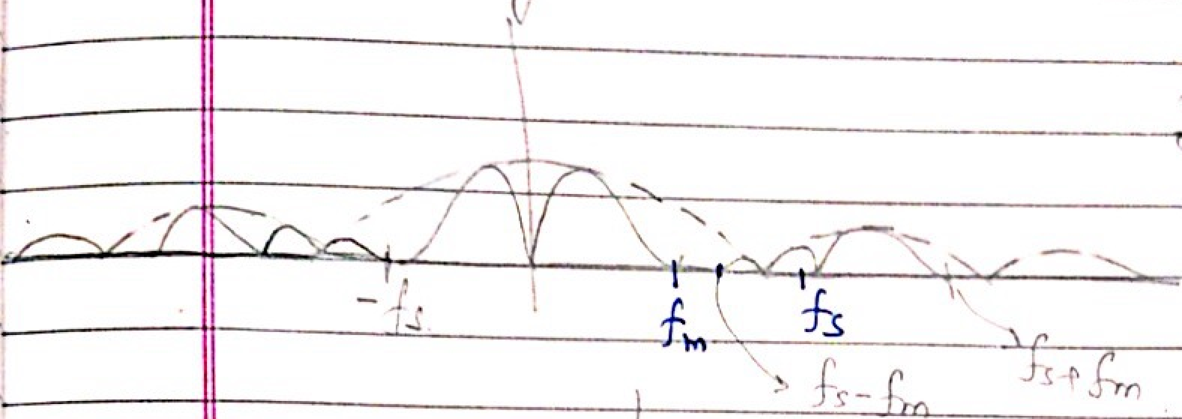


∘ M(f)

∘ $M_s(f)$



So, PAM signal will be

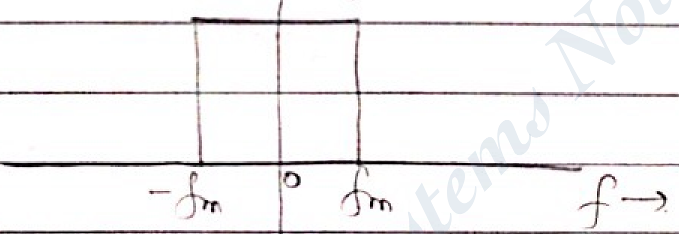


: $G_s(f)$

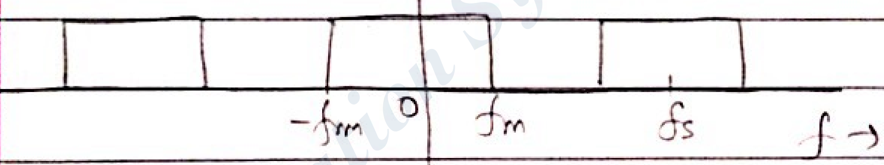
↳ again, $f_s - f_m > f_m$ or $f_s \geq 2f_m$

✓ Aliasing can occur

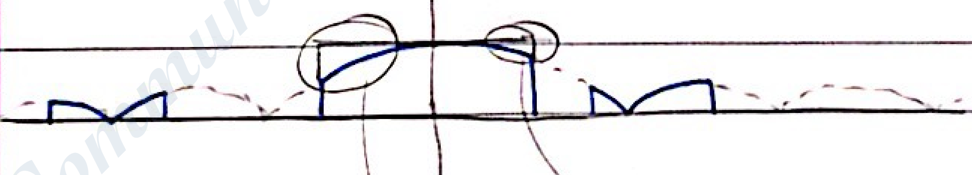
Consider another signal:



: $M(f)$



: $M_s(f)$

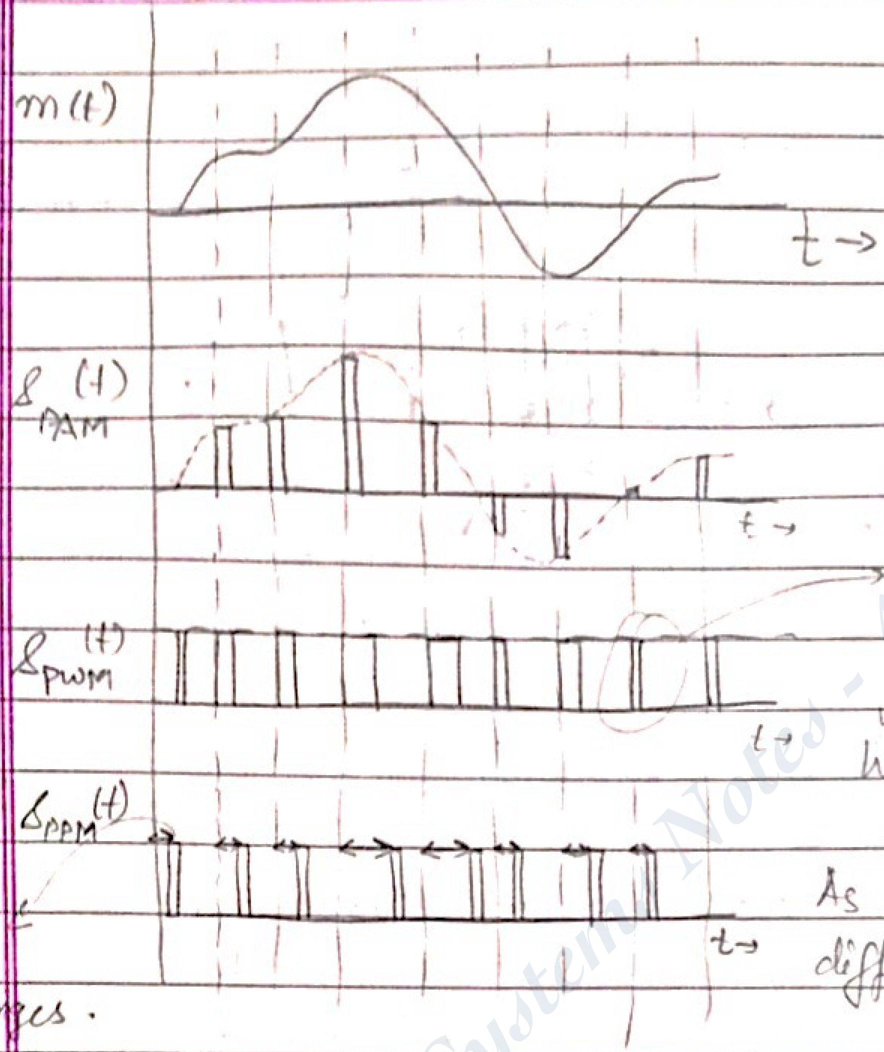


: $G_s(f)$

→ Attenuation is happening at higher freq. signal. This effect is called APPERTURE EFFECT.

Additional effect seen in PAM

- how to prevent apperture effect: decrease width (δ).
  ↳ If $\delta < 0.1 T_s$, apperture effect can be ignored.
  ↳ On doing this: BW ↓, but more no. of signals can come in spectrum.



Assuming this as reference width, when m(t) = 0
When m(t) ↑, width ↑

As m(t) ↑, phase difference ↑

Phase changes.

✯ PULSE CODE MODULATION (PCM)

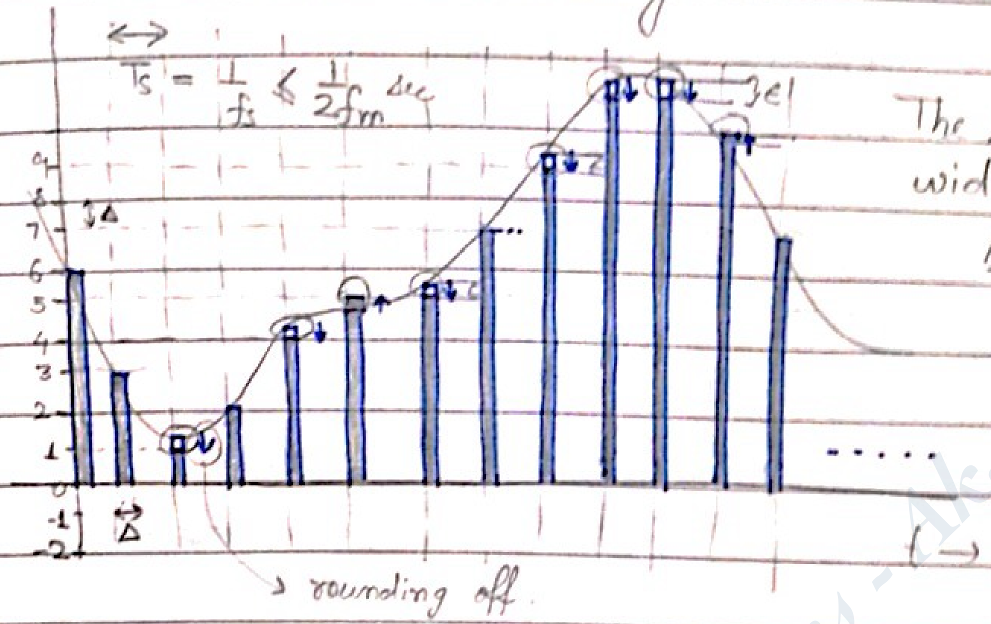
Analog Baseband signal, m(t) → Anti-Aliasing filter → Sampler → Quan-tizer → Binary Encoder (or M-ary) → PCM Signal

Anti-Aliasing filter: (LPF), 0, fm — So that the signal isn't seen after the max. freq

Sampler: ∵ if samples are sent, the gaps in b/w can be used to mix samples from other signals (multiplexing)

Sampler output: cts amplitude (∵ Amplitude change ✱) discrete time signal

Consider a continuous time signal



The sampling width
$\Delta \leq 0.1 T_s$.

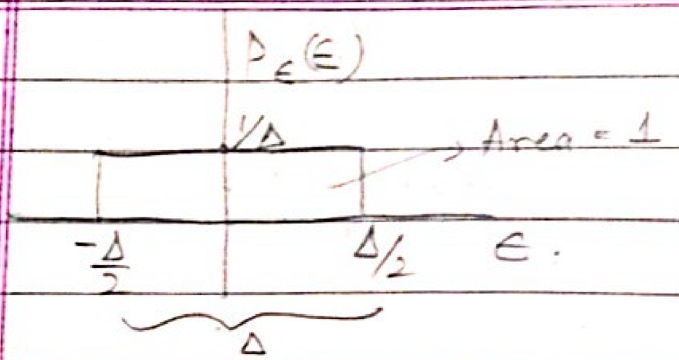
Taking samples of the signal so that we can generate the signal from its samples.
Taking flat-top sampling, as shown

Now, After sampling, we get a cts amplitude, discrete time signal.
∵ No. of amplitudes we have → ∞

Now, doing quantiz^n — dividing y axis in different parts with { constt separation (done here) / varying separation

In practical case, [signal] : This is the actual signal with noise. So, breaking up the signal y axis is req^d.
Suppose at $t = 2$, $m(t) = 2.4\ V \pm 0.5\ V$
↳ due to noise

So, we take 3 V levels in y axis. (0, 3, 6, 9 ...., eg)
Now, suppose I'm quantizing values at 0V, 1V, 2V, 3V ..... So, If I have $m(t)$ as 2.6 V, say, it is rounded off to 3V.
Due to rounding off, ∃ errors. Let the error be ϵ.
If the distance b/w levels is Δ, then $-\frac{\Delta}{2} \leq \epsilon \leq +\frac{\Delta}{2}$



PDF : Probability Density Function

Mean of ϵ = 1st moment of ϵ $= \overline{\epsilon} = \int_{-\infty}^{\infty} \epsilon^1 P_\epsilon(\epsilon)\, d\epsilon$

$$= \int_{-\frac{\Delta}{2}}^{\Delta/2} \epsilon\left(\frac{1}{\Delta}\right) d\epsilon$$

$$= \frac{1}{\Delta}\left(\frac{\epsilon^2}{2}\right)_{-\Delta/2}^{\Delta/2} = 0$$

Quantiz^n noise power, 2nd moment of ϵ

$$= \overline{\epsilon}^2 = \int_{-\infty}^{\infty} \epsilon^2 P_\epsilon(\epsilon)\, d\epsilon$$

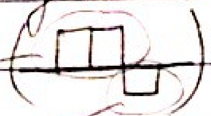
$$= \int_{-\frac{\Delta}{2}}^{\Delta/2} \epsilon^2 \left(\frac{1}{\Delta}\right) d\epsilon = \frac{1}{\Delta}\left(\frac{\epsilon^3}{3}\right)_{-\Delta/2}^{\Delta/2}$$

$$\Rightarrow \overline{\epsilon}^2 = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{\Delta^2}{12} V^2$$

$$\text{So, } \overline{\epsilon}^2 \propto \Delta^2$$

Now after quantiz^n, comes encoding.

Suppose $\exists$ 8 levels. So, in binary encoding, each level can be represented by 3 bits ($2^3 = 8$).
Suppose $m(t) = 6$ V. So, signal is not sent, its code is sent (easier to interpret – you have to detect only 1's & 0's) So, I send :



+ve pulse for 1
–ve pulse corresponding to 0, say

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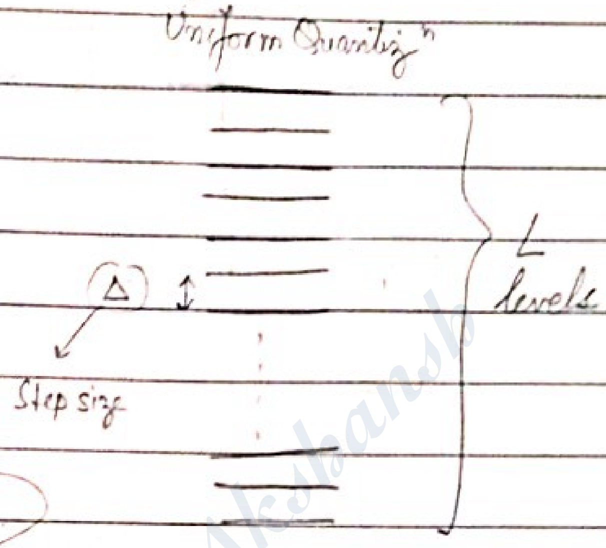
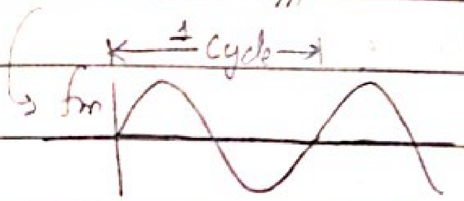
* Signal to Quantization noise power ratio (SQNR)

Consider single tone case :

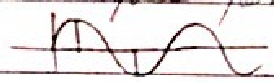


$m(t) = A_m \cos\omega_m t$



Sampling freq. should be : $f_s \geq 2 f_m$

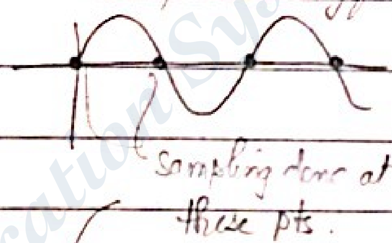
i.e, 2 samples per cycle



large value of f_s so that sampling isn't done exactly at Nyquist rate.

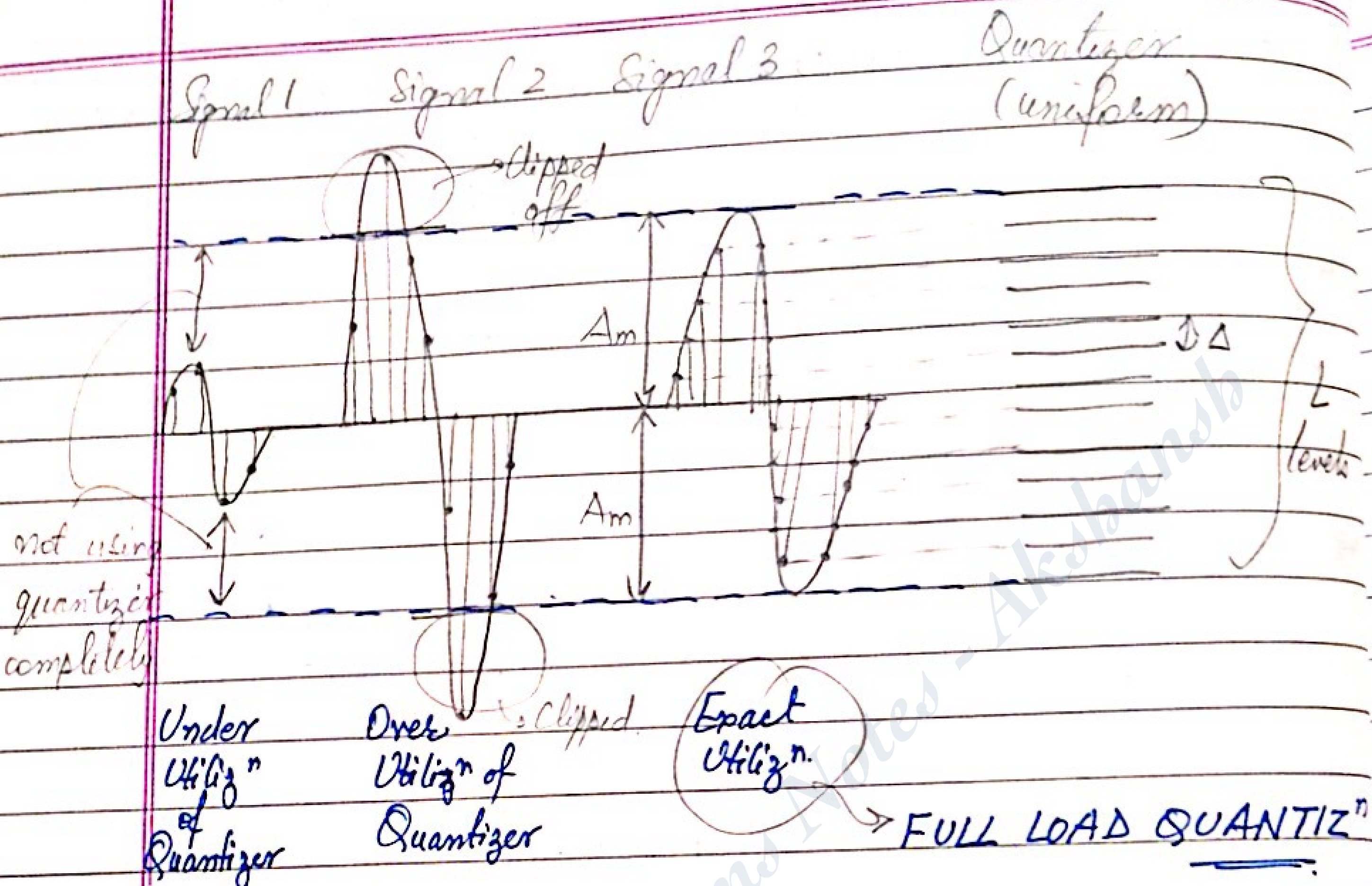
$$\boxed{S = \frac{A_m^2}{2}}$$

i.e, suppose I sample at Nyquist rate :-



So, o/p is zero. Hence, original signal cannot be detected by receiver

Hence, we take larger value of f_s



* for "Full Load Quantiz^n",

$$\Delta = \frac{2A_m}{L-1}$$

$$\simeq \frac{2A_m}{L} \quad ; \; L >> 1$$

$$N_q = \frac{\Delta^2}{12} = \left(\frac{2A_m}{L}\right)^2 \times \frac{1}{12} = \frac{A_m^2}{3L^2} = \frac{A_m^2}{3 \cdot 2^{2n}}$$

Noise power due to Quantiz^n

Assuming binary encoding.
So, $L = 2^n$

Now,

$$SQNR = \frac{A_m^2}{2} \cdot \frac{3 \cdot 2^{2n}}{A_m^2} = \left(\frac{3}{2}\right) \times \left(2^{2n}\right)$$

↳ expressing in dB :

$$\left(\frac{S}{N_q}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N_q}\right)$$

* We should have large SQNR
eg: SQNR ≥ 60 dB: almost negligible noise

$$= 10 \log \left[\frac{3}{2} \times 2^{2n}\right].$$

$$= 10 \log \left(\frac{3}{2}\right) + 10 \log \left(2^{2n}\right)$$

$$= (1.77) + 20n \cdot \underbrace{\log_{10} 2}_{0.301}$$

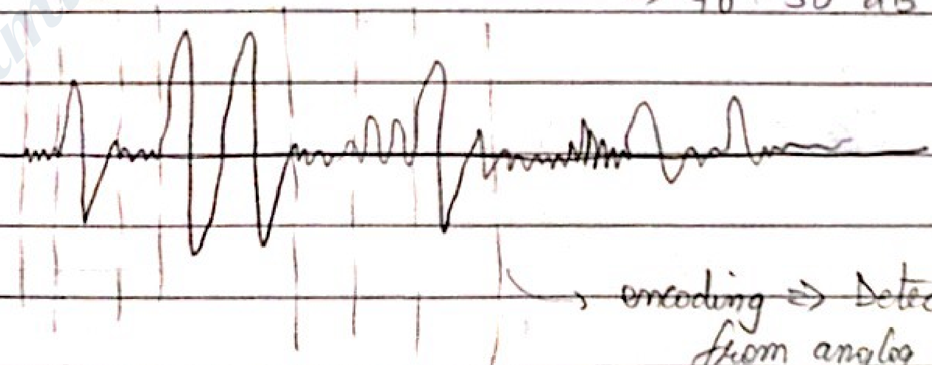
$$\Rightarrow \left(\frac{S}{N_q}\right)_{dB} = (1.77 + 6.02n)\ dB$$

↳ called as: 6-dB RULE OF PCM
↳ every add^nal bit used to encode, increases amplitude by 6 dB.

