

ELECTRICAL SCIENCES

FIRST YEAR
NOTES

-AKSHANSH CHAUDHARY



Electrical Science Notes, First Edition

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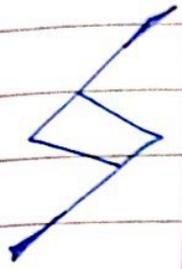


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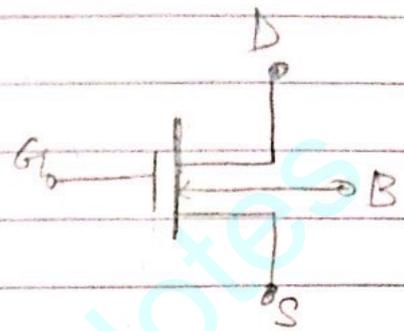
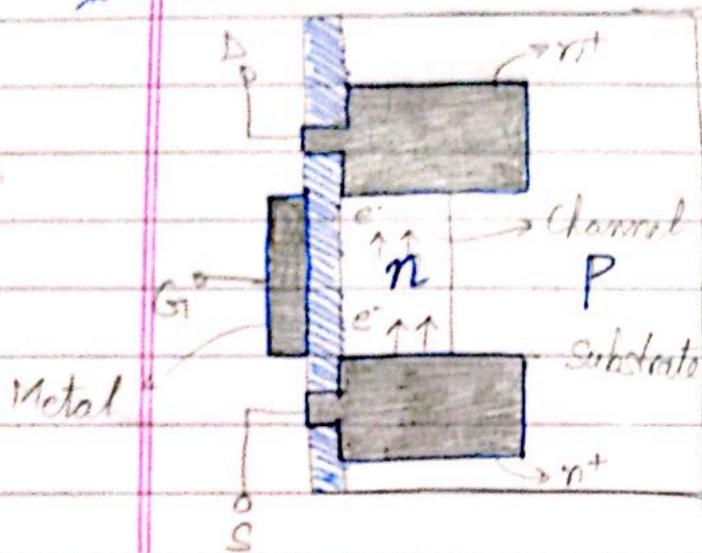
MOSFET

Metal oxide Semiconductor Field Effect Transistors

Depletion MOSFET

Enhancement MOSFET

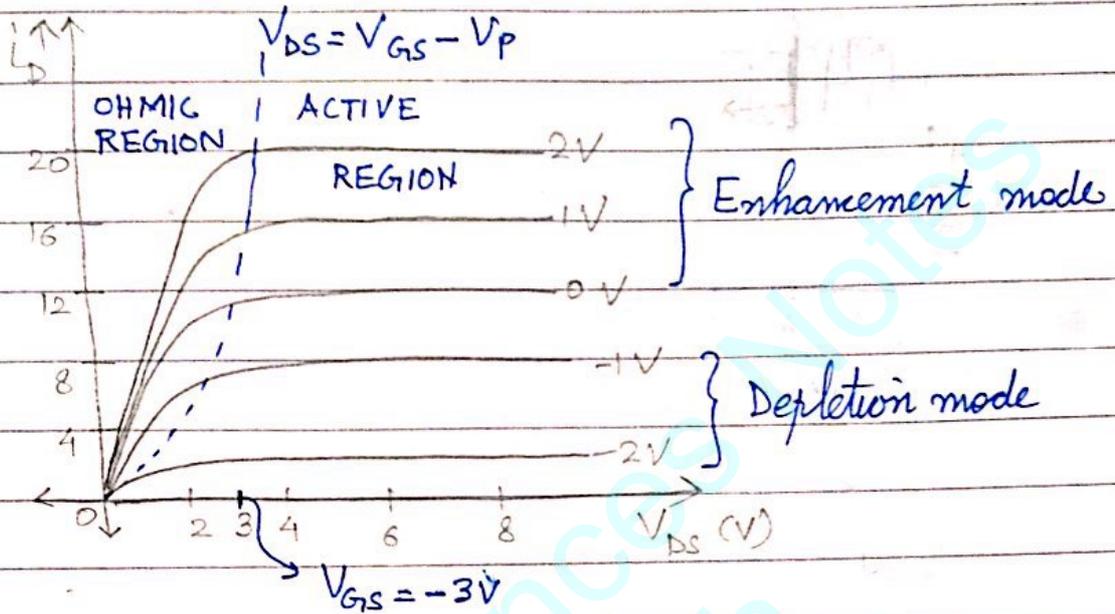
- * Apart from S, D & G, a new component called as substrate or body is introduced.
- Substrate is a p-type Si & forms 2 heavily doped n type regions denoted by n^+ , signifies heavily doped
- An n type channel is formed b/w 2 n^+ regions.
- Channel is covered with a thin insulating oxide layer of SiO_2 ($0.1 \mu\text{m}$).