If 2 bits are used to encode,
SQNR = 1.77 + 6.02(2) ≃ 14 dB
$n = 3$
SQNR = 1.77 + 6.02(3) ≃ 20 dB
So, dB ↑ by 6 dB.

* Speech signal: comes in "WIDE DYNAMIC RANGE"
↳ 40 − 50 dB

Speech signal:



→ encoding ⇒ Detecting speech from analog signal.

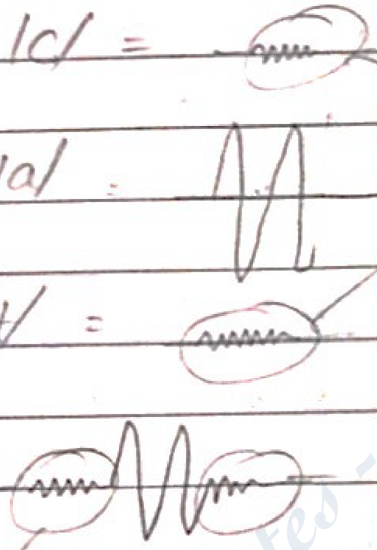
Above is any speech signal.
Whenever I talk to someone - i.e., a speech signal, ∃ some background noise. So, the noise amt till which I can hear the other person is called Intelligibility.

eg: consider words like bat, cat, mat ...,
If very high noise is there, it'll difficult to differentiate b/w words

Consider cat , /c/ =

/a/ :

/t/ =



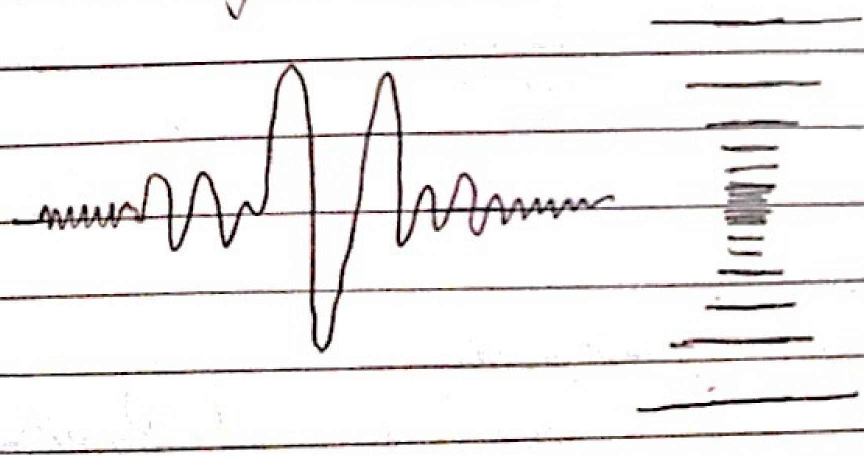
⇒ energy associated with /c/ & /t/ is very less. SQNR is low.

So,
cat =

defines intelligibility

☆ Ventriloquism :- sort of puppet action, i.e,
(V) one person says, other is just doing lip movement

☆ In a speech signal, we find some places where SQNR is low, somewhere its high. This is undesirable.
We want SQNR nearly constt. How to achieve?
Use : NON-UNIFORM Quantization.
i.e. something like :-



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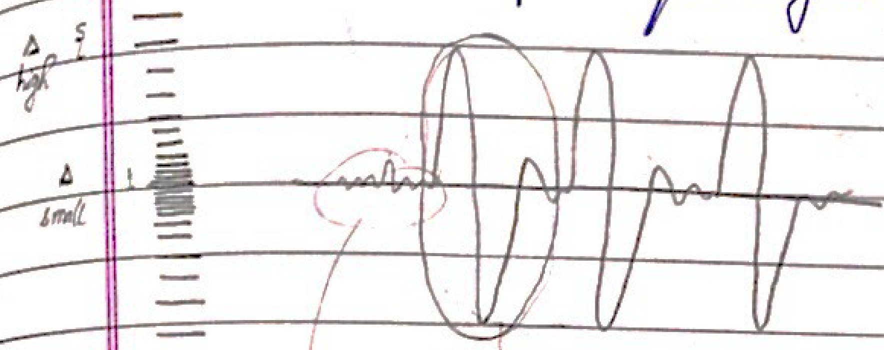
Companding in PCM
(non-uniform quantization)



$\frac{S}{N_q}$ → S → small; $N_q \propto \Delta^2$

$\Delta \downarrow$ so, $N_q \downarrow$

So, $\frac{S}{N_q} \simeq$ constt

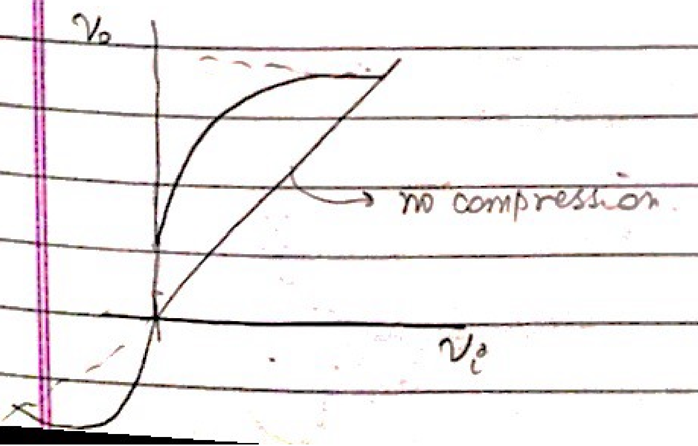
$\frac{S}{N_q}$ → S → high; N_q → Δ^2 is more

So, N_q increases

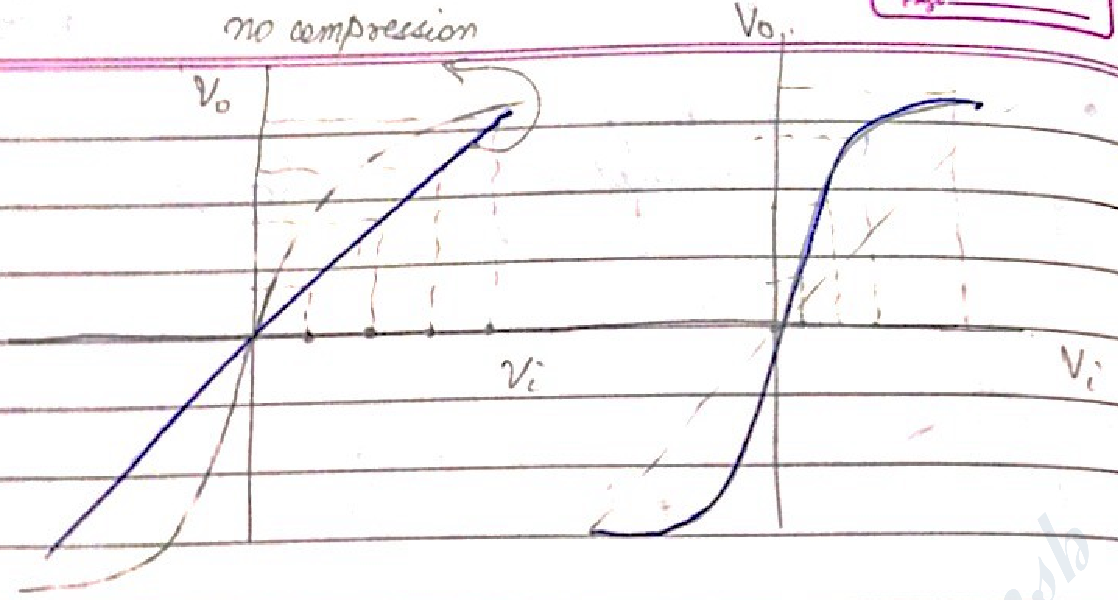
So, $\frac{S}{N_q} \simeq$ constt

☆ Using Uniform Quantizⁿ to get nearly constt SQNR i.e, implementing non-uniform quantizⁿ using Uniform Quantizer :-

$$m(t) \rightarrow \underbrace{\boxed{\text{Compressor}} \rightarrow \boxed{\text{Uniform Quantizer}}}_{\text{Non uniform Quantizer}} \rightarrow \boxed{\text{Encoding}} \rightarrow \text{PCM}$$

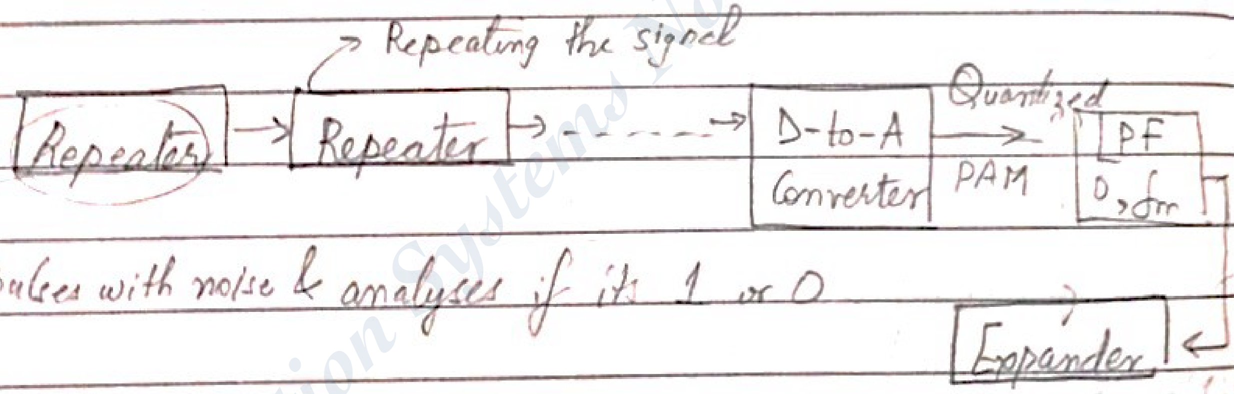


Idea : compress the larger signal to equalise the level



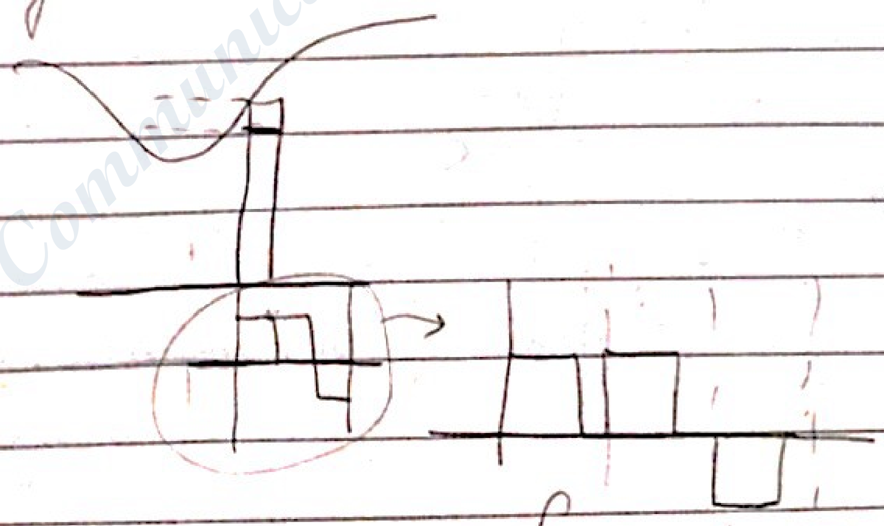
Levels increase linearly

At low V_i, we get more levels, at high V_i, we get less levels & so, non uniform.



Take i/p pulses with noise & analyses if its 1 or 0

Signal



So, signal actually received

Integrator : Smoothens the signal : levelling
Differentiator : Makes the signal sharp.

• LAW of Operⁿˢ :

1. A-law compression.
2. μ - law ~~pa~~ ~~cont~~ compression.

→ Formula PM.

→ Formula - reading assignment

★ PLL : reading modulⁿ

This process of compression & expanding.

§ DELTA MODULATION (extension of PCM)

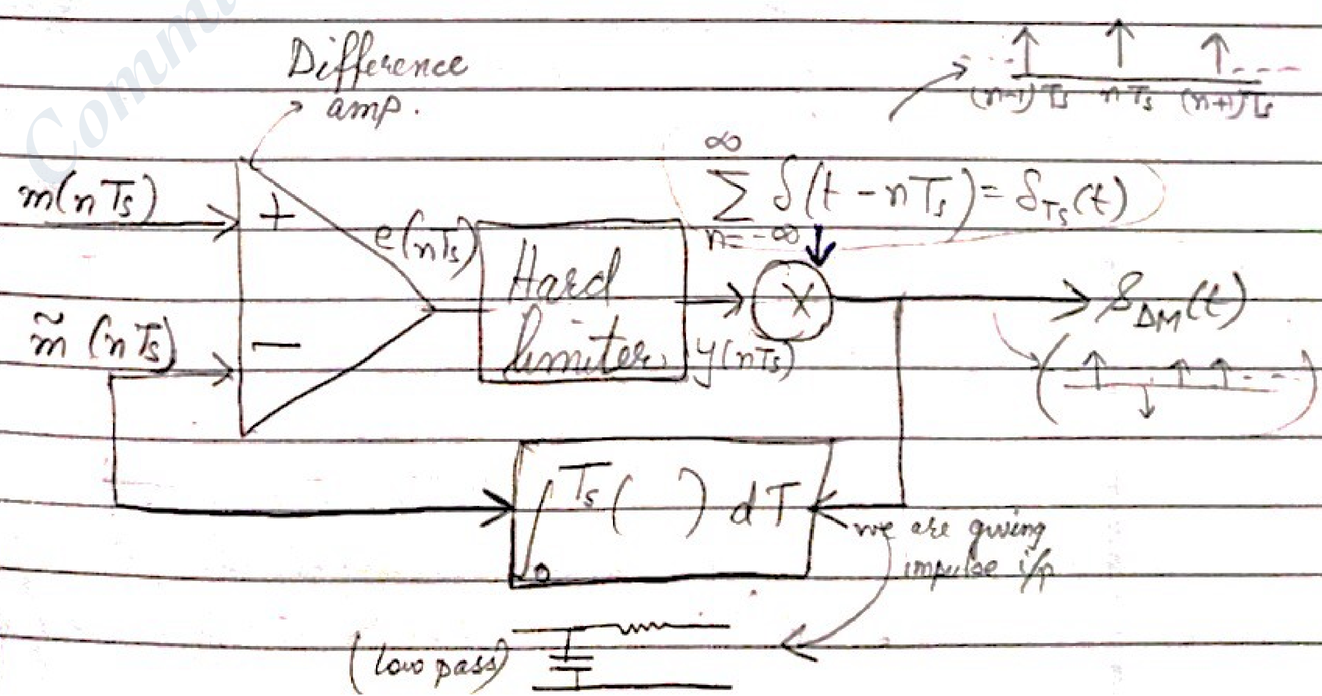
→ 2-level PCM

★ No. of levels $= L = 2^{(\text{no. of bits})}$ → using Binary ($L = M^n$, using M-ary)

here, $L = 2$

& $L = 2^n \Rightarrow n = 1$

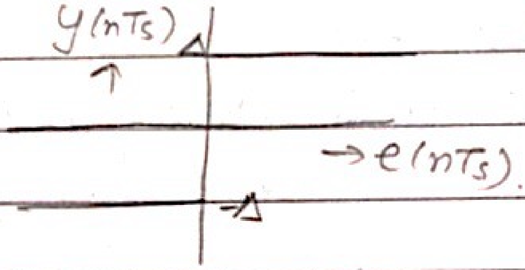
So, its 1-bit PCM



$$\delta(t) = \frac{d}{dt} u(t)$$

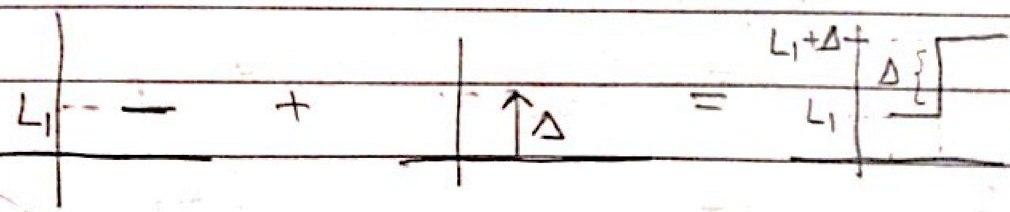
- In delta modulⁿ, encoding is less, levels are 2
  / we are sampling holding & quantizing $e(nT_s)$
- In PCM, encoding is more, levels are more
  ✓ we are sampling, holding & quantizing $m(nT_s)$

- hard limiter: $y(nT_s) = \begin{cases} \Delta, & e(nT_s) \geq 0 \\ -\Delta, & e(nT_s) < 0 \end{cases}$



———— x ————

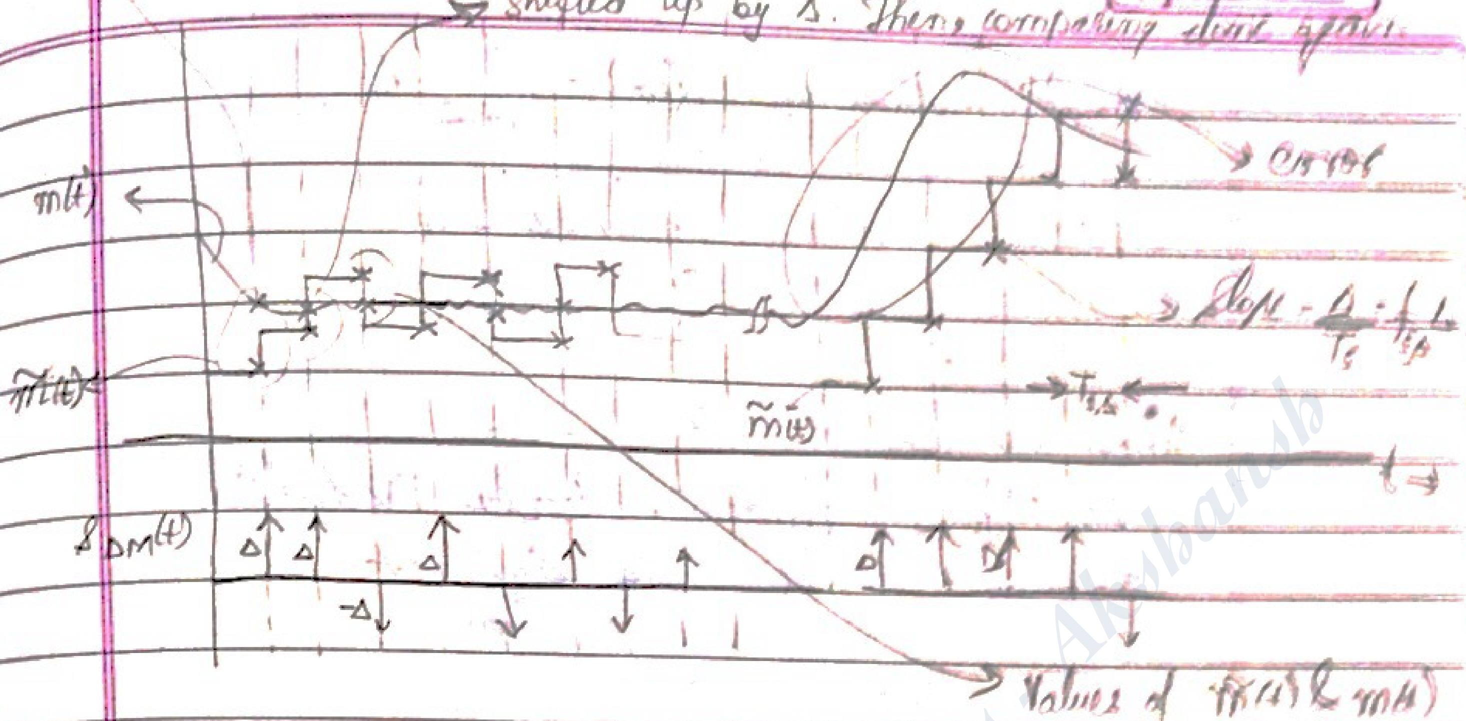
From fig. on prev. page, we saw that impulse i/p is coming to integrator (at feedback)
Now, Integrator levels the signal
Say, level of integrator = L_1. If impulse i/p is given, we have :-



↳ Integrator is also a storage element

These 2 values are compared, $m(t) - \tilde{m}(t) > 0$. So, o/p $= +\Delta$

shifted up by Δ. Then, comparing done again.