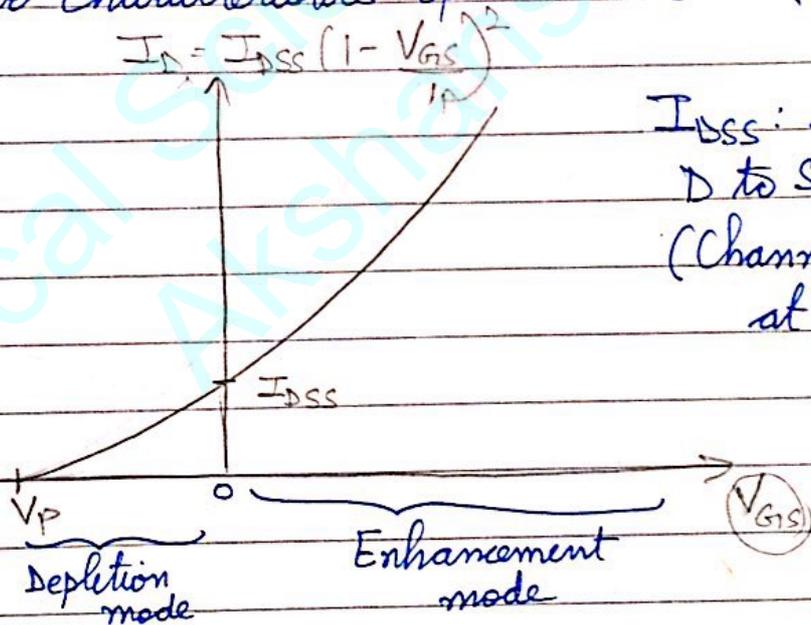
Circuit symbol

- Gate is obtained by depositing a metal on the oxide layer
- Such device is called MOSFET, also called NMOS.
- If n & p are interchanged, its called PMOS.
- \therefore \exists insulating layer b/w gate & channel, its called INSULATED GATE FET.
- \therefore of insulⁿ b/w G1 & S, resistance is extremely high $= 10^{10}$ to $10^{15} \Omega$, hence, $i_{G1} = 0$ for MOSFET (same as JFET)
- Typically the source S & body B are connected
- V_{GS} is made -ve, +ve charges are induced in the n-type channel, thereby, effectively narrowing it, as in case of JFET.
- A depletion MOSFET has characteristics similar to JFET.
- When V_{GS} is made +ve, \exists more -ve charges induced into n channel, thereby, widening the channel.
- When channel is widened with majority charge carriers, we say its an enhancement mode.

* O/P CHARACTERISTICS of a MOSFET

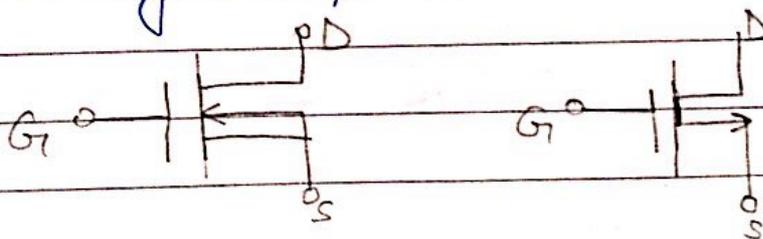


* Transfer Characteristics of a MOSFET

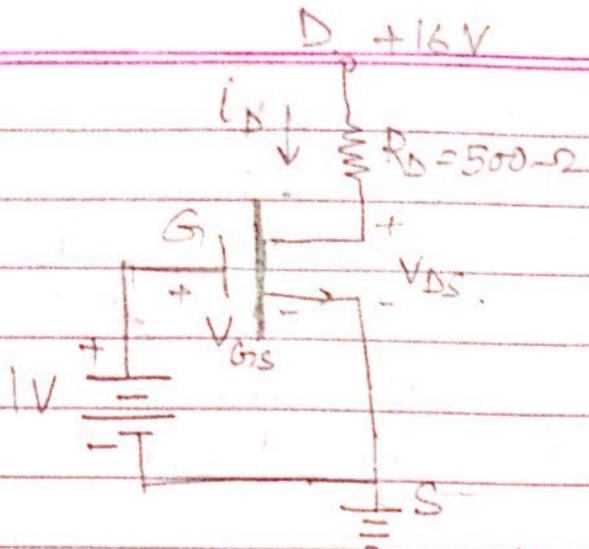


I_{DSS} : current from D to S for $V_{DS} = 0$.
(Channel pinch off at V_P).

* Circuit symbol for n-channel depletion MOSFET



Q



NMOS

$I_{DSS} = 8 \text{ mA}$

$V_P = -2 \text{ V}$

(a)

Find: V_{GS} , I_D & V_{DS}

(b) If $R_D = 750 \Omega$

instead, is it still in active region.

(a) $V_{GS} = 1 \text{ V}$ (inspection).

For active region.

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

8 mA 1 V -2 V

$\Rightarrow I_D = 18 \text{ mA}$

KVL $16 - V_{DS} - 500 I_D = 0$

$\Rightarrow V_{DS} = 7 \text{ V}$

$V_{DS} > V_{GS} - V_P \Rightarrow 7 > 3 \text{ V}$

\Rightarrow MOSFET is in active.

(b) $I_D = 18 \text{ mA}$ (same)

KVL $16 - V_{DS} - 750 I_D = 2.5 \text{ V}$

$2.5 \text{ V} < 3 \text{ V}$ $\rightarrow V_{GS} - V_P$

\therefore MOSFET is not in active region

It is in Ohmic region.