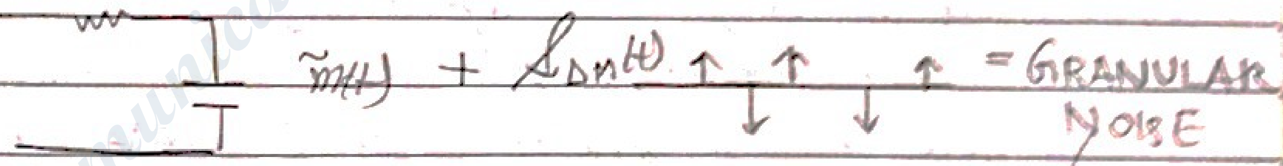


Hunting:

Delta modulation is hunting for a signal.

Consider :-

Delta modulated i/p



↑↓ intensity of ↑ & ↑ with granular noise

Δ : step size

↳ $\propto$ Granular noise

So, Hunting leads to Granular noise in the receiver.

* **Slope overload distortion:**

the approximation is not able to catch up with actual signal.

So, avg. slope of $\tilde{m}(t) = f_{s,\Delta}\,\Delta$.

Now, to avoid error of step overload :-

M1) Increase Δ

Avoiding Slope overload error
↳ Way 1 : Adaptive Delta Modul^n.
↳ Way 2 : Select step size s.t. Δ is bearable

✓ how come Slope overload distortion (SOD) come?
We did $\frac{d}{dt}\tilde{m}(t)$ & $\frac{d}{dt}m(t)$

✓ Solving without using adaptation :
We find from graph, Slope$(m(t))$ > Slope$(\tilde{m}(t))$
So, we want $\left|\frac{d}{dt}\tilde{m}(t)\right| \geq \left|\frac{d}{dt}(m(t))\right|$

✓ Consider single tone signal :
$m(t) = A_m \cos \omega_m t$
max. possible value of slope of $m(t) = \left|\frac{d}{dt}m(t)\right|_{max} = A_m\omega_m$

Writing directly, $A_m \leq \left(\frac{f_s \cdot \Delta}{\omega_m}\right)$.

✓ For speech signal, std. are :
$f_m = 800\,Hz$.

continuously variable slope

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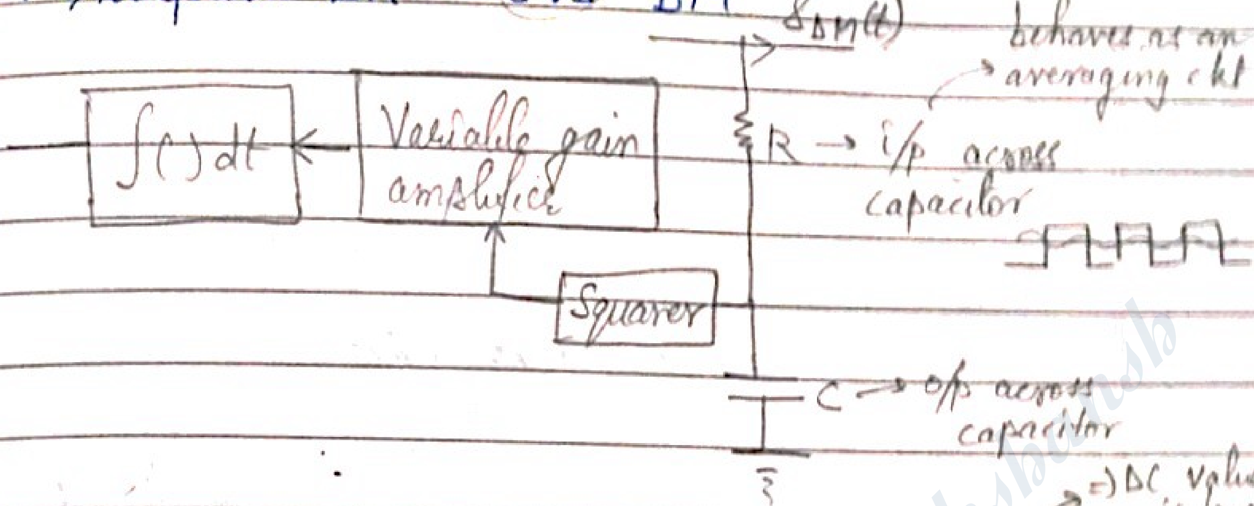
Way 1 : Adaptive DM ~ CVS DM



⇒ DC value is low

Hunting & Alternative pulses ↑ ↓ ↑ ↓ → avg is low

Slope overload distortion : ↑↑↑↑ : avg. value high +ve

or

↓↓↓↓ : avg. value high −ve

Granular noise : reduced by reducing value of Δ.

Consider above ckt &

Hunting :- avg. value low

squarer gives a low value.

SOD : variable gain amplitude should always get a +ve i/p

✓ gain goes high.

Variable gain amplifier :

i/p is low ⇒ gain is low

i/p is high ⇒ gain is high

⇒ slope changes continuously.

The above type of modul^n is baseband modul^m

PCM : Baseband spectrum

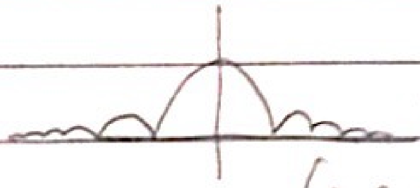
↳ not amenable to free space transmission

↳ requires RF freq.

PCM: 

made up of pulse which is +ve or -ve

PCM will have a spectrum like - sinc f^n.



→ lowpass spectrum

(we cannot transmit such a signal)

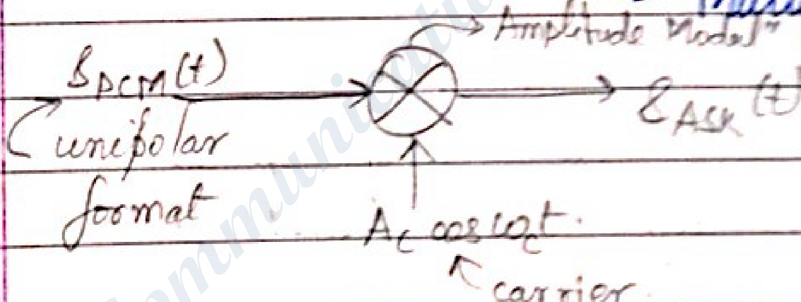
↳ (best in AM)

To send through free space → convert to RF freq. range

Amplitude Shift Keying (ASK)

∴ We have to give the PCM signal has i/p to AM (amplitude modulation) ckt. Then we get amplitude shift keying modulation. This type is called as RF digital modulation.

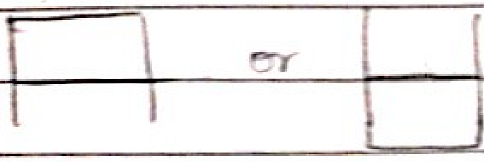
Basic AM modulator { Balanced modulator / Multiplier



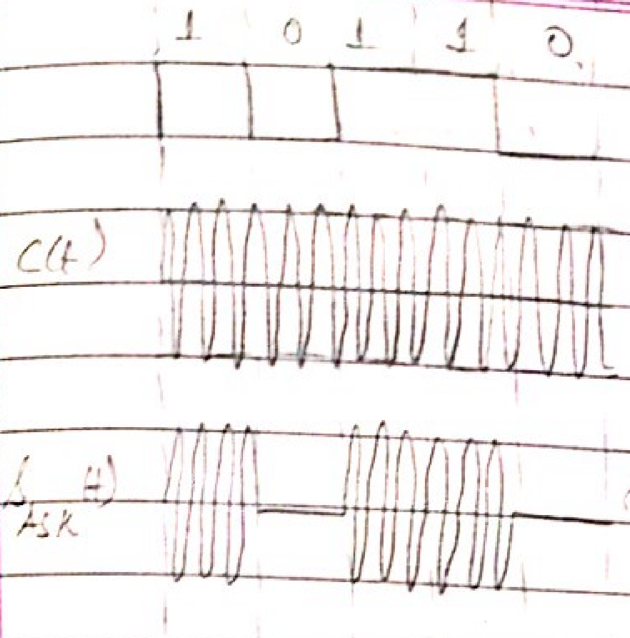
Unipolar Data Format: Polarity in one dirⁿ

If sending '0' : 

If sending '1' :



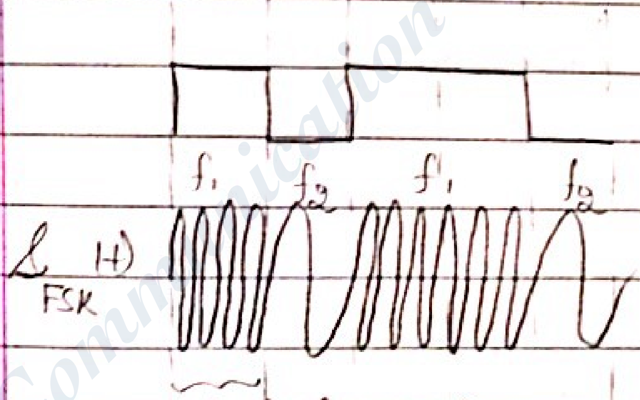
If we are using ASK, we have to use unipolar data format. Zero essentially has to be zero.



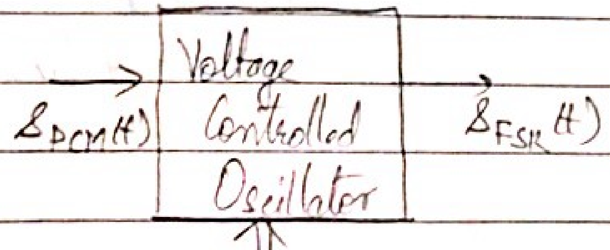
→ on-off keying

* Telegraphic signal using morse code : similar to ON-OFF keying
  ⇒ Digital communicⁿ preceded analog communicⁿ.

§ Frequency Shift Keying (FSK)



We can use direct method to generate FM : Voltage Controlled Oscillator

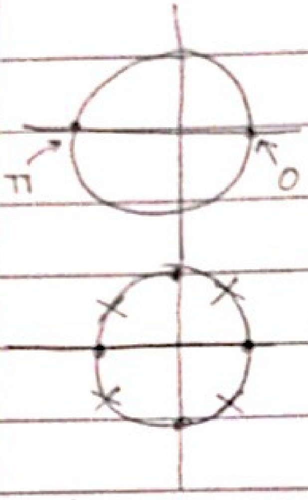


FM generator

* The selection of freq. f_1 & f_2 should not be arbitrary.
* In one bit cycle, no. of cycles of f_1 or f_2 should be an integer
  ↳ Integer no. of cycles

8) Phase Shift Keying (PSK)

If '1' → one phase value
If '0' → another phase value

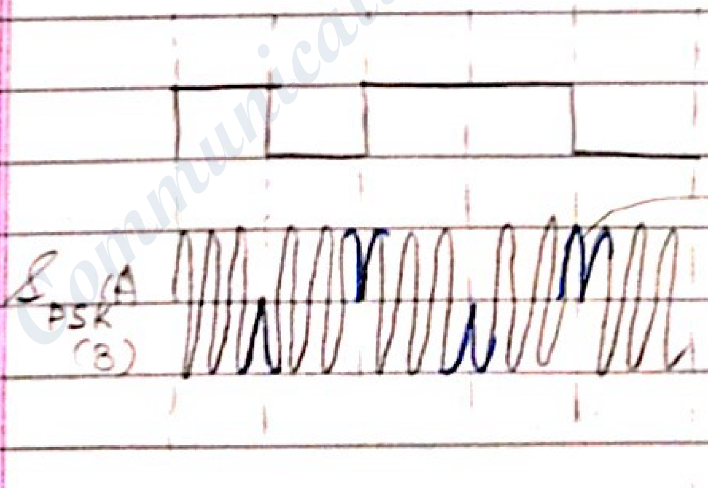


PSK ← BPSK ↓ binary → 2 phase values

or

QPSK ↓ quartenary → 4 phase values

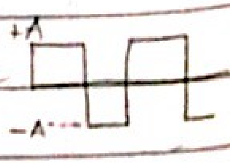
To represent 2 phase values, we need 1 bit
In QPSK → informⁿ is sent 2 bits at a time
(advantage over BPSK)
→ each symbol conveys 2 bit of info compared to one bit in BPSK
⇒ speed up process of transmission
⇒ double the info is sent



→ Phase changes depending on 1 or 0.
Can be any direction.
Phase shift should be 180°
↳ ∴ its BPSK

$S_{PCM}(t)$ → [Balanced Modulator] → $S_{PSK}(t)$
Polar PCM signal

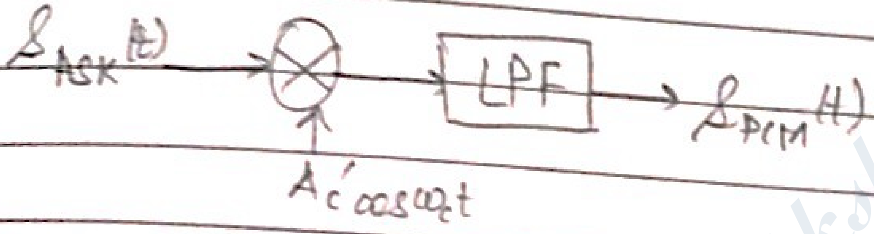
→ i/p signal will be like



• how to demodulate these signals?

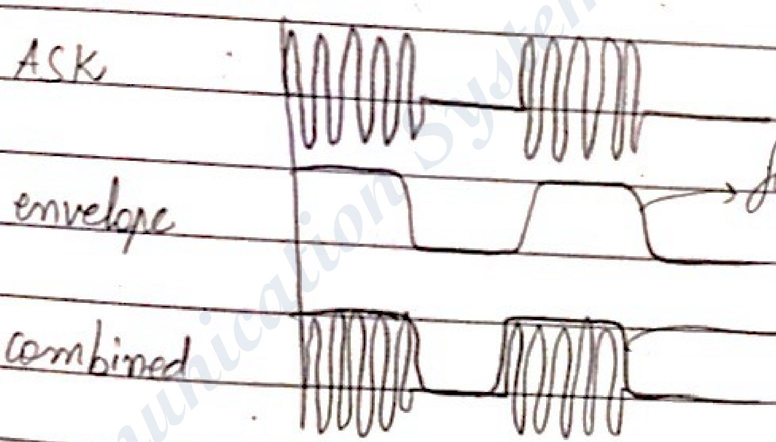
➤ ASK: Coherent Detector

Incoming ASK multiplied by high freq. carrier of same phase & pass through LPF



demodulⁿ: to convert $s_{PCM}(t) \rightarrow m(t)$: Use D/A converter.

• Non Coherent Detector will also work
↳ envelope detector



→ for ASK (got just like as in AM)

→ If envelope exactly traced, we get back original signal.
So, It'll work

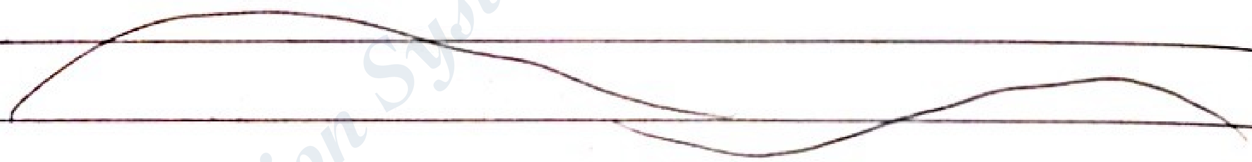
➤ PSK :

If we pass it through an envelope detector, we will have a constt "1". So, it wont work.
Only Coherent detector will work.
↳ Since its a costly process, Differential PSK (DPSK) came into picture.

★ DPSK : manage with simple multiplier & delay signals.
: we don't need accurate phase carrier etc.

FSK :

Coherent / Non Coherent Detection .



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Ch: Information Theory

- Concept of Information.
  general : anything that conveys sense
  Communicⁿ pt. of view : Something that may/may not convey sense.
- Idea : we want to measure / quantify info.

Depending on probability of occurence of event — less/more info is conveyed

↳ More the probability, LESS is the info conveyed. (& vice versa)

$$\text{Inform}^n,\ I = f(\text{Probability}) = f(P)$$

eg Suppose I go to shopkeeper to buy a chair.

So, Way ①

① I select model from 'm' models
So, probability of selection → $P_1 = \frac{1}{m}$.

Info conveyed, $I_1 = f(P_1) = f(\frac{1}{m})$

② I select color from 'n' colors of that model
Prob., $P_2 = \frac{1}{n}$

⇒ Info conveyed, $I_2 = f(\frac{1}{n})$.

Way ②

I tell him to display all models & colors. So, $I = f(\frac{1}{mn})$

It is seen that info conveyed in way 1 = way 2

$$f\left(\frac{1}{mn}\right) = f\left(\frac{1}{m}\right) + f\left(\frac{1}{n}\right)$$

$$\left(\equiv f(ab) = f(a) + f(b) \right)$$

$$\equiv \log f^n.$$

So,

$$I = \log(P)$$

usually in decimals

So, $\log(P) = -ve$.

So, instead of saying info = −ve, we take $I = -\log(P)$.

So, $\boxed{I = -\log(P)}$

- called as: Self Info associated with message.
- Probability of occurence of event → P
- General units: **bit**.

$\log_{(\cdot)}(P)$ — log base	Unit
2	bits
e	Nats
10	Hartleys

* Discrete Memoryless Source: (DMS)

↳ capable of sending messages, say $m_1 \longrightarrow m_N$ with probabilities $P_1, P_2 ---- P_N$

m_1,	m_2,	----,	m_N
P_1,	P_2	----	P_N

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* Probability of occurence/use of letters in English is not same
Statistically, occurence of letter E is max. (13%)

Now, trying to average out the info sent by source ?

(i.e., suppose a class of students having heights 5'8", 5'9", 6' - - -
Can we say, avg. height is 5'8" ?)

Approach:

These many bits of info. is conveyed for every occurence of m_1

Message	Prob.			
m_1	P_1	$-\log_2 P_1$	$P_1 L$	$-P_1 L \log_2 P_1$
m_2	P_2	$-\log_2 P_2$	$P_2 L$	$-P_2 L \log_2 P_2$
⋮	⋮	⋮	⋮	⋮
m_N	P_N	$-\log_2 P_N$	$P_N L$	$-P_N L \log_2 P_N$

$\sum_{i=1}^{N} P_i = 1$

$P_N L$: Total occurrence of message m_N in L messages

$-P_N L \log_2 P_N$: Info. conveyed for total occurence of msg. P_N

eg: Suppose $\exists$ 2 messages:-
m_1 0.2
m_2 0.8

If $\exists$ 1000 messages. How many messages occ of m_1
$= 0.2 \times 1000 = 200$
IIIy. $m_2 = 0.8 \times 1000 = 800$.

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Total info. in L messages $= -\sum_{i=1}^{N} P_i L \log_2 P_i$ bits

per conveyed by

→ Avg. info / message of source,

$$H(S) = -\sum_{i=1}^{N} P_i \log_2 P_i \quad \text{bits/message.}$$

name of source

entropy of source

$\equiv$ weighted summation of info i.e. weighting every self info ($-\log_2 P_i$) with its probability (P_i)

(called Entropy, SIMPLY b/c it looks like entropy formula in Thermodynamics)

has only 2 messages

* Binary Source

(eg: Digital source, say PCM source)

Let 2 messages & prob. →

$m_1 \rightarrow P_1$

$m_2 \rightarrow P_2$

$\therefore$ avg. info per message = entropy

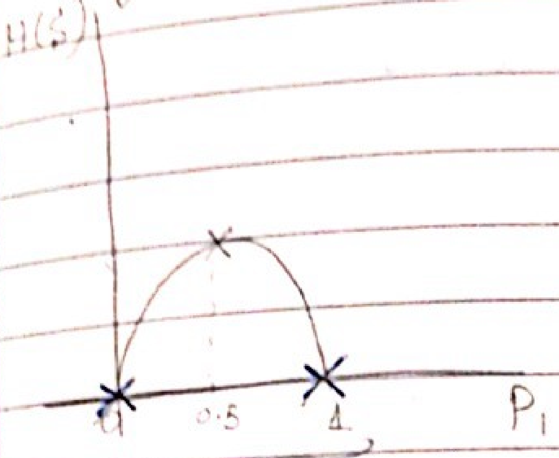
$$= H(S) = -\sum_{i=1}^{2} P_i \log_2 P_i$$

$$= -P_1 \log_2 P_1 - P_2 \log_2 P_2$$

$$(P_1 + P_2 = 1 \Rightarrow P_2 = 1 - P_1)$$

$$= -P_1 \log_2 P_1 - (1-P_1) \log_2 (1-P_1)$$

* Plotting entropy of binary source vs P_1



Prob. lies only b/w 0 & 1.

let $P_1 = 0$ ($\Rightarrow P_2 = 1$)

So, only m_2 will come out

So, source is conveying no. info. because I know its 0 or 1 always.

So, Entropy = 0 (at $P_1 = 0$ & 1)

Now, when both P_1 & P_2 have same chance of occuring I cant say which will come (i.e. $P_1 = P_2 = 0.5$)

In this case, max. info is being conveyed. So, entropy is max.

Ily, if no. of messages are 3, entropy is max. when all 3 messages have equal probability,

i.e., $P_1 = P_2 = P_3 = \frac{1}{3}$.

Ily, for more no. of messages.

Note ☆ $\log_b a = \dfrac{\log_c a}{\log_c b}$ (formula)

↳ So, $\log_2 x = \dfrac{\log_{10} x}{\log_{10} 2} = \dfrac{\ln(x)}{\ln(2)}$

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In general, limits on entropy :-

$$0 \leq H(S) \leq \log_2 N$$

→ for binary, $N = 2$

$$0 \leq H(S) \leq \log_2 2$$

$$\Rightarrow 0 \leq H(S) \leq 1$$

For letters of English alphabet, say & say all of them have equal probability,

So, $0 \leq H(S) \leq \log_2 26$

So, max. value of entropy lies b/w 4 & 5

In reality, however, the probability of occurrence of any event is not same.

Prove : Entropy is +ve.

Say, a source can send N messages, so, probabilities are:-

$$P_i \leq 1 \;, i = 1, 2, \ldots, N.$$

$$\text{So, } \frac{1}{P_i} \geq 1.$$

$$\log_2\left(\frac{1}{P_i}\right) \geq \log_2 1$$

$$\Rightarrow -\log_2(P_i) \geq 0 \quad (\log_{\underbrace{()}_{\text{anything}}} 1 = 0)$$

$\times P_i$, both sides

$$\Rightarrow \underbrace{-P_i \log_2 P_i}_{\text{for a single message}} \geq 0$$

Repeating for $P_1, P_2, \ldots \rightarrow$ we get

$$-\sum_{i=1}^{N} P_i \log_2 P_i \geq 0$$

So, $H(S) \geq 0$

So, entropy is always +ve.

Prove: entropy $H(S) \leq \log_2 N$

Consider $H(S) - \log_2 N$

$$H(S) - \log_2 N = H(S) - (1) \log_2 N$$

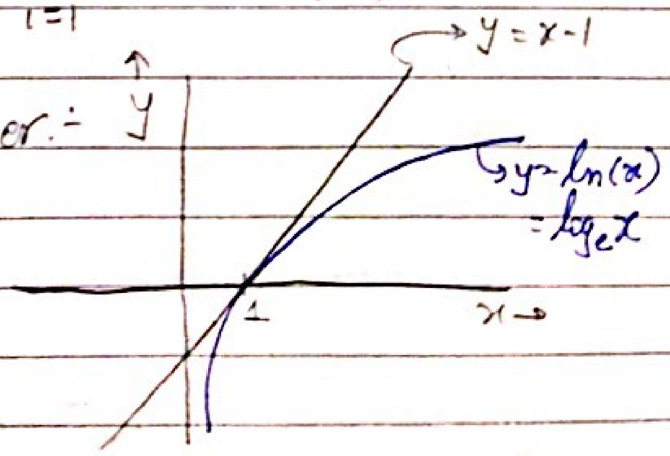
$$= -\sum_{i=1}^{N} P_i \log_2 P_i - \underbrace{\sum_{i=1}^{N} P_i}_{\text{Total Prob.} = 1} \log_2 N$$

$$= -\sum_{i=1}^{N} P_i \left[\log_2 P_i + \log_2 N \right]$$

$$= -\sum_{i=1}^{N} P_i \left[\log_2 P_i N \right]$$

$$= \sum_{i=1}^{N} P_i \log_2 \left(\frac{1}{P_i N} \right) \longrightarrow (1)$$

Now, consider:-



Clearly,

$x - 1 \geq \ln(x)$

(with equality at $x = 1$)

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So, $\ln x \leq x-1. \rightarrow (2)$

Using (2) in (1).

$H(S) - \log_2 N = \sum_{i=1}^{N} P_i \log_2\left(\frac{1}{P_i N}\right)$

$= \frac{\sum_{i=1}^{N} P_i \ln\left(\frac{1}{P_i N}\right)}{(\ln 2)}$

$= \frac{1}{\ln 2} \sum_{i=1}^{N} P_i \ln\left(\frac{1}{P_i N}\right)$

$\leq \frac{1}{P_i N} - 1$

$\leq \frac{1}{\ln 2} \sum_{i=1}^{N} P_i \left(\frac{1}{P_i N} - 1\right)$

$\leq \frac{1}{\ln 2}\left[\sum_{i=1}^{N} \frac{1}{N} - \underbrace{\sum_{i=1}^{N} P_i}_{=1}\right]$

$\leq \frac{1}{\ln 2}\left[\frac{1}{N}\sum_{i=1}^{N}(1) - \sum_{i=1}^{N} P_i\right]$

$\leq \frac{1}{\ln 2}\left[\frac{1}{N}\sum_{i=1}^{N}(1)^i - \sum_{i=1}^{N} P_i\right]$

$\leq \frac{1}{\ln 2}\left[\frac{1}{N}\left(1+1^2+1^3+\dots 1^N\right) - \sum_{i=1}^{N} P_i\right]$

* white noise : has all freq. with equal weightage.

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$$\leq \frac{1}{\ln 2}\left[\frac{1}{N}(N) - 1\right]$$

$$\leq 0.$$

So, $H(S) - \log_2 N \leq 0$

So, $H(S) \leq \log_2 N$

H.P

☆ Information Rate of a source (bits/sec)

We know, entropy $H(S)$ is in bits/message

message rate, r is in messages/sec

So, Information rate, $\boxed{R = r \cdot H(S) \text{ bits/sec}}$

§ CHANNEL CAPACITY, "C"

↳ Max. rate at which we can send info. through a channel

☆ We can send as much info. as we want through a channel. But, if we send more than Max value (Channel capacity), we don't get back the signal. So, send less than max.

- Shannon - Hartley give the channel, called as AWGN Channel

∵ noise keeps adding

Additive → White → Gaussian → Noise

• AWGN is based on Central Limit theorem of probability.

If ∃ ∞ no. of diff sources having uniform PDF (probability Density fn), once we add all these sources, the overall PDF is Gaussian PDF.

* Shannon - Hartly gave the following formula:

The max. weight which can be pushed through an AWGN channel, keeping in mind the following parameters - Signal Power (S), Noise Power (N), Bandwidth (B).

The channel capacity.

This max. weight i.e. Channel capacity, C, is given by,

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

Found by: Power spectral density × Bandwidth B

* how to increase C?

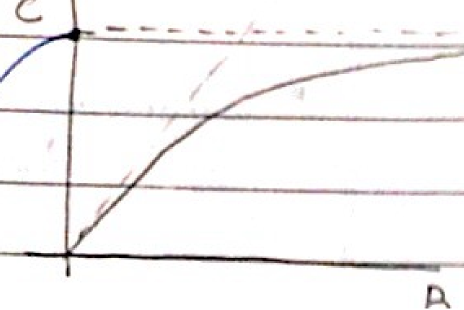
Can be done. But not much.

✓ Inc. B → costly (∵ BW is being increased)

✓ Inc. S → ∃ limit to inc. S ∵ S is transmitted power.

→ What happens when $B \to \infty$

i.e. $\lim_{B \to \infty} C = ?$



$1.44\, S/\eta$ ← derived later

White noise :

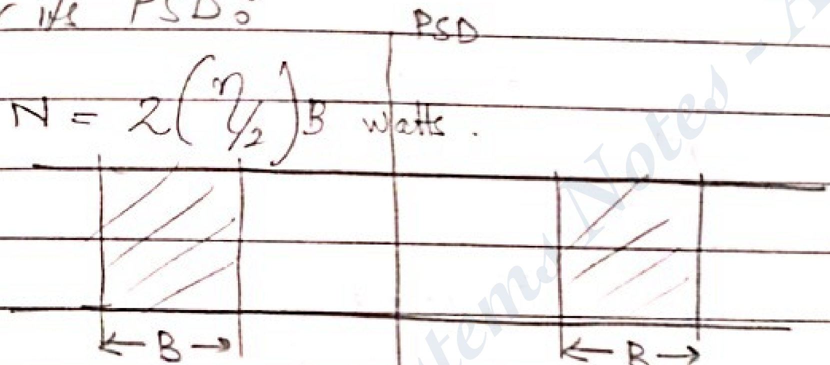
A random process defined as having a uniform / constt. power spectral density (PSD) of

$\eta/2$ Watts/Hz : 2 sided (freq. +ve or −ve)

η Watts/Hz : 1 sides (freq. +ve only)

Consider its PSD :

$$N = 2\left(\frac{\eta}{2}\right)B \text{ watts.}$$



So, $N = \eta B$

So, $$C = B \log_2\left(1 + \frac{S}{\eta B}\right)$$

$$= \left(\frac{\eta B}{S}\right)\left(\frac{S}{\eta}\right)\log_2\left(1 + \underbrace{\left(\frac{S}{\eta B}\right)}_{x}\right)$$

Let $x = \frac{S}{\eta B} \Rightarrow B \to \infty \Rightarrow x \to 0$.

So, $\lim\limits_{B\to\infty} \rightarrow \lim\limits_{x\to 0}$.

$$\Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)\left(\frac{S}{\eta}\right)\log_2(1+x)$$

$$= \frac{S}{\eta}\lim_{x\to 0}\left(\frac{1}{x}\right)\log_2(1+x)$$

$$\Rightarrow \lim_{B \to \infty} C = \lim_{x \to 0} \frac{S}{\eta} \log_2 (1+x)^{1/x}$$

We know:

$$\lim_{x \to 0} (1+x)^{1/x} = e$$

$$\Rightarrow \lim_{B \to \infty} C = \frac{S}{\eta} \lim_{x \to 0} \log_2 e$$

$$\Rightarrow \lim_{B \to \infty} C = \frac{S}{\eta} \log_2 e$$

$$\boxed{\log_{B \to \infty} C = 1.44 \frac{S}{\eta}}$$

SHANNON'S LAW

If $R \leq C$, probability of error in received signal $= 0$

If $R > C$, $\exists$ leakage of signal $\rightarrow$ i.e., error

Puffin
Date: 5/12/13

About → Trying to represent any signal in
1, 0 → in Binary coding
0, 1, ... M-1 → M-ary coding.

CODING

used → for privacy
for efficient transmission
using min. no. of bits

Source Encoding

having a source of finite no. of messages (DMS), how to encode that (using binary – in syllabus)

- It succeeds only when channel is NOISELESS.

We → Binary Encoding (0, 1)

Channel Encoding

Trying to secure the signal in presence of noise

Properties of a GOOD CODE

(eg) ① consider a source sending following messages & its codes, say :-

Message	Code 1	Code 2	Code 3	Code 4
A	0	00	1	1
B	01	01	10	01
C	111	10	100	001
D	0	11	1000	0001

Assume we are getting a sequence of messages → 01001101 0 - - -

We want to decode :

0 1 0 0 1 1 0 1 0

Code 1 → A B
D

Code 1 not a good code

∃ 2 messages giving same code. So, confusion.

Code should be uniquely decodable.

011 00 11 01 0. . . —

Code 2 $\underbrace{B \quad A \quad D \quad B}$. —

uniquely decodable

(i.e everyone using it will decode it in this way only)

1 00 11 01 0

Code 3: A×

B×

C ✓

I see 1st bit = 1 → A

So, go to next bit = 10 → B

go to next bit = 100 = 100 → C

Now, I go to next bit = 1.

So, stop at C.

Everyone using this code will stop at C.

So, here, I have to wait for coming bits.

Hence, its not Instantaneously decodable

So, code ~~is~~ ~~uniquebl~~ should be uniquely decodable.

1 0 0 1 1 0 1 0 - - —

Code 4 A

wait → wait → stop. → stop wait → stop

$\underbrace{\text{wait} \rightarrow \text{wait} \rightarrow \text{stop}}_{C}$ $\underbrace{\rightarrow \text{stop}}_{A}$ $\underbrace{\text{wait} \rightarrow \text{stop}}_{B}$

Bit → "1" indicates end of selection / stopping. So, I don't need to wait. So, its desirable.

V.Imp ✱ Higher the occurence of a message — use min. no. of bits to represent it. (& Vice versa)

Hence, codes 2 & 4 are desirable.
Now, choose between them:

Average length of a code.

→ ✓ for code 2 : avg. length = 2.

✗ for code 4 : avg. length = $\frac{1 + (2) + (3) + (4)}{4}$ (01, 001, 0001) $= \frac{10}{4} = 2.5$.

→ Wrong idea

↳ ∵ Probability of occurence of all bits isn't same.

Say,

Code 2	Code 4	Probability of occurence
00	1	0.5
01	01	0.35
10	001	0.10
11	0001	0.05.

Avg. length $\dfrac{(2\text{ bits} \times 0.5) + (2 \times 0.35) + (2 \times 0.10) + (2 \times 0.05)}{4}$, $\dfrac{1 \times 0.5 + 2 \times 0.35 + 3 \times 0.10 + 4 \times 0.05}{4}$

⇒ Avg. Length. = 2 bits = 1.7 bits

So, although it doesn't seem to be, code 4 is more efficient. Mathematically,

Efficiency of a code, $\eta = \dfrac{H(S)}{L_{av}}$

* Redundancy : Any info, if got missed, based on what signal is coming, we get to know that we missed

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* Theorem in info. theory :

$$L_{av} \geq \frac{H(S)}{\log_2 r} \quad \text{(always)}$$

→ r : size of code alphabet

→ Binary : r = 2

$\Rightarrow L_{av} \geq H(S)$ ☆

So, for binary,

$$\eta = \frac{H(S)}{L_{av}} < 1 \text{, mostly}$$

max. efficiency when

$L_{av} = H(S)$

(difficult to achieve)

☆ Redundancy $= 1 - \eta$

- Consider a sequence of messages. We have to rearrange them to make a good code.
  - S1) Arrange them in decreasing order of probability.
  - S2) Give no. of bits to it (More probability ⇒ less no. of bits)
  - S3) See the idea of instantaneous decodability (stopping automatically → FLAG) on giving bits.

* An Instantaneously Decodable (ID) code should satisfy PREFIX PROPERTY

→ No code word should be a prefix of another code word

eg: 1

1 0 → prefix

0 0 1

⋮

prefix prop. not satisfied. Its not ID code.

Now, for messages :-

m_1	0
m_2	1 0
m_3	1 1 0
m_4	1 1 1 0

not a good method of coding

SOURCE ENCODING METHODS

Method 1) Shannon - Fano method :

Assume we have to encode 6 messages using binary encoding :

$\{m_1, m_2, m_3, m_4, m_5, m_6\}$

Probability of occurence : $\{0.10, 0.35, 0.15, 0.25, 0.10, 0.05\}$

S1) Now, arrange messages in decreasing order of probabilities

Message	Probability
m_2	0.35
m_4	0.25
m_3	0.15
m_1	0.10
m_5	0.10
m_6	0.05

→ Same probability for both. So, choose anyone first, say m_1.

here, encoding is binary. So, no. of letters in code alphabet, $r = 2$. → no. of symbols/letters

{Note: for ternary code (i.e. how the o/p signal is got: 0, 1, 2 ; A, B, C ; Red, Black, Blue etc.)
for quarternary encoding: Make groups of 4}

S2) Now, make groups of messages s.t. total probability in group 1 = group 2 (or nearly same)
$0.5 = 0.5$ (if possible)
$0.6, 0.4$ (close)

So, here we group m_2, m_4 in group 1 $(0.35 + 0.25 = 0.6)$
& rest is in group 2.

Now, give 0 to group 1 & 1 to group 2.
Next further divide elements of each group & keep doing like this until further division isn't possible.

As the table (compiled) on next page shows, prefix property is satisfied.
Observation: We find ∃ lesser no. of bits with message of higher probability.

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Message	Probability	Code 1	Code 2	Code 3	Final Code
m_2	0.35	0	0 0	00	0 0
m_4	0.25	0	0 1	0 1	0 1
m_3	0.15	1	1 0	10	10
m_1	0.10	1	1 1	110	110
m_5	0.10	1	1 1	111	1110
m_6	0.05	1	1 1	111	1111

length = 1 length = 2 length = 3 length = 4

s3) Finding efficiency of the code

$$\text{Efficiency, } \eta = \frac{H(S)/\log_2 r}{L_{av}} = \frac{H(S)}{L_{av}} \quad ; r = 2$$

$$\rightarrow L_{av} = \sum_{i=1}^{N} P_i l_i = \begin{pmatrix} 0.35\times2 + 0.25\times2 + \\ 0.15\times2 + 0.10\times3 + \\ 0.10\times3 + 0.05\times4 \end{pmatrix}$$

bits/message

$\rightarrow H(S) =$ avg. info. per message of source

$$= -\sum_{i=1}^{N} P_i \log_2 P_i$$

$$= -\left(0.35 \log_2 0.35 + 0.25 \log_2 0.25 + \cdots 0.05 \log_2 0.05\right)$$

using calculator →
$$\frac{0.35 \times \log_{10} 0.35}{\log_{10} 2}$$

bits/message

→ if $m_1, m_2, ..$ are called messages (can also be called SYMBOLS)

Say, we get $\eta = .95$ or 95%

So, remaining 5% is redundancy in the code.

Method 2) Huffman's method

(usually gives higher efficiency than Shannon-Fano method)

Considering sequence of 6 messages again:

$\{m_1, m_2, m_3, m_4, m_5, m_6\}$

Probability: $\{0.10, 0.35, 0.15, 0.25, 0.10, 0.05\}$

S1) Again arrange them in decreasing order of probability

Msg.	Prob	Final Code	Reduced Source 1		Reduced source 2		Reduced source 3		Reduced Source 4	Code
m_2	0.35	00	0.35	00	0.35	00	0.40	1	0.60	0
m_4	0.25	01	0.25	01	0.25	01	0.35	00	0.40.	1
m_3	0.15	11	0.15	11	0.25	10	0.25	01		
m_1	0.10	101	0.15	100	0.15	11				
m_5	0.10	1000	0.10	101						
m_6	0.05	1001								

Again, $r = 2$ & alphabets are $(0, 1)$

In the rearranged sequence, go to the last row. Select last r messages (i.e select 2 messages). Now, collapse them (add their probabilities ($m_5 + m_6 = 0.15$))

Now, leave a column for final code & make columns for Reduced source 1.

Now, we got $m_5 + m_6 = 0.15$. We already have $m_3 = 0.15$. So, Put that accordingly & revise the sequence

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Idea: Start from back make

0.6	0
0.4	1

Now, 0.4 is in reduced source 3 so, its 1.

& 0.35   0 0

0.25   0 1

for 0.6 = 0

higher = 0

lower = 1

So, 0 0

0 1

Next 0.4 was = 0.25 + 0.15

So, make their start as 1 0 → 0.25

1 1 → 0.15

0.35 & 0.25 remain same as 00 & 01 as above.

Next 0.25 = 10

it was composed of 0.15   10 0

0.10   10 1

0.15 stays 11. So do 0.35 & 0.25.

Next, 0.15 is composed 0.10 & 0.05

So, make 0.10   100 0

0.15   100 1.

rest remain same. So, we get final code.

Next find efficiency, just like in (M1) & find optimum Ans.