Drill Ex: $I_{DSS} = 8 \text{ mA}$, $V_P = -2 \text{ V}$.

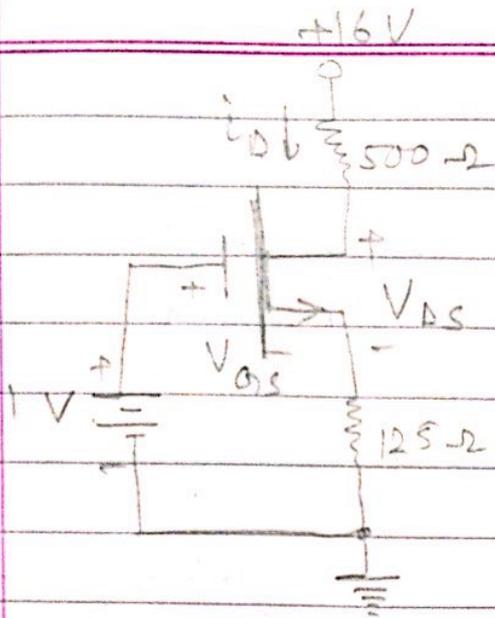
8.3

Find: V_{GS} , I_D , V_{DS}

Solⁿ: Active region $\rightarrow 8 \text{ mA}$

$\Rightarrow I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = I_{DSS} \left(1 + \frac{V_{GS}^2}{4} + \frac{V_{GS}}{2}\right)$

\rightarrow



Also, By KVL

$$V_{GS} + 125 i_D - 1 = 0$$

$$\Rightarrow i_D = \frac{1 - V_{GS}}{125} \quad \text{--- (2)}$$

From (1) & (2)

$$\Rightarrow \frac{1 - V_{GS}}{125} = (8 \times 10^{-3}) \left(1 + V_{GS} + \frac{V_{GS}^2}{4} \right)$$

$$\Rightarrow 1 - V_{GS} = 1 + V_{GS} + \frac{V_{GS}^2}{4}$$

$$\Rightarrow 2V_{GS} + \frac{V_{GS}^2}{4} = 0 \Rightarrow V_{GS}^2 + 8V_{GS} = 0$$

$$\Rightarrow V_{GS} = 0 \text{ or } V_{GS} = -8 \text{ V.}$$

$$\therefore V_{GS} > V_P (= -2 \text{ V})$$

$$\Rightarrow V_{GS} = 0 \text{ V, (not } -8 \text{ V)}$$

$$\therefore i_D = \frac{1 - V_{GS}}{125} = \frac{1 - 0}{125} = 8 \text{ mA}$$

By KVL, $V_{DS} + 500 i_D + 125 i_D - 16 = 0$,

$$\Rightarrow V_{DS} = 11 \text{ V.}$$

Now, $V_{DS} > V_{GS} - V_P$, $\Rightarrow 11 > 2 \text{ V}$

\Rightarrow Active region confirmed.

Chapter - 14.

ELECTROMAGNETICS

* Total flux (ϕ) passing through a given area is

$B \equiv \frac{\phi}{A}$

magnetic flux density (T: Tesla) * \rightarrow magnetic flux (Wb: Weber)

$\phi = \int \vec{B} \cdot d\vec{S}$

* Area \equiv

* $\vec{F} = q (\vec{v} \times \vec{B})$ \rightarrow LORENTZ FORCE

$|\vec{F}| = q v B \sin \theta$

Dirⁿ of force: RHTR (Right Hand Thumb Rule).
 : Flemming's Left Hand Rule.

* Current carrying conductor produces magnetic field: given by RHTR.

$B = \frac{\mu}{2\pi d} i$; \vec{B} for a straight current carrying coil at a distance 'd' from it.

$\frac{\mu}{4\pi} \left(\frac{2i}{r} \right)$ \equiv permeability of material.

$B = \frac{\mu N i}{l}$; \vec{B} for a solenoid of length 'l' & N no. of turns ($l \gg r$ (radius))

$\frac{\mu (2\pi i) N}{4\pi l} \equiv$

$\phi = \vec{B} \cdot d\vec{S}$
 $= B(\text{Area})$
 $\Rightarrow \phi = \frac{\mu N i}{l} (\pi r^2)$

* Relative permeability (μ_r) = $\frac{\mu}{\mu_0}$ (Free space: $4\pi \times 10^{-7}$)

Q. $A = 0.01 \text{ m}$
 $l = 0.2 \text{ m}$
 $i = 1 \text{ A}$
 $B = 0.1 \text{ T}$

Find :- N : when core material is
 (a) Air
 (b) Iron with $\mu_r = 1200$

$$B = \frac{\mu N i}{l} \Rightarrow N = \frac{B l}{\mu i}$$

$$\Rightarrow N = \frac{0.1 \times 0.2}{\mu \times 1}$$

(a) $4\pi \times 10^{-7}$ (μ_0)
 (b) $\mu_0 \mu_r = 4\pi \times 10^{-7} \times 1200$

*** TOROID**

l : length $\approx 2\pi r$
 r : radius

$$\vec{B} = \frac{\mu \cdot N \cdot i}{2\pi R} \equiv \frac{\mu}{4\pi} \left(\frac{2i}{R} \right) N$$

length of loop
 Radius of toroid ($l = 2\pi R$)

$$\phi = \vec{B} \cdot d\vec{s} = \frac{\mu N i}{2\pi R} \times \pi r^2 = \frac{\mu N r^2}{2R} i$$

Q. $N = 500$
 $\mu_r = 1500$
 $R = 0.1 \text{ m}$
 $A = 0.02 \text{ m}$
 $B = 0.5 \text{ T}$

Find $i = ?$

$$B = \frac{\mu N i}{2\pi R} \Rightarrow i = \frac{2\pi R \times B}{\mu N}$$

$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1500$
 $N = 500$

*** For a toroid, $\phi = \frac{\mu_0 N r^2}{2R} i$**

$$= \frac{(\mu_0 N) (\pi r^2)}{(2\pi R)} i = \frac{\mu_0 N A}{l} i$$

$$\Rightarrow \phi \propto N$$

$$\propto i$$

$$\propto Ni$$

*** * Ni : MAGNETOMOTIVE FORCE (mmf), written as.**
 $\mathcal{F} = Ni$; \mathcal{F} units : Amp-turns (A-t)

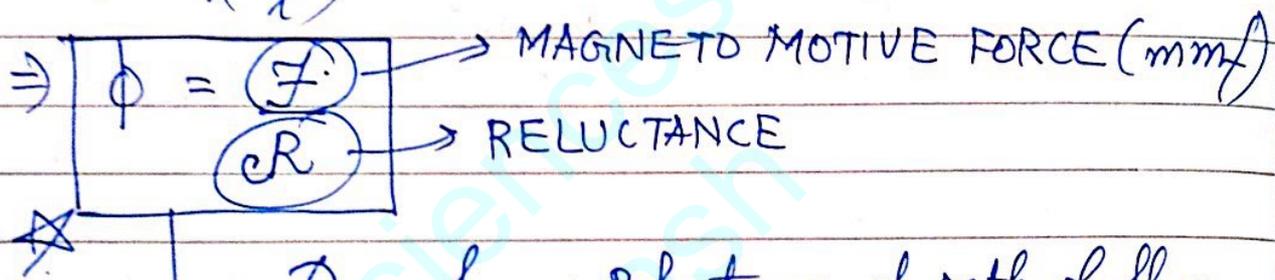
★ MAGNETIC CIRCUITS

we know, $\mathcal{F} = Ni$ & $\phi = \frac{\mu Ni}{l} \cdot A$

$$\Rightarrow \phi = \left(\frac{\mu NA}{l} \right) i$$

$$= \left(\frac{\mu A}{l} \right) (Ni)$$

$$\phi = \left(\frac{\mu A}{l} \right) \mathcal{F}$$



$\mathcal{R} = \frac{l}{\mu A}$: Reluctance of path of flux through the core

units : Ampere-turns per Weber
(A-t/Wb)

$$\Rightarrow \boxed{\mathcal{F} = \mathcal{R} \phi}$$

From previous example.

eg: - $B = 0.5 \text{ T}$, $i = 0.33 \text{ A}$, $A = 0.02$, $N = 500$, $\mathcal{R} = 0.1$

$$\Rightarrow \phi = B \cdot AS = B \cdot A = (0.5)(0.02)^2 \checkmark$$

$$\text{So, } \mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu(\pi r^2)} = \frac{2(0.1)}{(6\pi \times 10^{-4})(0.02)^2} \checkmark$$

$$\text{mmf} = \mathcal{F} = \mathcal{R} \phi \checkmark$$

$$i = \frac{\mathcal{F}}{N} = \frac{\mathcal{F}}{500} = 0.33 \text{ A}$$

(= same as i in previous ex.)

Sol ex
14.1

Find ϕ

$$r = 0.01 \text{ m}, \quad l = 0.2 \text{ m}, \quad N = 100, \quad i = 0.1 \text{ A}$$

(a) core material: Air

(b) core material: Iron; $\mu_r = 1500$

$$\Rightarrow \mu = \mu_0 \mu_r$$

$$(a) B = \frac{\mu Ni}{l} = \frac{\mu_0 Ni}{l}$$

$$\Rightarrow \phi = B \cdot dS = B (\pi r^2)$$

$$(b) B = \frac{\mu Ni}{l} = \frac{\mu_0 \mu_r Ni}{l}$$

$$\phi = B \cdot dS = \left(\frac{\mu_0 \mu_r Ni}{l} \right) (\pi r^2)$$

14.2. Toroid

$$R = 0.1 \text{ m}, \quad l_c = 0.02 \text{ m}$$

core radius

(cross-sectional radius)