★ Note :-

DRAWING SPECTRUM

Given a signal :-

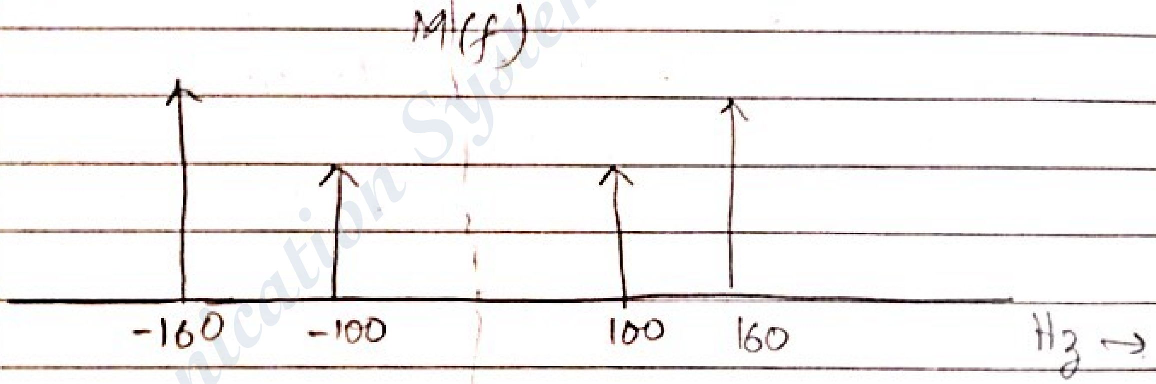
$m(t) = \cos 200\pi t + 4\cos 320\pi t$

$f_s = 250\,Hz$

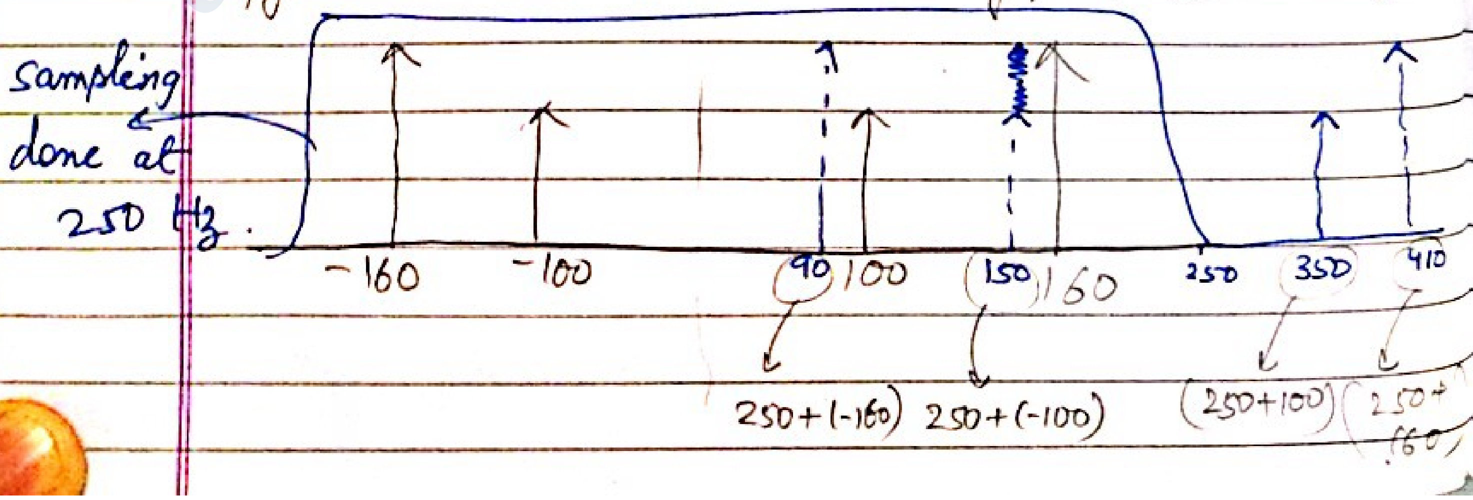
$f_1 = \frac{200\pi}{2\pi} = 100\,Hz$

$f_2 = \frac{320\pi}{2\pi} = 160\,Hz$

Drawing 2 impulses :-



Sampling means, repeat the same spectrum & copy it ∞ times, with the gap of f_s (250 Hz)



Considering the +ve part, the freq. coming within f_s (250 Hz) =

90, 100, 150, 160 Hz

Important : Portion for Compre :

✓ Information theory

✓ Coding techniques.

✓ Noise in AM/FM

✓ Matched filter.

CHANNEL CODING

Source encoding : Assumption : Channel is noiseless
↳ we find : average length of code
: efficiency of code (by increasing)
: by reducing redundancy

↳ We talk in terms of DISTANCE between Code Words (called as HAMMING DISTANCE)

no. of bit posns in which the 2 code words differ

Consider

(C_1) code word 1 : 0 1 1 0 1 0

(C_2) " " 2 : 1 0 1 1 1 0

different ← ✓ ✓ ✓
C_1 & C_2

$$d(C_1, C_2) = 1+1+1 = 3$$

Consider 3 bit code

So, in binary, 2^3 sequences are possible (000 --- 111)

let them correspond to 8 distinct messages (m_1 --- m_8)

Automatic Request for Retransmission : ARQ

m_1	000
m_2	001
m_3	010
m_4	011
m_5	100
m_6	101
m_7	110
m_8	111

Consider channel noise (good amt of it) s.t at receiver side $\exists$ one bit error.

So, suppose sender sends

Sender	Receiver gets
m_4 →	
011	0(0)1

error has occured

So receiver thinks m_2 (001) was sent. So, he won't get to know error occurs.

So, the way of coding this way is improper because receiver cant know error.

★ Burst errors :
Errors which had lower probability (say 1 in 1000000) but, it occured many times in a short interval.

★ Error detection :
Providing enough structure so that receiver knows when error has occured.

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In previous page, we saw that 3 bits were making more chances that receiver is NOT able to detect error.

So,

now choose 4 bits & any 8 message out of 16 combin^ns (say, we choose any 8)

m_1	0000	hamming distance = min. of 2 bits b/w adjacent messages
m_2	0011	
m_3	0101	
m_4	0110	
m_5	1001	
m_6	1010	
m_7	1100	
m_8	1111	

Hamming weight : no. of 1's in a code word.

Finding hamming distance : min. hamming distance is the min. hamming weight (ignoring hamming weight = 0)

		HW → Hamming weight
m_1	0000	0 → ignore
m_2	0011	2
m_3	0101	2
m_4	0110	2
m_5	1001	2
m_6	1010	2
m_7	1100	2
m_8	1111	4

So min hamming distance = 2

FEC : forward error correcting codes

Suppose I send m_3 from sender side.

$m_3 = 0101$

Now, considering 1 bit error in m_3

errors in m_3 (1 bit) :

1101
0001
0111
0100

→ If receiver receives any of these messages, it doesn't match any of $m_1 - - m_8$. So, he will know that something is wrong & he will send an ARQ signal.

Similarly, this error can be detected if any other message is sent from sender.

ERROR CORRECTION :

If the added structure is sufficiently detailed to allow pinpointing of locⁿ of error, then, code is error correcting code.

Such structures/codes come under :

Forward Error Correction Codes (FEC codes)

Idea : Add Redundancy

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* Redundancy :
Increase hamming distance b/w pair of code words

One of the many set of codes :

ace : Algebraic codes
↳ bits are added in blocks (called Block codes)
Consider message of k bits
Add redundancy bits → (m bits)
So, we get

* n-bit code word = $\underbrace{k\text{-bits}}_{\text{inform}}$ + $\underbrace{m\text{-bits}}_{\text{redundancy}}$

redundancy ↓ "Check bits"

So, basically,

* code word = $c_1, c_2, \ldots, c_m, v_1, v_2, \ldots, v_k$

↳ "Systematic Code"

eq^n :

$$\bar{v} = \bar{u}\,[G]$$

$\bar{u}$: message vector
$[G]$: Generator matrix

↳ $u \rightarrow (1 \times k)$ vector : Info. bits
$v \rightarrow (1 \times n)$ row vector : Code word
$G \rightarrow (k \times n)$ matrix : Generating matrix.

(basically I'm expanding my message "u" by multiplying with "G")

So, our code word is

(n, k) linear code

$n \to$ code word size

$k \to$ message block size.

eg :- Consider $(7,4)$ code

$$
\begin{array}{c}
\begin{array}{ccccccc} c_1 & c_2 & c_3 & & & & \end{array} \\
[G] = \left[\begin{array}{ccc:cccc}
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1
\end{array}\right]_{4\times 7}
\end{array}
$$

Identity matrix

$7-4 = 3$

So, 3 check bits

$\to$ 4 message bits, 3 check bits $\}$ 7 bit code

If $\exists$ identity matrix in G matrix, its Systematic code (So, i.e, message bits are in correct sequence)

Inform^n. bits				Code	Word	H (Hamming weight)
0	0	0	0	000	0000	0
0	0	0	1	101	0001	3
0	0	1	0	111	0010	4
0	0	1	1	010	0011	3
0	1	0	0	011	0100	3
0	1	0	1	110	0101	4
0	1	1	0	100	0100	3
0	1	1	1	001	0111	4
1	0	0	0	110	1000	3
1	0	0	1	011	1001	4
1	0	1	0	001	1010	3
1	0	1	1	100	1011	4
1	1	0	0	101	1100	4
1	1	0	1	001	1101	3
1	1	1	0	010	1110	4
1	1	1	1	111	1111	7

first 3 bits are Check bits $c_1\, c_2\, c_3$

Comments :-

> First parity bit provides even parity when combined with 1st 3rd & 4th info bits

> 2nd parity bit provides even parity when combined with 1st, 2nd & 3rd info bits

> 3rd parity bit provides even parity when combined 2nd 3rd & 4th info bits

PTO

Generating code word for each message

Say, [0 1 1 1], we multiply it with G matrix

Here, MODULO-2-ADDITION is applied
$$\begin{cases} 0 \oplus 0 = 1 \oplus 1 = 0 \\ 0 \oplus 1 = 1 \oplus 0 = 1 \end{cases}$$

So,

$$[0\,1\,1\,1]\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

① $(0\times1)\oplus(1\times0)\oplus(1\times1)\oplus(1\times1) = 0\oplus0\oplus1\oplus1$
$= 0$

② $(0\times1)\oplus(1\times1)\oplus(1\times1)\oplus(1\times0) = 0\oplus1\oplus1\oplus0$
$= 0$

- - - - - -

$= [0\ 0\ 1\ 0\ 1\ 1\ 1]$

* If d_{min} is the min. distance b/w code words, so, $(d_{min}-1)$ errors can be detected.

i.e.,
So, for $d_{min} = 1$ bit, $d_{min}-1 = 0$. So, we can't detect any error. So, $d_{min} = 2$ atleast

* If d_{min} = even :- Correction up to $\frac{d_{min}-2}{2}$ errors
* If d_{min} = odd :- $\frac{d_{min}-1}{2}$ errors can be corrected

Finding d_{min} :-

Write Info bits & Corresponding Hamming weights. Ignore (0000) & see the min. hamming weight. That is d_{min}

For increasing d_{min}, we add additional bits.

Seeing even parity code :-

$C_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 1st, 2nd, 3rd, 4th info bit

So, C_1 can show its parity for 1st 3rd & 4th info bit.

So for any code word say

for 0011 → $\overset{C_1 C_2 C_3}{000}$ 0011

first parity bit, 2nd parity bit, 3rd parity bit.

Now combining C_1 with 0011

i.e 0 0011 → even 1's.

So, even parity.

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* Parity Check Matrix [H]

Consider a (6, 3) code.

no. of bits in a block of message

no. of parity check bits = 6 - 3 = 3 = $C_1 \; C_2 \; C_3$

messages = $U_1 \; U_2 \; U_3$

So, code word is given by :

$$\underbrace{[C_1 \; C_2 \; C_3 \; U_1 \; U_2 \; U_3]}_{\bar{V}} = \underbrace{[U_1 \; U_2 \; U_3]}_{\bar{u}} \underbrace{\begin{bmatrix} g_{11} & g_{12} & g_{13} & 1 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & 0 & 1 & 0 \\ g_{31} & g_{32} & g_{33} & 0 & 0 & 1 \end{bmatrix}}_{\text{Generator matrix } [G]}$$

Solving LHS & RHS

$\Rightarrow$

$$\left.\begin{aligned} C_1 &= U_1 g_{11} + U_2 g_{21} + U_3 g_{31} \\ C_2 &= U_1 g_{12} + U_2 g_{22} + U_3 g_{32} \\ C_3 &= U_1 g_{13} + U_2 g_{23} + U_3 g_{33} \end{aligned}\right\} \rightarrow (A)$$

Now, doing modulo-2 addition to eq^n (A).

$$C_1 + C_1 = C_1 + U_1 g_{11} + U_2 g_{21} + U_3 g_{31}$$
$$C_2 + C_2 = C_2 + U_1 g_{12} + U_2 g_{22} + U_3 g_{32}$$
$$C_3 + C_3 = C_3 + U_1 g_{13} + U_2 g_{23} + U_3 g_{33}$$

Now, for modulo-2 addition,

$1+1 = 0$ & $0+0 = 0$.

& so, $C_1 + C_1 = 0 = C_2 + C_2 = C_3 + C_3$

$\Rightarrow$ we get,

$$C_1 + U_1 g_{11} + U_2 g_{21} + U_3 g_{31} = 0$$
$$C_2 + U_1 g_{12} + U_2 g_{22} + U_3 g_{32} = 0$$
$$C_3 + U_1 g_{13} + U_2 g_{23} + U_3 g_{33} = 0$$

Putting it again in matrix form

$$[c_1\ c_2\ c_3\ u_1\ u_2\ u_3]_{1\times 6} \times \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{23} & g_{33} \end{bmatrix}_{6\times 3}} = [0\ 0\ 0] = \bar{0}$$

☆ Transpose of parity check matrix $[H]^T$

hence, parity check matrix, $[H]$ is

☆ $$\boxed{[H] = \begin{bmatrix} 1 & 0 & 0 & g_{11} & g_{21} & g_{31} \\ 0 & 1 & 0 & g_{12} & g_{22} & g_{23} \\ 0 & 0 & 1 & g_{13} & g_{23} & g_{33} \end{bmatrix}}$$

☆ At the receiver's side, this matrix is used to give error correction capability of code.

* Consider previously taken generator matrix

$$[G] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

taking one of the code words (out of 16 written before):

$$\bar{v} = [1\ 0\ 0\ 1\ 0\ 1\ 1]$$

assuming we got this at receiver. $\bar{r}$

H^T Taken from generator matrix

$$\bar{V} \times [H]^T = \underset{1\times 7}{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}} \underset{7\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}$$

Checking if the coded word is same in receiver side i.e. $\bar{r} \times [H]^T$

$$= \begin{bmatrix} 1+1+1+1 & 0+0+0+1+0+1+0 & 0+0+0+0+0+1+1 \end{bmatrix}$$

modulo addition

$$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

it's a valid code word

$$\boxed{\bar{V} \cdot [H]^T = \bar{O}}$$

If this cond^n is satisfied, it is a valid CODE WORD.

Now, suppose $\exists$ 1 bit error in code

Say,

$$\bar{V} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

1 bit error

$$\bar{r} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Now,

$$\bar{r} = \bar{V} + \bar{e} \quad \text{(can be written)}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

formula ★ $d_{min} \geq 2t + 1$

↳ t : amount of error correction
↳ i.e., for 1 bit error $t = 1$

Now,

$$\bar{r}\,[H]^T = [\bar{v} + \bar{e}]\,[H]^T$$

$$= \bar{v}\,[H]^T + \bar{e}\,[H]^T$$

$$= \bar{0} + \bar{e}\,[H]^T$$

$$= \underbrace{\bar{e}\,[H]^T}$$

→ will match one of the rows of $[H]^T$ corresponding to error position

Now, seeing this error

$$\bar{r} \times [H]^T = [1\;0\;1\;\;1\;0\;1\;1] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= [1+1+1+1 \quad 1+1 \quad 1+1+1]$$

$$= \underbrace{[0 \quad 0 \quad 1]}$$

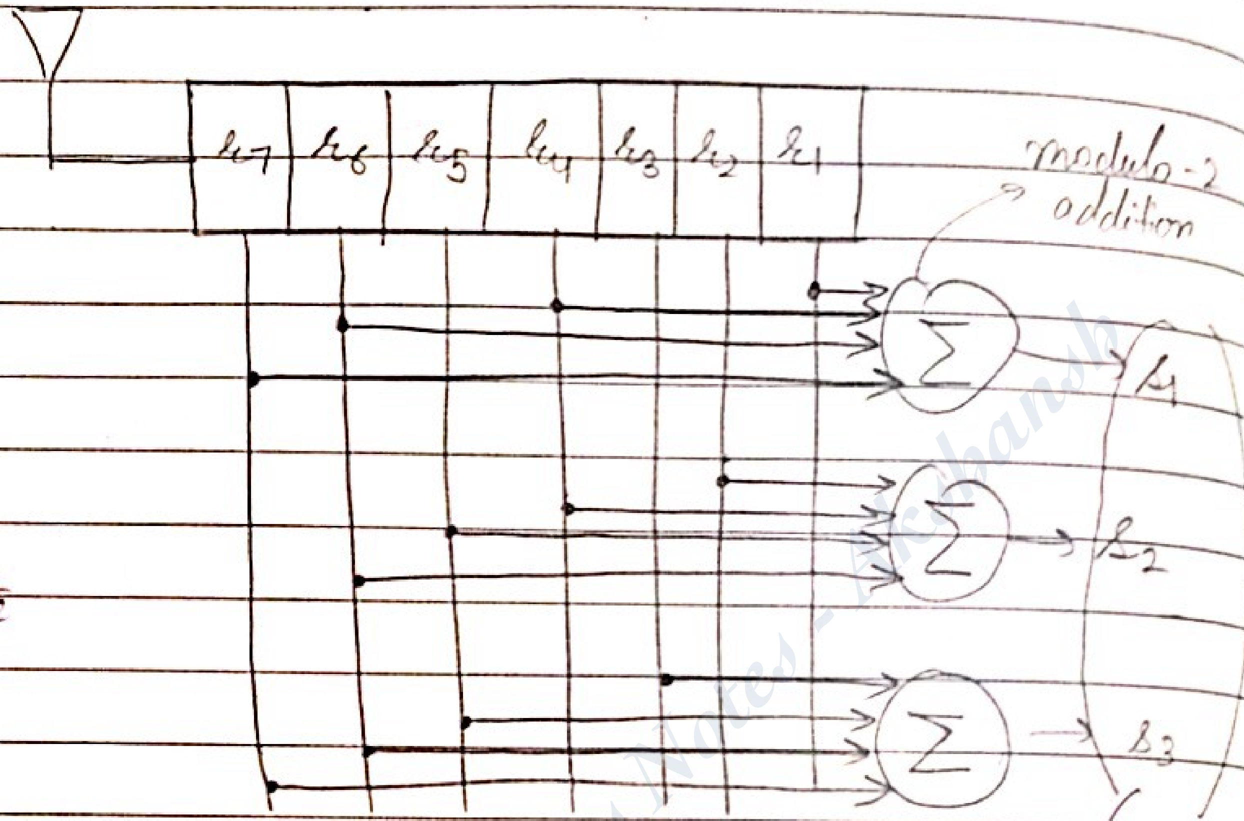
→ This matches with 3rd row in $[H]^T$ matrix. So, we have to check the 3rd bit & then error can be corrected.

★ $\bar{r} \times [H]^T$ is called SYNDROME VECTOR

$= \bar{0}$: ∃ no error

$\neq 0$: ∃ error (see $[H]^T$)

Seeing receiver's side :



Tapping appropriate bits of $\bar{r}$.

So, this is $\bar{r} \times [H]^T$ (multiplication done using shift registers)

We had

$$[H]^T = \begin{array}{c}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}\end{array}\begin{array}{l} \text{1st bit} \\ \text{2nd} \\ \text{3rd} \\ \text{4th} \\ \text{5th} \\ \text{6th} \\ \text{7th} \end{array}$$

1st, 4th, 6th & 7th have to be tapped.

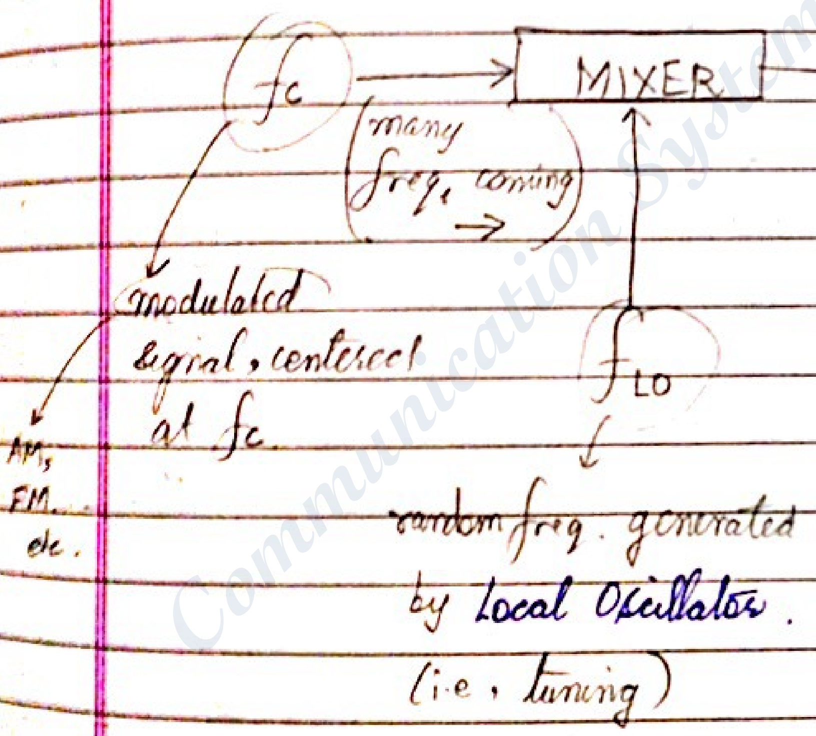
2nd, 4th, 5th & 6th bit have to be tapped

3rd, 5th, 6th & 7th have to be tapped

5 Noise in analog communication systems

As mentioned in beginning, all receivers follow Heterodyne principle.