$$\mu_r = 1200, \quad \phi = 1 \text{ Wb}, \quad i = 1 \text{ A}$$

Find: N

$$\phi = \vec{B} \cdot d\vec{S} = \left(\frac{\mu Ni}{2\pi R} \right) \pi r^2$$

$$= \left(\frac{\mu_0 \mu_r Ni}{2\pi R} \right) (\pi r^2) \checkmark$$

$$\Rightarrow N = \frac{\phi (2R)}{\mu_0 \mu_r i r^2} \checkmark$$

14.3 Toroid $R = 0.1 \text{ m}$
 $r = 0.02 \text{ m}$
 $\mu_r = 1200$
 $\phi = 1 \times 10^{-3} \text{ Wb}$
 $N = 332$

Find: (a) \mathcal{R}
 (b) mmf (\mathcal{F})
 (c) i

$$(a) \mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu A} = \frac{2\pi R}{\mu_0 \mu_r (\pi r^2)} =$$

$$\Rightarrow \mathcal{R} = 331740.976$$

$$(b) \mathcal{F} = \mathcal{R} \phi = 331.740976$$

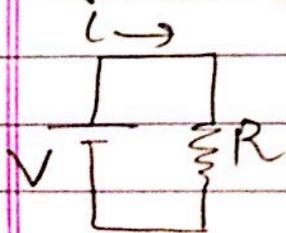
$$(c) i = \frac{\mathcal{F}}{N} \approx 0.9992198072 \text{ A}$$

* Analogies for Electric and Magnetic Circuits.

Electric

- Current (i)
- Voltage or emf (V)
- Resistance (R)
- $V = Ri$ (Ohm's Law)

- Electric circuit

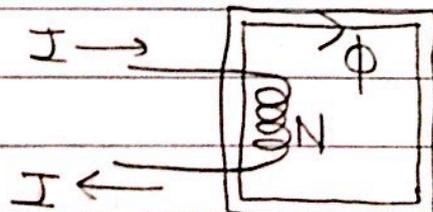


$$V = iR$$

Magnetic

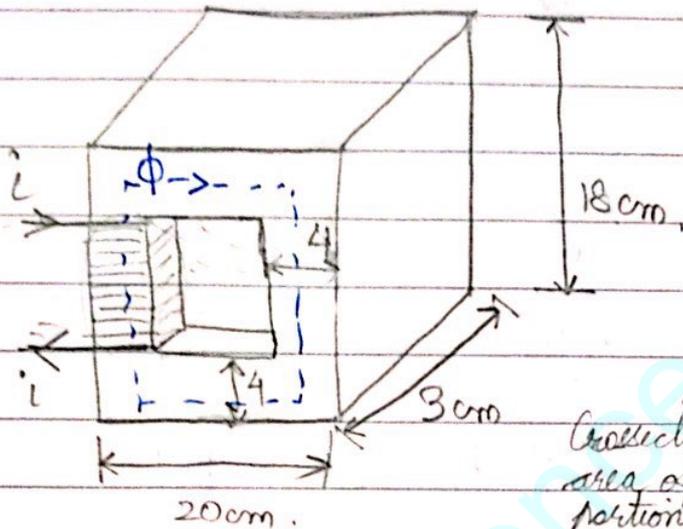
- Flux (ϕ)
- mmf (\mathcal{F})
- Reluctance (\mathcal{R})
- $\mathcal{F} = \mathcal{R} \phi$

- Magnetic circuit



$$\mathcal{F} = \phi \mathcal{R}$$

ex 14.4 Rectangular iron core; $\mu_r = 1500$
 Find: \mathcal{R} , Φ $N = 200$
 $i = 2A$



$$\text{Mean length } (l) = 16 + 14 + 16 + 14 = 60 \text{ cm}$$

$$\Rightarrow l = 0.6 \text{ m}$$

$$\text{Area} = 4 \text{ cm} \times 3 \text{ cm}$$

$$(A) = 0.04 \times 0.03$$

$$= 0.0012 \text{ m}^2$$

Cross-sectional area of inside portion

$$(a) \mathcal{R} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A} \checkmark$$

$$(b) \mathcal{F} = Ni = 200 \times 2 = 400 \text{ A-t}$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \checkmark$$

X

Drill 14.4 Rectangular iron core; $\mu_r = 1500$

$$i = 1A$$

$$N = ?$$

$$\Phi = 3 \text{ mWb}; (l, A)$$

ex 14.4

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}} = \frac{Ni}{l/\mu_0 \mu_r A}$$

$$\Rightarrow N = \frac{\Phi l}{(\mu_0 \mu_r A) i} \checkmark$$

X

* Magnetizⁿ Curves.

We know,

$$\text{Magnetic flux density } (B) = \frac{\phi}{A}$$

$$\& \phi = \frac{F}{R} \quad \& \quad R = \frac{l}{\mu A}$$

$$\Rightarrow B = \frac{F}{RA} = \frac{F}{A \frac{l}{\mu A}} = \frac{\mu F}{l}$$

So, $B = \mu H$; $H = \frac{F}{l} = \frac{Ni}{l}$

← Magnetic field
or

Magnetic flux
density.