(Heterodyne ≡ mixing)

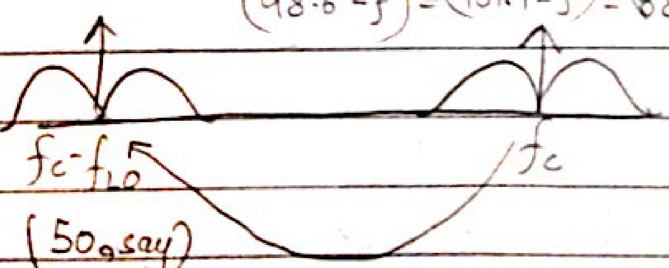


Suppose 3 freq's are coming from i/p → 98.6, 101.4, 88.8

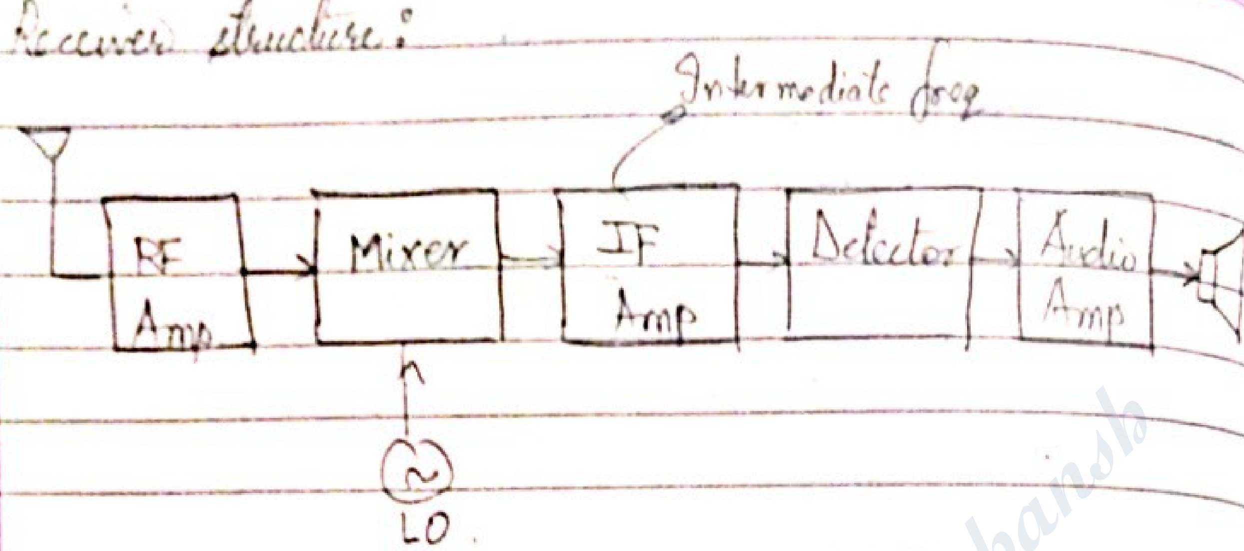
Now, Suppose any freq, say 50 whichever freq I want to choose that is shifted to same ft. f s.t

$(98.6 - f) = (101.4 - f) = (88.8 - f) = 50.$

from spectrum :



Receiver structure:



Idea: As told before, what I'm doing is, I am making a good amp. circuit at 455 kHz (std. value in practical purpose) → f_{IF}, say.

Now, whatever signal comes from i/p, say f_c, RF amplifier and Local Oscillator coordinate with each other such that freq given by LO is $f_c + f_{IF}$

Now, what this does is, the o/p of mixer is always f_{IF} (455 kHz).

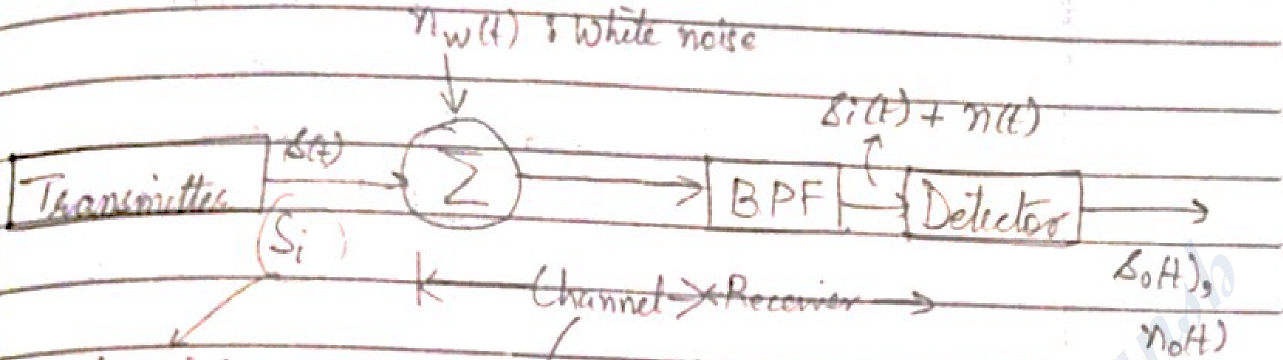
I have made proper arrangement at f_{IF} so the got i/p signal can be properly amplified & heard.

* Few values:

$\frac{S}{N}$ ratio = 30-35 dB → Telephone
50 dB → FM

White Noise :
* Any random process which is assumed to have constt power spectral density.

BANDPASS Noise



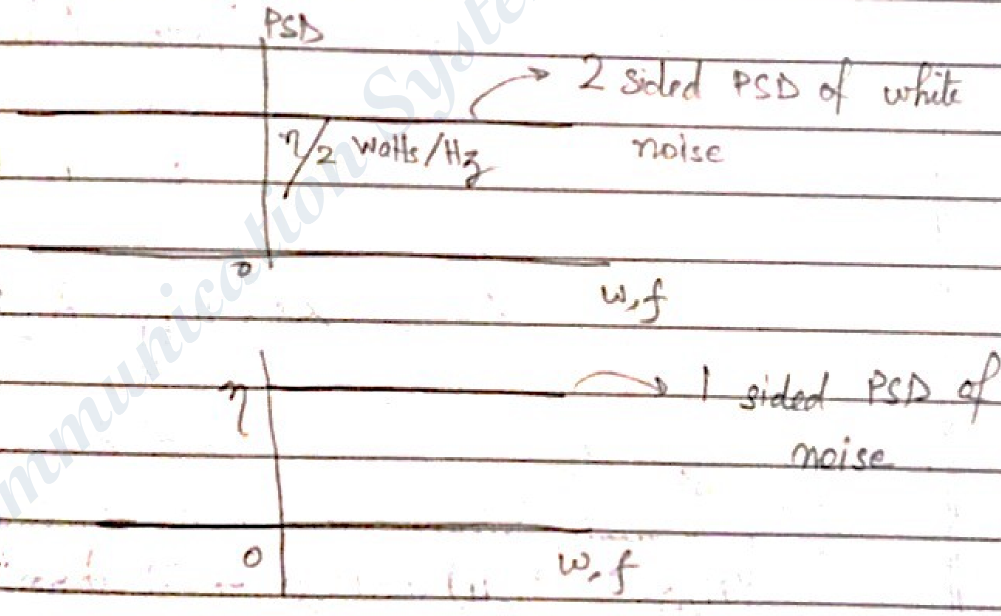
i/p signal power

AWGN Channel
Additive White Gaussian Noise

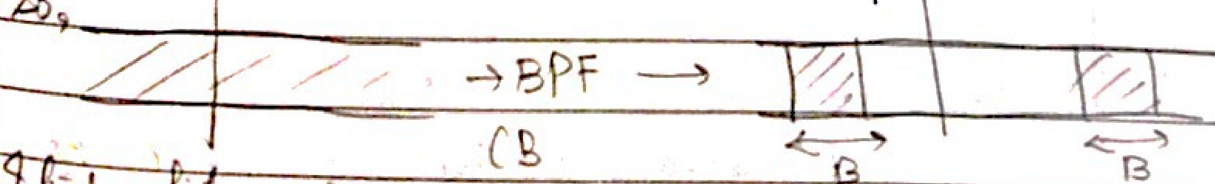
S_o, N_o

* Power Spectral Density (PSD)
A curve / f^n, area under which gives power (over area of interest → BW)

Note :



So, White Noise + BPF = Bandpass Noise



Infinite white noise power, $n_w(t)$

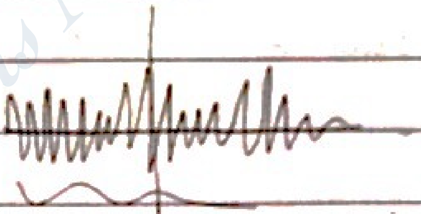
(B band width, say)

Finite White Noise power, $n(t)$ (taken by detector)

* Noise Generators are available commercially
(White noise + BPF $\longrightarrow$ Some part of white noise left. So, now, doesn't remain white
(VIBGYOR) $\overset{\text{Total}}{=}$ white
↳ yellow noise
↳ Pink noise
Depending upon how much we are taking

* LAB Observation

White noise $\longrightarrow$ BPF $\longrightarrow$ Bandpass Noise,
$n_w(t)$ $\qquad\qquad\qquad\qquad$ $n(t)$



↳ A cosine f^n with random amplitude & phase.

So, $n_{(t)} = A_{(t)} \cos(\omega_c t + \phi_{(t)})$

So, $n_{(t)} = A_{(t)} \cos(\phi_{(t)}) \cos\omega_c t - A_{(t)} \sin(\phi_{(t)}) \sin\omega_c t$

$\Rightarrow \boxed{n_{(t)} = n_c(t) \cos\omega_c t - n_s(t) \sin\omega_c t}$.

↳ expression for BP noise

Recall, $s(t) = m(t)\cos\omega_c t - \hat{m}(t) \sin\omega_c t$ is expression of SSB-SC. So, this is similar to BP noise expression. So, any thing coming out of BPF has same expression, be it signal/noise.

* Properties of Bandpass Noise.

P① Mean squared value of BP noise = Mean squared value of individual components.

i.e., $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$ ($= \eta B$, for white noise)

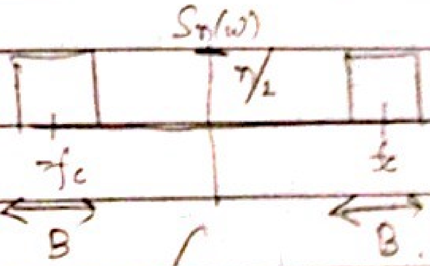
↳ where $n(t)$ = bandpass process
$n_c(t), n_s(t)$ = lowpass process

BP is at ω_c & my expression is $n_c(t)\cos\omega_c t$. If $n(t)$ is at ω_c, then multiplic with $\cos\omega_c t$ will make spectrum move to $2\omega_c$ & so, to at 0.

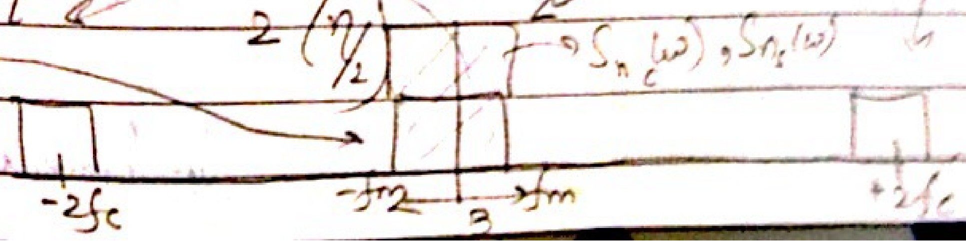
P②:-

$$S_{n_c}(\omega) = S_{n_s}(\omega) = \begin{cases} S_n(\omega+\omega_c) + S_n(\omega-\omega_c) & ; |\omega| < \omega_m \\ 0, & \text{elsewhere} \end{cases}$$

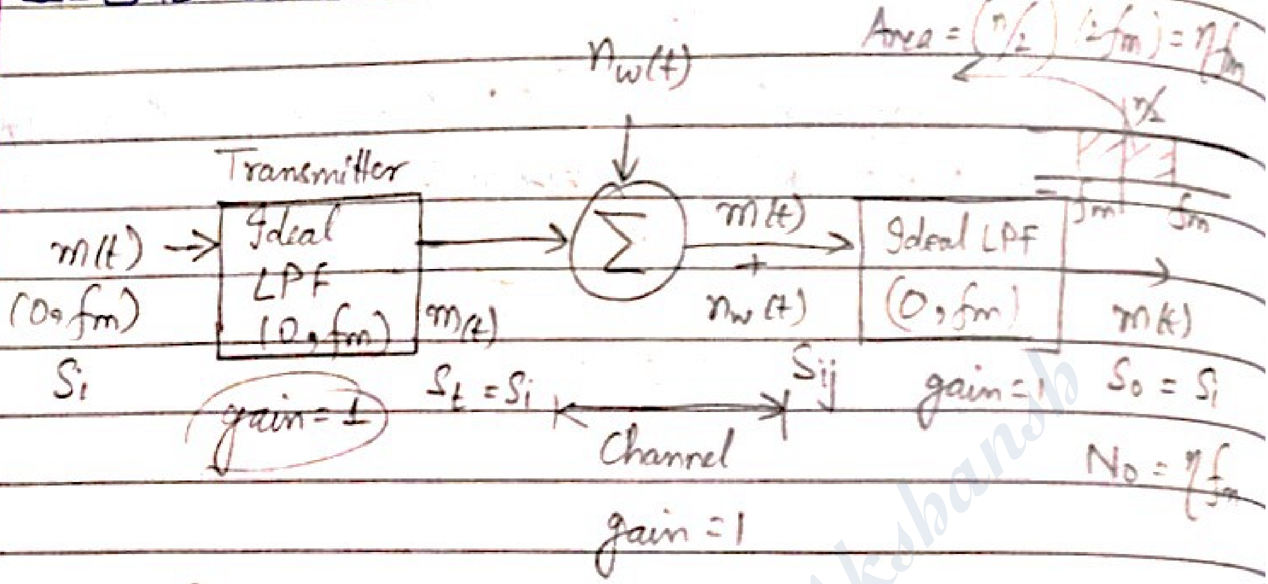
I have

$S_n(\omega) =$ 

Area = ηB



IDEAL BASEBAND COMMUNICATION SYSTEM

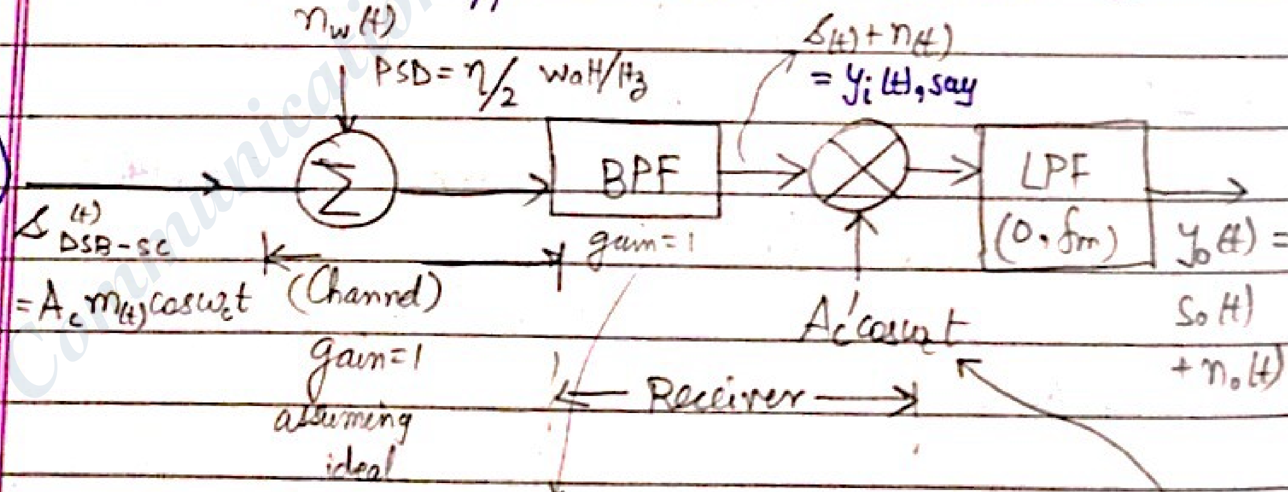


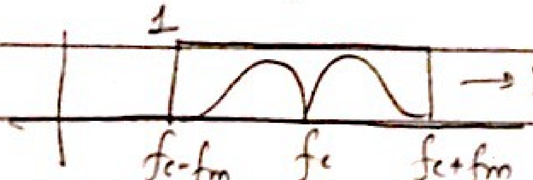
So, $\boxed{\frac{S_o}{N_o} = \frac{S_i}{\eta f_m}} = \gamma$, gamma

↳ Signal to noise ratio for ideal baseband communicatⁿ sys.

⟨1⟩

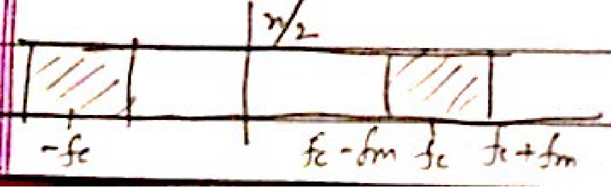
Noise Analysis for Double-sideband suppressed Carrier (DSB-SC)





→ Detection done by Coherent/Synchronous detector

So, this analysis is said as Synchronous/Coherent detection



Note. $s(t) = s_{DSB-SC}(t)$ (∵ gain = 1 in movement)

So,

$$y_i(t) = A_c m(t)\cos\omega_c t + n(t)$$

$$= \underbrace{A_c m(t)\cos\omega_c t}_{\text{Power, (mean sq. value)}} + \underbrace{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t}_{\text{Quadrature represent}^n\text{ of noise}}$$

$n_c(t)$: In-phase component of noise
$n_s(t)$: Quadrature component of noise

$$S_i = \overline{\left[A_c m(t)\cos\omega_c t\right]^2} \qquad N_i = \overline{n^2(t)}$$

$$= (A_c^2)\left(\overline{m^2(t)}\right)\left(\tfrac{1}{2}\right) \qquad = \left(\eta/2\right)\left(f_c+f_m-(f_c-f_m)\right) \times 2$$

$$= 2\times \eta/2 \times 2f_m$$

$$= 2\eta f_m$$

Now,

$$y_o(t) = \underset{(0,f_m)}{LPF}\left[y_i(t)\times A_c'\cos\omega_c t\right]$$

$$= \underset{(0,f_m)}{LPF}\left\{\begin{array}{c} A_c A_c' m(t)\underbrace{\cos^2\omega_c t}_{\frac{1}{2}(1+\cos 2\omega_c t)} \\ + \\ A_c' n_c(t)\underbrace{\cos^2\omega_c t}_{\frac{1}{2}(1+\cos 2\omega_c t)} \\ + \\ - A_c' n_s(t)\underbrace{\cos\omega_c t\,\sin\omega_c t}_{\frac{1}{2}\sin 2\omega_c t} \end{array}\right\}$$

Now, expand it and see which spectrum will be retained by LPF.