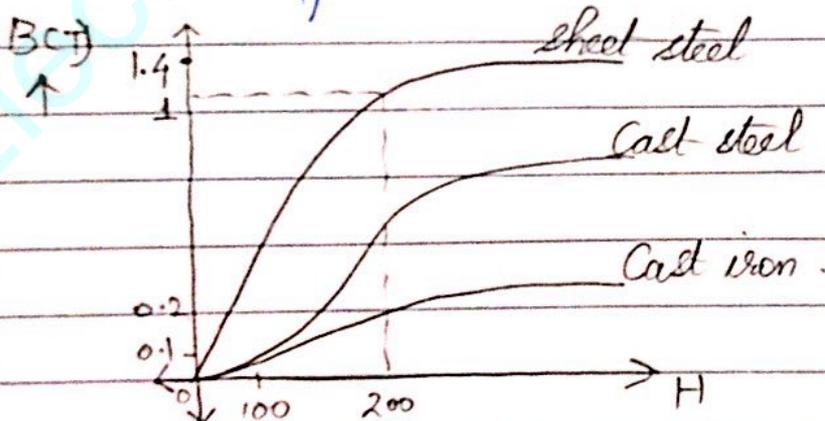
→ magnetic field intensity or magnetizⁿ force.

$$H \propto N$$

$$\propto i$$

$$\propto 1/l$$

- * B-H curve or Magnetizⁿ curve :- Plot of B vs H.
- * B-H curve for cast iron, cast steel, sheet steel →



ex 14.6 Toroid

$$B = 1 \text{ T}, R = 0.1 \text{ m}, r = 0.02 \text{ m}, N = 500$$

sheet steel

Find :- (a) μ_r
(b) i } (B-H curve is given)

$$\rightarrow H = 200$$

(a) $B = \mu H \Rightarrow \mu = \frac{B}{H}$ ✓ (Given)

$\Rightarrow \mu_r = \frac{\mu}{\mu_0} = \frac{\mu}{4\pi \times 10^{-7}} \checkmark (3980.89)$, 200 (graph)

(b) $i = \frac{H l}{N} = \frac{H \times 2\pi R}{N} = \frac{200 \times 2 \times 3.14 \times 0.1}{500} \checkmark$
(0.2512)

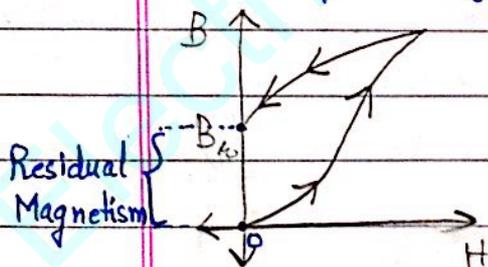
Drill 14.6 Toroid $B = 0.8 T$, $R = 0.1 m$, $l = 0.02 m$, $N = 500$
Cast steel $H = 500$ (from graph \rightarrow given)

Find) (a) μ_r , (b) i

1273.885. \rightarrow 0.628 A

HYSTERESIS

- a demagnetized material is one whose $B = 0$ when no ext. magnetizing force ($H = 0$) is applied
- If magnetising force is applied, resulting $B-H$ curve



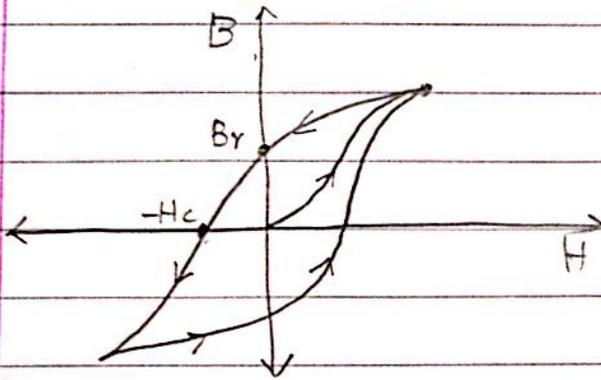
- * For small H , curve is relatively st.
- * As $H \uparrow$, curve bends, $B \uparrow$ & flattens out & further \uparrow in H yields no \uparrow in B . Hence, material is saturated.

* If after saturⁿ, H is reduced to zero, $B \downarrow$, but, doesn't follow the same path.

* This value of B obtained when $H = 0$ is $B = B_r$: Residual Magnetism.

* Now, H is decreased from 0 so that for some $H = H_c$, $B = 0$. This H_c : coercive force.

- * Now, H is \downarrow further, material saturates again, but, in opp. dirⁿ (-ve region).
- * Now, H is \uparrow & made large enough, material saturates again, but in +ve region.
- * Resulting loop :-



Note :- Some materials have rectangular hysteresis loops. They are called SQUARE LOOP MATERIAL \Rightarrow a small change in H , makes large change in B (used in digital computers).

* TRANSFORMERS

- * devices used to transfer electrical energy efficiently.
- * They Step up & step down voltages - used in power distribⁿ - to accomplish max. power transfer b/w source & load.
- They are employed to couple circuits without running a conducting electrical connection.
- They are insulating one circuit from another.