$So,$

$$y_0(t) = \underset{(0,f_m)}{LPF}\left\{ \underbrace{\frac{1}{2}A_cA_c' m(t)}_{b/w\ 0\ to\ f_m} + \overbrace{\frac{1}{2}A_cA_c' m(t)\cos 2w_ct}^{at\ 2w_c \to blocked} + \underbrace{\frac{1}{2}A_c' n_c(t)}_{b/w\ 0\ to\ f_m} + \overbrace{\frac{1}{2}A_c' n_c(t)\cos 2w_ct}^{at\ 2w_c \to blocked\ by\ LPF} + \underbrace{-\frac{1}{2}A_c' n_s(t)\sin 2w_ct}_{at\ 2w_c \to blocked\ by\ LPF} \right.$$

$\because$ $n_c(t)$ has LP nature

$$\Rightarrow y_0(t) = \underbrace{\frac{1}{2}A_cA_c' m(t)}_{\substack{signal\\component\\s_0(t)}} + \underbrace{\frac{1}{2}A_c' n_c(t)}_{\substack{noise\\component\\n_0(t)}}$$

O/p power: Signal, $S_0 = \overline{\left[\frac{1}{2}A_cA_c' m(t)\right]^2}$

$$= \left(\frac{1}{4}\right)(A_c^2)(A_c')^2\ \overline{m^2(t)}$$

Noise, $N_0 = \overline{\left[\frac{1}{2}A_c' n_c(t)\right]^2}$

$$= \left(\frac{1}{4}\right)(A_c')^2\ \overline{n_c^2(t)}$$

$$= \left(\frac{1}{4}\right)(A_c')^2\ \overline{n^2(t)} \quad \text{from PD}$$

$$= \left(\frac{1}{4}\right)(A_c')^2\ \underbrace{(2\eta f_m)}_{N_i}$$

Now, $$\frac{S_0}{N_0} = \frac{\frac{1}{4}A_c^2(A_c')^2\ \overline{m^2(t)}}{\frac{1}{2}(A_c')^2(\eta f_m)}$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{A_c^2\,\overline{m^2(t)}}{2\eta f_m}$$

Now, $\frac{A_c^2\,\overline{m^2(t)}}{2} = S_i$

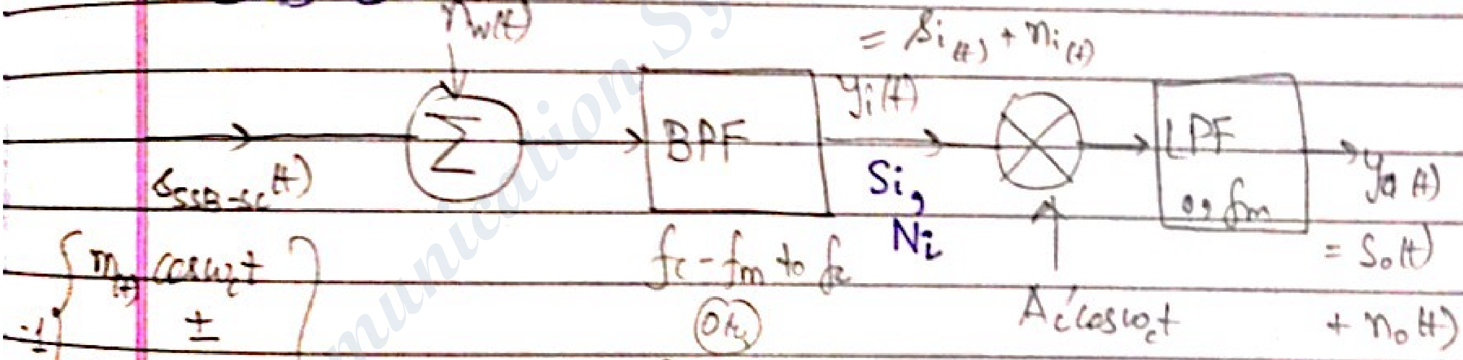
$$\Rightarrow \frac{S_o}{N_o} = \frac{S_i}{\eta f_m}$$

$$\Rightarrow \frac{S_o}{N_o} = \gamma$$

↳ same as of ideal baseband communicⁿ sys.

(2) Noise Analysis for SSB-SC:

→ The width of BPF has reduced to half as from DSB-SC ($2f_m \to f_m$)



$$s_{SSB-SC}(t) = \frac{1}{2}\left\{ m(t)\cos\omega_c t \pm \tilde{m}(t)\sin\omega_c t \right\}$$

$S_i = \overline{[m(t)\cos\omega_c t]^2} \rightarrow ①$

$+ \overline{[\tilde{m}(t)\sin\omega_c t]^2} \rightarrow ②$

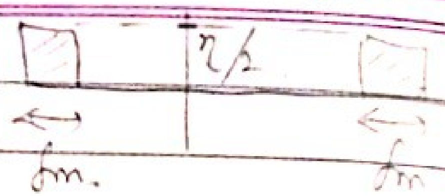
$+ \overline{[2m(t)\tilde{m}(t)\cos\omega_c t\,\sin\omega_c t]^2} \rightarrow 0$ (as its product of even & odd fⁿ = odd fⁿ's mean = 0)

② is Hilbert transform of ①.

$$\Rightarrow S_i = \overline{m^2(t)}\left(\tfrac{1}{2}\right) + \overline{\tilde{m}^2(t)}\left(\tfrac{1}{2}\right)$$

$$S_i = \overline{m^2(t)}$$

$$N_i = 2\left(\frac{\eta}{2}\right)(f_m)$$



$$\Rightarrow N_i = \eta f_m$$

$$y_i(t) = \underbrace{m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t}_{\substack{\text{message} \\ s_i(t)}} + \underbrace{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t}_{\substack{\text{noise} \\ n_i(t)}}$$

Now

$$y_o(t) = \underset{(0,f_m)}{LPF}\left[y_i(t) \times A_c'\cos\omega_c t\right]$$

$$= \underset{(0,f_m)}{LPF}\left\{\begin{array}{l} A_c' m(t)\underbrace{\cos^2\omega_c t}_{\frac{1}{2}(1+\cos 2\omega_c t)} - A_c' \hat{m}(t)\underbrace{(\sin\omega_c t\cos\omega_c t)}_{\frac{1}{2}(\sin 2\omega_c t)} \\ + \\ A_c' n_c(t)\cos^2\omega_c t - A_c' n_s(t)\underbrace{(\cos\omega_c t\sin\omega_c t)}_{\frac{1}{2}(\sin 2\omega_c t)} \end{array}\right.$$

$$= \underbrace{\frac{1}{2}A_c' m(t)}_{s_o(t)} + \underbrace{\frac{1}{2}A_c' n_c(t)}_{n_o(t)}$$

$$S_o = \frac{1}{4}(A_c')^2\,\overline{m^2(t)} \qquad N_o = \frac{1}{4}(A_c')^2\,\overline{n_c^2(t)}$$

$$= \frac{1}{4}(A_c')^2(\eta f_m)$$

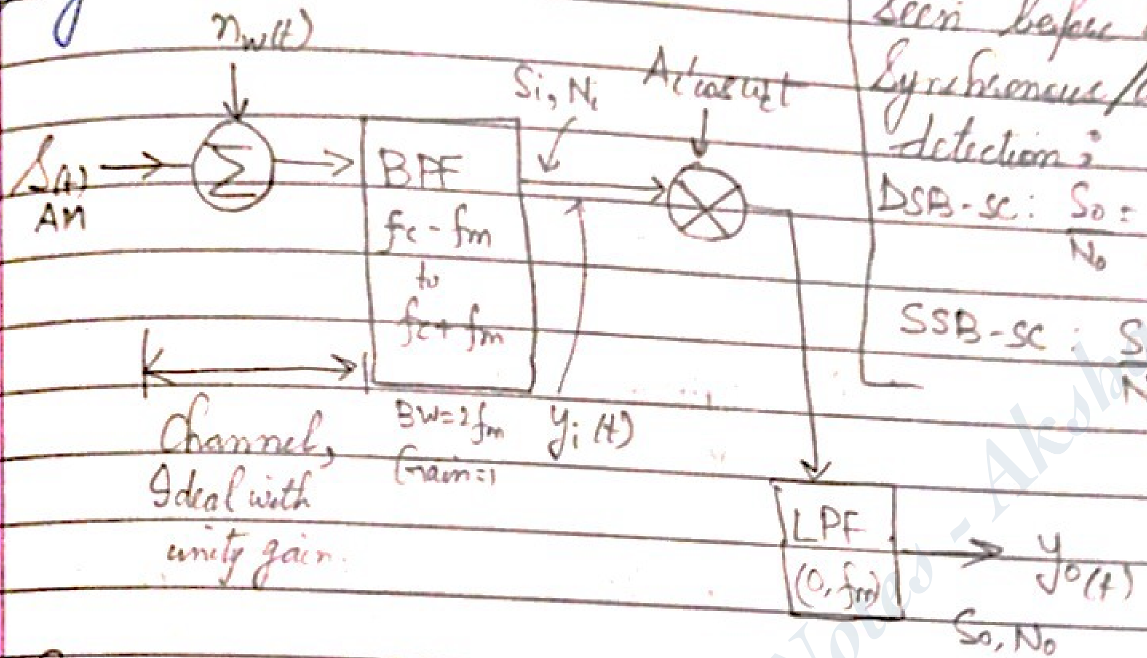
So,

$$\frac{S_o}{N_o} = \frac{\cancel{\left(\frac{1}{4}\right)}(A_c')^2\,\overline{m^2(t)}}{\cancel{\left(\frac{1}{4}\right)}(A_c')^2(\eta f_m)} = \frac{S_i}{\eta f_m} = \gamma$$

So, SSB-SC is as good as DSB-SC is as good as Ideal Baseband Comm. Sys.

⟨3⟩ Noise Analysis for Conventional AM.

(A) Synchronous detection.



Seen before: Synchronous/coherent detection:

DSB-SC: $\dfrac{S_o}{N_o} = \dfrac{S_i}{\eta f_m} = \gamma$

SSB-SC: $\dfrac{S_o}{N_o} = \gamma$

here.

$$y_i(t) = \underbrace{A_c\left[1 + k_a m(t)\right]\cos\omega_c t}_{s_i(t)} + n(t)$$

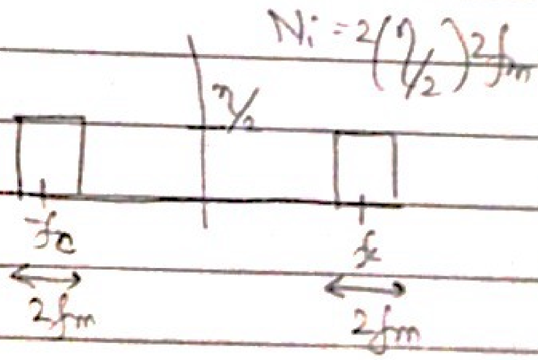
Now,

$$S_i = \overline{\left(A_c(1 + k_a m(t))\cos\omega_c t\right)^2}$$

$$= (A_c^2)\left(1 + k_a^2\,\overline{m^2(t)} + 2k_a\,\overline{m(t)}\right)\frac{1}{2}$$

Taking only voice & music signal, the mean of signal $= 0$, i.e, $\overline{m(t)} = 0$.

$$\Rightarrow S_i = \frac{A_c^2}{2}\left[1 + k_a^2\,\overline{m^2(t)}\right].$$



$$\& \quad N_i = 2\eta f_m = \overline{n^2(t)}.$$

$$y_0(t) = \underset{(0, f_m)}{LPF}\left[y_r(t) \times A_c' \cos\omega_c t\right]$$

$$= \underset{(0, f_m)}{LPF}\Big[A_c A_c' [1 + k_a m(t)] \underbrace{\cos^2 \omega_c t}_{\frac{1}{2}(1+\cos 2\omega_c t)}$$

$$+ A_c' n_c(t) \cos^2 \omega_c t - A_c' n_s(t) \sin\omega_c t \cos\omega_c t \Big]$$

$$\left(\because n(t) = n_c(t) \cos\omega_c t - n_s(t) \sin\omega_c t\right)$$

$$= \frac{A_c A_c'}{2}\left(1 + k_a m(t)\right) + \underbrace{\frac{A_c A_c'}{2}\left(1 + k_a m(t)\right) \cos 2\omega_c t}_{\to 0 \text{ (removed by LPF)}}$$

$$+ \frac{A_c'}{2} n_c(t) + \underbrace{\frac{A_c'}{2} n_c(t) \cos 2\omega_c t}_{\to 0 \text{ (removed by LPF)}} - \underbrace{\frac{A_c'}{2} n_s \sin 2\omega_c t}_{\to 0 \text{ (removed by LPF)}}$$

(LPF is b/w 0 to f_m. So, anything beyond that is removed)

$$= \frac{A_c A_c'}{2}\left(1 + k_a m(t)\right) + \frac{A_c'}{2} n_c(t)$$

Now, my signal is zero mean. So, ∄ no DC component in signal

$$= \underbrace{\frac{A_c A_c'}{2} k_a m(t)}_{s_0(t)} + \underbrace{\frac{A_c A_c'}{2}}_{\text{DC component (can be removed using capacitor)}} + \frac{A_c'}{2} n_c(t)$$

So, o/p signal power $= S_0 = \dfrac{A_c^2 (A_c')^2 k_a^2 \overline{m^2(t)}}{4}$

& $N_0 = \dfrac{(A_c')^2}{4} \overline{n_c^2(t)} = \dfrac{(A_c')^2}{4} \overline{n^2(t)}$

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Date
Page

$\Rightarrow N_0 = \frac{(A_c')^2}{4} \times 2\eta f_m$

$N_0 = \frac{(A_c')^2}{2}\eta f_m$

So,

o/p SNR $= \frac{S_0}{N_0} = \frac{\frac{A_c^2 (A_c')^2}{4} k_a^2 \overline{m^2(t)}}{\frac{(A_c')^2}{2}\eta f_m}$

$$\frac{S_0}{N_0} = \frac{A_c^2 k_a^2 \overline{m^2(t)}}{2\eta f_m} \rightarrow (1)$$

We know,

$$S_i = \frac{A_c^2}{2}\left(1 + k_a^2 \overline{m^2(t)}\right)$$

$$\Rightarrow \frac{A_c^2}{2} = \frac{S_i}{\left(1+k_a^2 \overline{m^2(t)}\right)}$$

$$\Rightarrow \frac{S_0}{N_0} = \underbrace{\frac{S_i}{\eta f_m}}_{\gamma} \; \frac{k_a^2 \overline{m^2(t)}}{\left(1+k_a^2 \overline{m^2(t)}\right)}$$

→ Modulation efficiency of AM, η_{AM}

$$\Rightarrow \frac{S_0}{N_0} = \eta_{AM}\,\gamma$$

$\eta_{AM} < 1$

Inference : $\left(\frac{S_0}{N_0}\right)_{AM} < \left(\frac{S_0}{N_0}\right)_{DSB\text{-}SC \,\&\, SSB\text{-}SC}$

So, efficiency of AM < DSB-SC & SSB-SC

Considering single tone AM:

$m(t) = A_m \cos \omega_m t$

$$\overline{m^2(t)} = \frac{A_m^2}{2}$$

So,

$$\eta_{AM} = \frac{k_a^2 A_m^2/2}{1 + k_a^2 A_m^2/2} = \frac{k_a^2 A_m^2}{2 + k_a^2 A_m^2}$$

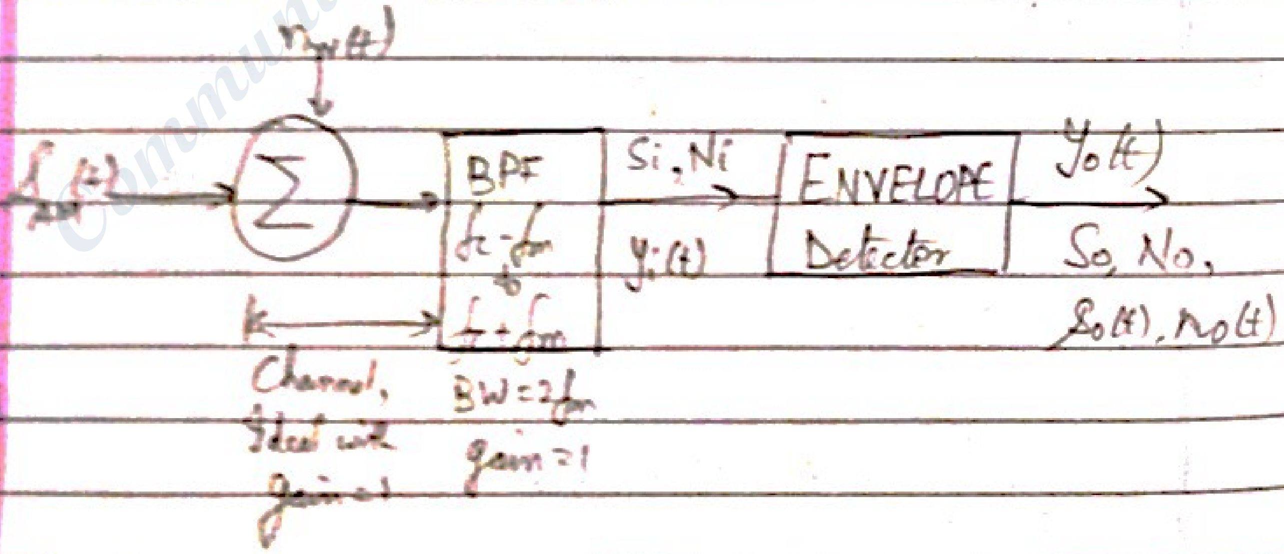
But, $k_a A_m \triangleq \mu$ (seen before)

$$\Rightarrow \eta_{AM} = \frac{\mu^2}{2+\mu^2}$$

So $$\left(\frac{S_o}{N_o}\right)_{max,\ \mu=1} = \frac{1}{3}\gamma$$

→ again performance is lower as compared to DSB-SC & SSB-SC

(B) ENVELOPE DETECTION



Just as in synchronous detection, S_i & N_i remain same. So, $S_i = (A_c^2/2)(1 + k_a^2 \overline{m^2(t)})$

$N_i = 2\eta f_m$

Now,

$$y_i(t) = A_c(1+k_a m(t))\cos\omega_c t + \underbrace{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t}_{n(t)}$$

&

$$y_o(t) = \text{Envelope of}\left\{\underbrace{\left[A_c(1+k_a m(t)) + n_c(t)\right]}_{A}\cos\omega_c t + \underbrace{\left[-n_s(t)\right]}_{B}\sin\omega_c t\right\}$$

[Note: for envelope detector, with i/p as

$$A\cos\theta + B\sin\theta \rightarrow \boxed{\text{Envelope}} \xrightarrow{\sqrt{A^2+B^2}}$$

or $\sqrt{A^2+B^2}\cos(\theta+\phi)$

(doesn't take θ & ϕ into account, only takes magnitude)

$$\Rightarrow y_o(t) = \sqrt{\left\{A_c(1+k_a m(t)) + n_c(t)\right\}^2 + (-n_s(t))^2}$$

$$\Rightarrow y_o(t) = \sqrt{A_c^2(1+k_a m(t))^2 + 2A_c n_c(t)(1+k_a m(t)) + n_c^2(t) + n_s^2(t)}$$

→ Case (1): Small noise, consider

$$\Rightarrow A_c(1+k_a m(t)) >> n_c(t),\ n_s(t)$$

(So, its square is also true)

$$y_o(t) = \sqrt{A_c^2(1+k_a m(t))^2 + 2A_c n_c(t)(1+k_a m(t))}$$

$$\Rightarrow y_o(t) \simeq A_c(1+k_a m(t))\left[\underbrace{\sqrt{1+\frac{2n_c(t)}{1+k_a m(t)}}}_{(1+x)^{1/2}\ ;\ x \ll 1}\right]$$

So,
$$(1+x)^n = 1 + nx + {}^nC_2 x^2 + \cdots$$
$$= 1 + nx, \text{ under approxim}^n, \because x \ll 1$$

$$\Rightarrow y_0(t) = A_c(1 + k_a m(t))\left(1 + \left(\frac{1}{2}\right)\left(\frac{2 n_c(t)}{A_c(1 + k_a m(t))}\right)\right)$$

$$\Rightarrow y_0(t) \simeq A_c(1 + k_a m(t)) + n_c(t)$$

$$\simeq A_c k_a m(t) + A_c + n_c(t)$$

→ DC component, needs to be removed using capacitor ($\because$ we have assumed music signal. DC component $\simeq 0$)

$$\simeq \underbrace{A_c k_a m(t)}_{S_0(t)} + \underbrace{n_c(t)}_{n_0(t)}$$

So, $S_0 = A_c^2 k_a^2 \overline{m^2(t)}$

$N_0 = \overline{n_c^2(t)} = \overline{n^2(t)} = 2\eta f_m$

So $\dfrac{S_0}{N_0} = \dfrac{A_c^2}{2} \dfrac{k_a^2 \overline{m^2(t)}}{\eta f_m} = \eta_{AM}\, \gamma$

→ same as what we got as in synchronous detector

→ Under small noise cond^n. Envelope detector performs identically as coherent detector.