* MAGNETIC INDUCTION

- Faraday's Law :-
$$e.m.f = N \frac{d\phi}{dt}$$

★

Let $N\phi = \lambda$; λ : no. of flux linkages.

$$\therefore \text{emf} = \frac{d\lambda}{dt}$$

[Now, $v = L \frac{di}{dt}$, for an inductor]

$$\left[\begin{array}{l} \because \phi \propto i, N\phi = Li \\ \Rightarrow N \frac{d\phi}{dt} = L \frac{di}{dt} \\ \Rightarrow \text{emf} = v = L \frac{di}{dt} \end{array} \right]$$

equating v and emf

$$\Rightarrow L \frac{di}{dt} = - \frac{d\lambda}{dt}$$

$$\Rightarrow L \frac{di}{dt} \left(\frac{dt}{dt} \right) = \frac{d\lambda}{dt} \left(\frac{dt}{d\lambda} \right)$$

$$\Rightarrow \boxed{L = \frac{d\lambda}{di}}$$

$$\text{or } L = N \frac{d\phi}{dt}$$

Now, $\mathcal{F} = Ni$, $\mathcal{F} = R\phi$

$$\Rightarrow Ni = R\phi$$

$$\Rightarrow \phi = \frac{Ni}{R}$$

$$\Rightarrow \phi = \left(\frac{N}{R} \right) i \Rightarrow k = \frac{\phi}{i}$$

Also, $\frac{d\phi}{dt} = k$ (Taking derivative)

$$\therefore \frac{d\phi}{di} = \frac{\phi}{i}$$

$$\therefore L = N \frac{d\phi}{di} = \frac{d\lambda}{di}$$

becomes $L = \frac{N\phi}{i} = \frac{\lambda}{i}$

Now, $\phi = \frac{Ni}{\mathcal{R}}$

$$\therefore L = \frac{N(Ni/\mathcal{R})}{i}$$

$$\Rightarrow L = \frac{N^2}{\mathcal{R}}$$

* $L \propto N^2$

ex 14.8 Toroid :- $\mu_r = 1500$, $R = 0.1\text{m}$, $h = 0.02\text{m}$, $\mathcal{R} = 2.65 \times 10^5$
 $N = 500$ A-t/Wb

Find (1) :- $L = \frac{N^2}{\mathcal{R}}$ ✓

* Sequence of Operation

In transformer design you would normally like to deal in terms of voltages on the windings. However, the key to understand what happens in a transformer (or other wound component) is to realise that what the transformer really cares about is the current in the windings & that everything follows on from that.

- The current in the winding produces mmf
 $F = Ni$ A-t

- m.m.f produces magnetic field

$$H = \frac{F}{l} ; \text{ A-t/m.}$$
- Field produces magnetic flux density

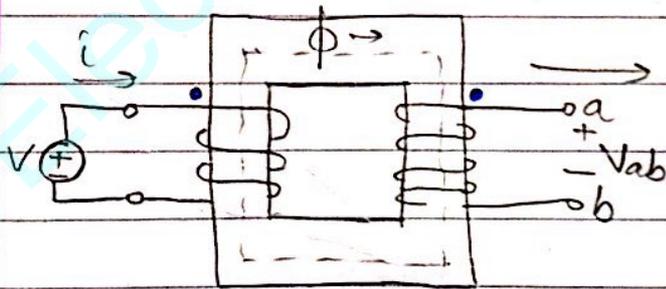
$$B = \mu H. \text{ T}$$
- Summed over the crosssectional area of the core, this equates to a total flux.

$$\phi = B \cdot dS \text{ (B.A) Wb.}$$
- Flux produces induced voltage (emf) :-

$$\text{emf} = N \frac{d\phi}{dt} \text{ V.}$$

* Lenz's Law:

Polarity of voltage induced by a changing flux tends to oppose the change in flux that produced the induced voltage



The dirⁿ of current in 1st hand coil makes the flux oppose the flux induced due to left hand coil, with current i .

* If $i \uparrow$ with time, $\phi \uparrow$ too ($\because Ni = \mathcal{R}\phi$)

* Resulting current gives potential drop V_{ab} b/w a & b

* $\bullet \rightarrow$ dots indicate that the dotted ends are +ve & -ve at same time.



Chapter - 15

* PRINCIPLE OF GENERATOR/MOTOR

PMI -
 Principle
 of
 Magnetic
 Induction

Whenever a conductor is moved within a magnetic field in such a way that the conductor cuts across magnetic lines of flux, voltage is generated in the conductor.

Amt. of voltage generated depends on

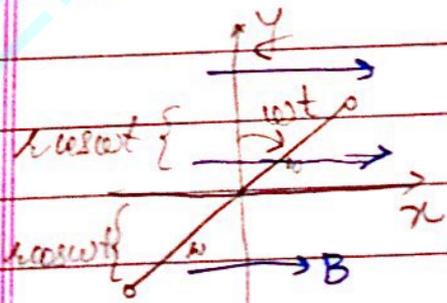
- (i) Strength of \vec{B}
- (ii) Angle at which conductor cuts the magnetic field.
- (iii) Speed at which conductor is moved.
- (iv) Length of conductor within magnetic field.