* Modern day receivers are PLL receivers
Phase Lock loop
These receivers can be used for both AM & FM (Sync. detection)

Case (2) : Large noise

$\Rightarrow A_c(1+k_a m(t)) << n_c(t), n_s(t)$

So, $$y_o(t) \simeq \sqrt{n_c^2(t) + n_s^2(t) + 2A_c n_c(t)[1+k_a m(t)]}$$

$$\simeq \sqrt{n_c^2(t) + n_s^2(t)} \left[1 + \underbrace{\frac{2A_c n_c(t)(1+k_a m(t))}{n_c^2(t)+n_s^2(t)}}\right]^{1/2}$$

unlike before, this time we have $n_c(t) \times m(t)$ & $\frac{\text{message}}{\text{noise}}$ ... → message is mutilated beyond detection ability. So, we can't get detect/differentiate b/w $S_o(t)$ & $n_c(t)$. This effect is

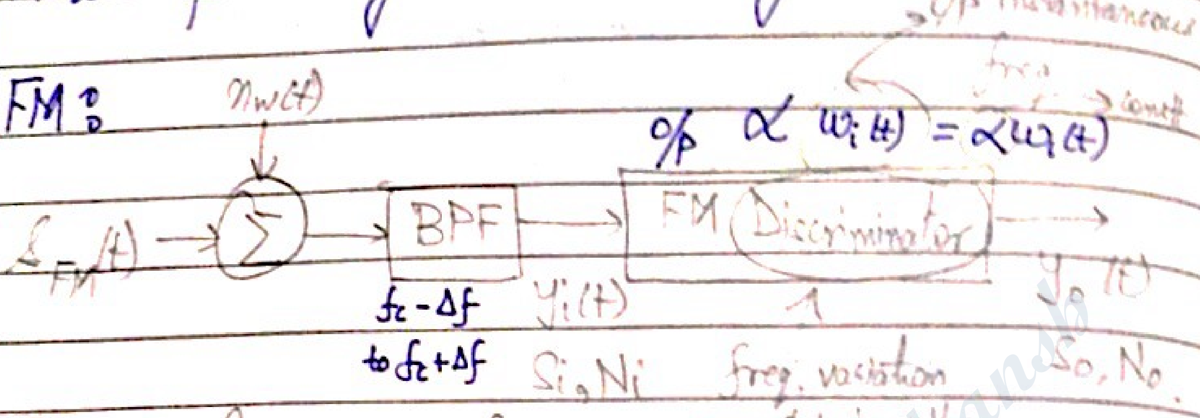
THRESHOLD EFFECT.

So, for large noise, envelope detection doesn't work.

* Discriminator : freq. to voltage converter.

(4) Noise for Angle Modulated Systems

(a) FM :



By similar approach,

$$y_i(t) = A_c \cos[\omega_c t + k_f \int m(t)\,dt] + n(t)$$

Karson's Formula (for Commercial FM)

$$B = 2(\Delta f + W)$$

$\hookrightarrow \Delta f = 75\ kHz$

$\hookrightarrow W = 15\ kHz$

here, assume, $BW \simeq 2\Delta f$

So, for BPF, it varies from $f_c - \Delta f$ to $f_c + \Delta f$ as shown

Now, Solving using Principle of Superposition

let $n(t) = 0$.

$$\Rightarrow y_i(t) = A_c \cos[\underbrace{\omega_c t + k_f \int m(t)\,dt}_{\theta(t)}]$$

$$S_i = \frac{A_c^2}{2} \rightarrow (1)$$ (Signal power : depends only on magnitude not on phase)

Now, $\omega_i(t) \triangleq \frac{d}{dt}\theta(t) = \omega_c + k_f m(t)$

Discriminator o/p $= \alpha\, \omega_i(t)$ $\rightarrow 2\omega$

$= \alpha \omega_c + \alpha k_f m(t)$

So, o/p signal power $= S_o = \alpha^2 k_f^2 \overline{m^2(t)} \rightarrow (2)$

(not taking for $\alpha\omega_c$, as its DC, ignored like before)

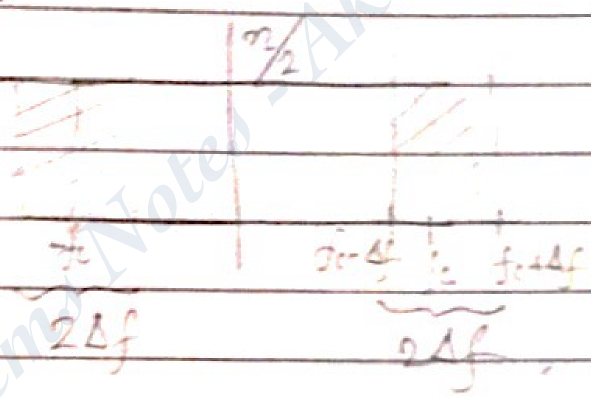
Now, taking $m(t) = 0$

$\Rightarrow y_i(t) = A_c \cos \omega_c t + n(t)$

So,

$N_i = \overline{n^2(t)}$

$\Rightarrow N_i = 2\eta \Delta f \rightarrow (3)$



So, $N_i = (2) \times \frac{\eta}{2} \times 2\Delta f$

$= 2\eta \Delta f$

Now,

finding $\underline{N_o}$:

We have, $y_i(t) = A_c \cos\omega_c t + n_c(t) \cos\omega_c t - n_s(t) \sin\omega_c t$

$= (A_c + n_c(t)) \cos\omega_c t - n_s(t) \sin\omega_c t$

$\approx A\cos\theta + B\sin\theta$

$= \sqrt{A^2 B^2} \cos(\theta + \phi)$

Now

o/p $= \alpha\, \omega_i(t)$

Finding $\omega_i(t)$ from $y_i(t)$

Idea : $\omega_i(t) = \frac{d}{dt}(\theta + \phi)$

Rearranging $y_i(t) = \sqrt{[A_c + n_c(t)]^2 + (-n_s(t))^2} \cos[\underbrace{\omega_c t + \phi}_{\theta(t)}]$

$\hookrightarrow \phi = -\tan^{-1}\left(\frac{n_s(t)}{A_c + n_c(t)}\right)$

So, $\theta(t)$ i.e $\theta + \phi$

$$= \omega_c t - \tan^{-1}\left(\frac{n_s(t)}{A_c + n_c(t)}\right)$$

Case ①: Consider small noise case

$$\Rightarrow A_c >> n_c(t), n_s(t)$$

So, $\phi(t) \approx -\tan^{-1}\left(\frac{n_s(t)}{A_c}\right)$

$$\simeq \frac{-\sin^{-1}\left(\frac{n_s(t)}{A_c}\right)}{\cos^{-1}\left(\frac{n_c(t)}{A_c}\right)}$$

(Small angle approximⁿ
$\sin\theta \simeq \theta$
$\cos\theta \simeq 1$)

$$\simeq -\frac{\frac{n_s(t)}{A_c}}{1} \simeq -\frac{n_s(t)}{A_c}$$

So,

$$\theta(t) = \omega_c t - \frac{n_s(t)}{A_c}$$

Now, $\omega_i(t) = \frac{d}{dt}\theta(t) = \omega_c - \frac{1}{A_c}\frac{d}{dt}n_s(t)$

So, o/p $= \alpha\,\omega_i(t)$

$\Rightarrow$ o/p. $= \alpha\omega_c - \frac{\alpha}{A_c}\frac{d}{dt}n_s(t)$

again, DC component ✗

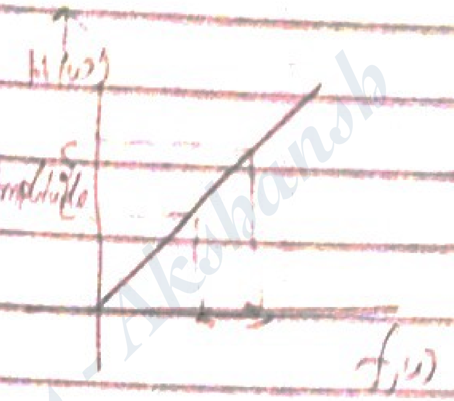
Now, $n_o(t) = -\frac{\alpha}{A_c}\frac{d\,n_s(t)}{dt}$.

We know $n_s(t)$ is low pass component of BPF
Its called quadrature component of noise.

Now,

$$n_s(t) \longrightarrow \boxed{d/dt} \longrightarrow \frac{d}{dt} n_s(t)$$

$$H(s) = s$$
$$H(\omega) = j\omega$$
$$|H(\omega)| = \omega$$



let PSD of $n_s(t) = S_{n_s}(\omega)$

& PSD of $\frac{d}{dt} n_s(t) = S_{\dot{n}_s}(\omega)$

∵ we are relating power, we use $|H(\omega)|^2$ of TF

$$\Rightarrow S_{\dot{n}_s}(\omega) = |H(\omega)|^2 S_{n_s}(\omega)$$

$$\Rightarrow S_{\dot{n}_s}(\omega) = \omega^2 S_{n_s}(\omega)$$

We had, $n_0(t) = \frac{-\alpha}{A_c} \frac{d}{dt} n_s(t)$. So, in terms of PSD

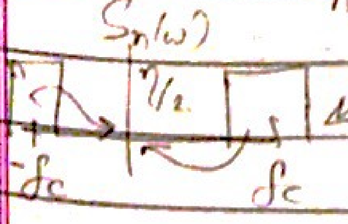
$$S_{n_0}(\omega) = \frac{\alpha^2}{A_c^2} S_{\dot{n}_s}(\omega)$$

$$\Rightarrow S_{n_0}(\omega) = \frac{\alpha^2}{A_c^2}\left(\omega^2 S_{n_s}(\omega)\right)$$

↳ PSD at o/p of FM receiver.

from previous knowledge,

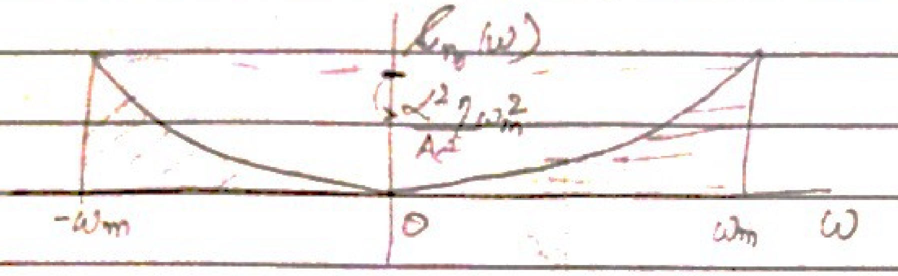
$$S_{n_s}(\omega) = \begin{cases} S_n(\omega+\omega_c) + S_n(\omega-\omega_c) & , |\omega| < \omega_m \\ 0 & , \text{otherwise} \end{cases}$$



$$= \begin{cases} \eta/2 + \eta/2 & ; |\omega| < \omega_m \\ 0 & , \text{otherwise} \end{cases}$$

So,

$$S_{n_o}(\omega) = \begin{cases} \dfrac{\alpha^2 \eta \omega^2}{A_c^2} & ; |\omega| < \omega_m \\ 0 & ; \text{otherwise} \end{cases}$$



So, we find : o/p noise PSD has a parabolic spectrum.

Now, No = Area under above curve

$$\Rightarrow N_o = 2\int_0^{f_m} S_{n_o}(f)\,df \text{ or } \frac{2\times 1}{2\pi}\int_0^{\omega_m} S_{n_o}(\omega)\,d\omega$$

$$= 2\int_0^{f_m} \frac{\alpha^2 \eta (2\pi f)^2}{A_c^2}\,df$$

$$\Rightarrow N_o = \frac{2}{3}\left(\frac{\alpha^2 \eta}{A_c^2}\right)\frac{\omega_m^3}{2\pi}$$

From previous part, we had got $S_o = \alpha^2 k_f^2 \overline{m_{(t)}^2}$

So,

$$\frac{S_o}{N_o} = \frac{3\pi A_c^2 k_f^2 \overline{m^2(t)}}{\eta \omega_m^3}$$

Now, $S_i = \frac{A_c^2}{2}$, $\gamma = \frac{S_i}{\eta f_m}$

$$\Rightarrow \frac{S_o}{N_o} = \frac{3\pi \overset{2S_i}{(A_c^2)} k_f^2 \overline{m^2(t)}}{\eta \omega_m^2 (2\pi f_m)} = \frac{3 k_f^2 \overline{m^2(t)}}{\omega_m^2} \times \gamma$$

for FM, max. freq. deviation $= \Delta\omega = k_f \, |m(t)|_{max}$

$$\Rightarrow k_f = \frac{\Delta\omega}{|m(t)|_{max}}$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{3 \, \frac{(\Delta\omega)^2}{m^2(t)|_{max}} \times \overline{m^2(t)}}{\omega_m^2} \times \gamma$$

$$= 3 \left(\frac{\Delta\omega}{\omega_m}\right)^2 \frac{\overline{m^2(t)}}{|m(t)|^2_{max}} \times \gamma$$

$$= 3 \, \beta^2 \left(\frac{\overline{m^2(t)}}{|m(t)|^2_{max}} \right) \gamma$$

$\hookrightarrow \frac{S_o}{N_o} \propto (\Delta\omega)^2$

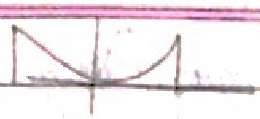
$\propto B_{FM}^2$ (square of BW $\because$ BW $= 2(\Delta\omega + \omega_m)$, ignoring ω_m $\Rightarrow$ BW $\propto \Delta\omega$)

$\hookrightarrow$ every doubling of BW increases o/p SNR by a factor of 4 (or 6 dB)

$\hookrightarrow$ valid only for small noise i.e. $A_c \gg n_s(t), n_c(t)$ should be satisfied

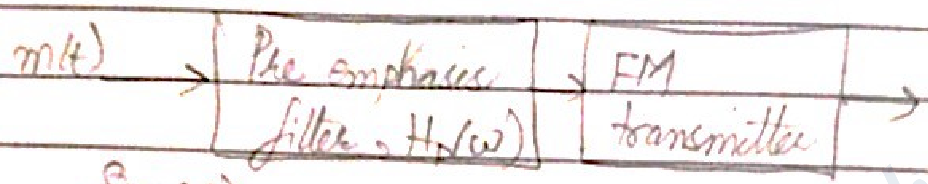
$\hookrightarrow$ This is called Threshold effect for FM.

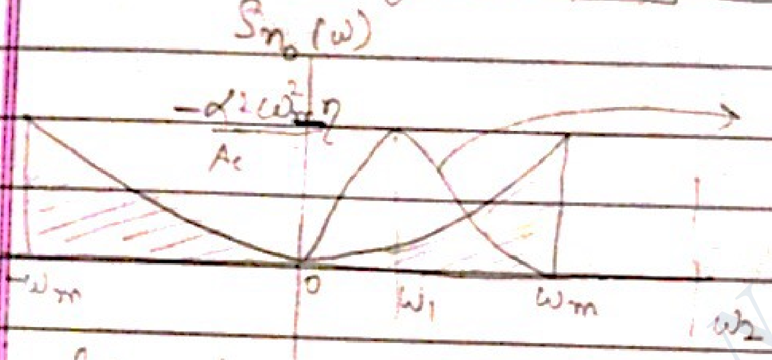
Our noise spectrum at o/p :



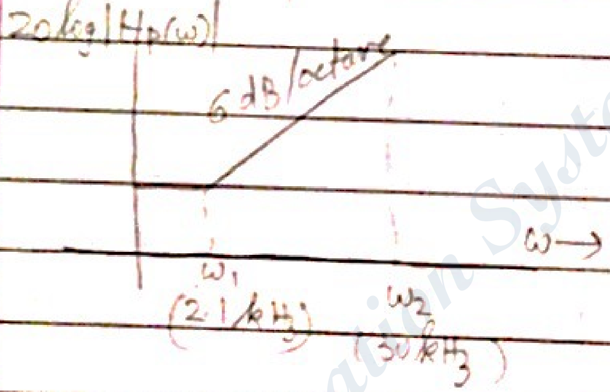
General speech spectrum:

So, at high freq , noise will dominate my signal
So, to make up for that , use Pre-emphasis filter.

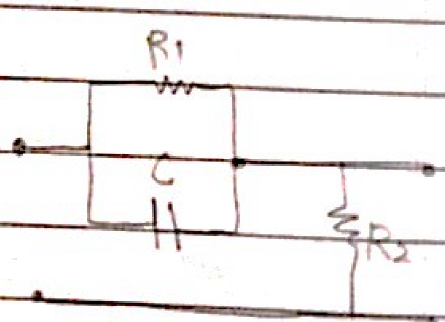




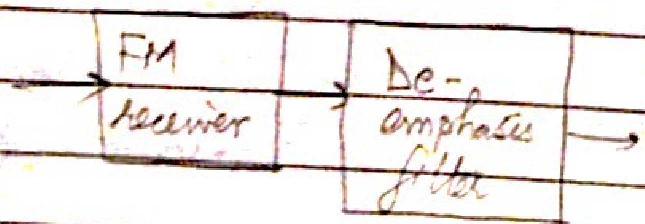
This fall is amplified with DC component so that noise doesn't suppress message

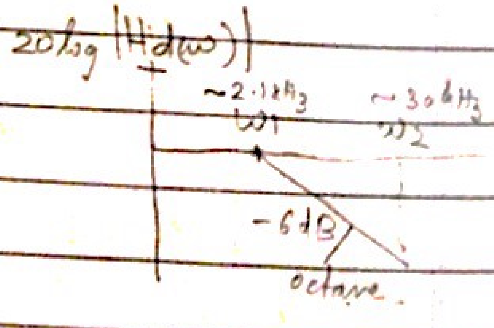


This pre-emphasis filter is :



Again, at receiver side, we need to remove what we added. So, use De-emphasis filter.





FM vs AM

For single tone case,

$\to m(t) = A_m \cos \omega_m t$

$\to \overline{m^2(t)} = A_m^2/2$

$\to |m(t)|_{max} = A_m$

$$\left(\frac{S_o}{N_o}\right)_{AM} = \left(\frac{\mu^2}{\mu^2+2}\right)\gamma = \frac{\gamma}{3} \quad \eta_{AM} \text{ max value.}$$

&

$$\left(\frac{S_o}{N_o}\right)_{FM} = 3\beta^2\left(\frac{\overline{m^2(t)}}{|m(t)|^2_{max}}\right)\gamma$$

$$= \frac{3}{2}\beta^2\gamma$$

So,
$$\frac{\left(\frac{S_o}{N_o}\right)_{FM}}{\left(\frac{S_o}{N_o}\right)_{AM}} = \frac{\frac{3}{2}\beta^2\gamma}{\gamma/3} = \frac{9}{2}\beta^2$$

For FM superior to AM,

$$\frac{9}{2}\beta^2 > 1$$

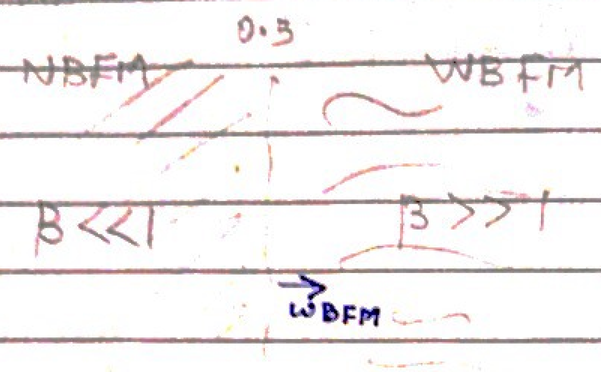
$$\Rightarrow \beta^2 > \frac{2}{9}$$

$$\Rightarrow \beta > \frac{\sqrt{2}}{3}$$

$$\Rightarrow \boxed{\beta \simeq 0.5}$$

$\hookrightarrow$ FM modulⁿ index

We saw, in FM

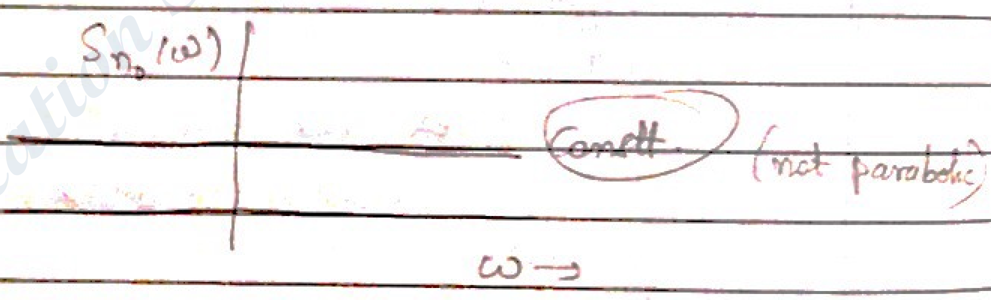


So, $\beta > 0.5$, $\Rightarrow$ we are going in WBFM

$\hookrightarrow$ So, improvement is possible for WBFM, not NBFM.

(b) PM (self)

After analysis, we find



*

end of course

* For Compre: Read through Lab experiments.