To be studied :- D.C. Machines

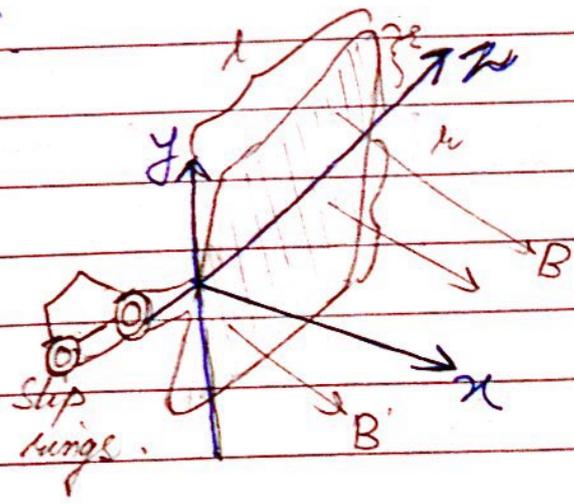
(i) Rotating Machines

(ii) Generators & Motors

* ROTATING MACHINES



(Side view)



Let A be the area of the loop & ϕ be the flux passing through the loop,

$$\text{So, } A = 2r(2r)l$$

$$\text{Also, } \phi = B \cdot ds = B(A)$$

$$= B(2rl)$$

Here, $r \rightarrow r \cos \omega t$

$$\Rightarrow \phi = B(2r \cos \omega t) l$$

$$= B \cos \omega t (2rl)$$

$$\Rightarrow \phi = BA \cos \omega t$$

According to Faraday's law, emf induced across brushes is:

$$e = \frac{d\phi}{dt} = -BA \omega \sin \omega t$$

↳ For N no. of turns,

Voltage:

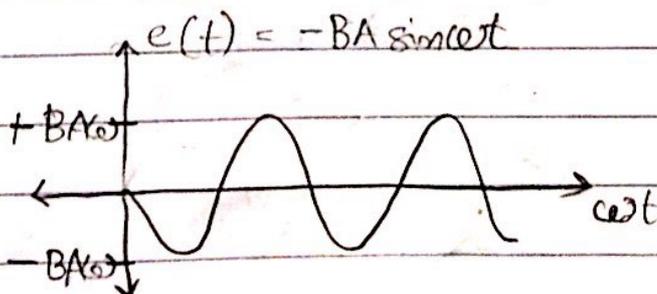
sinusoidal f°

$$emf = N \frac{d\phi}{dt} = -NBA \omega \sin \omega t$$

$$\left(\phi = NBA \cos \omega t \right)$$

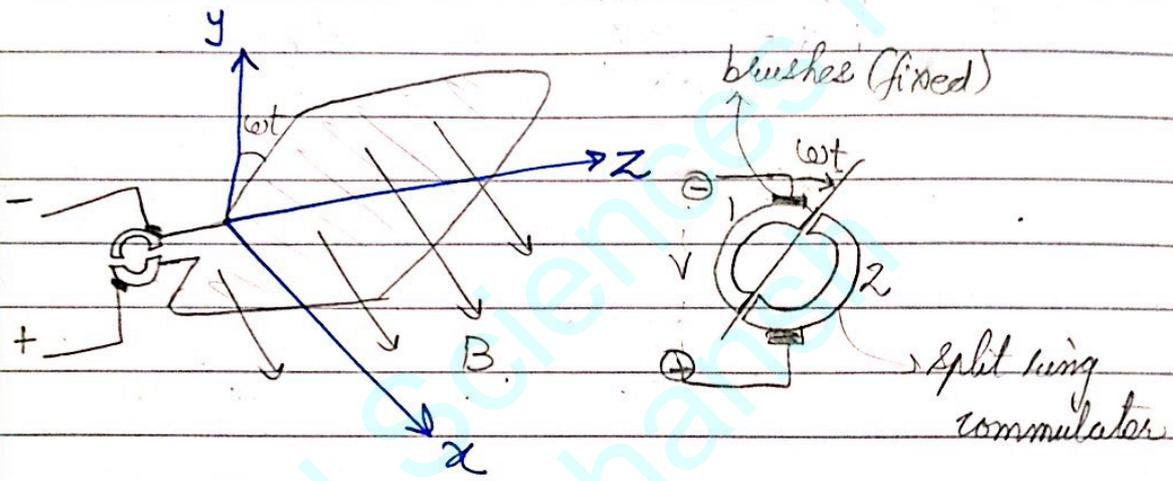
* turns

* This rotating machine produces sinusoidal waveform (voltage). So, it's an example of AC GENERATOR. also called ALTERNATOR.

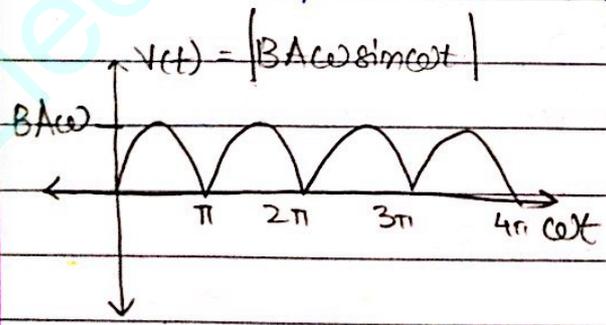


* Modify Simple Alternator: to DC Generator

- Replace slip rings with split ring commutator.
- Brushes on either side of split ring commutator.
- $e_{mf} = -BA\omega \sin \omega t$: voltage b/w halves of split ring commutator.

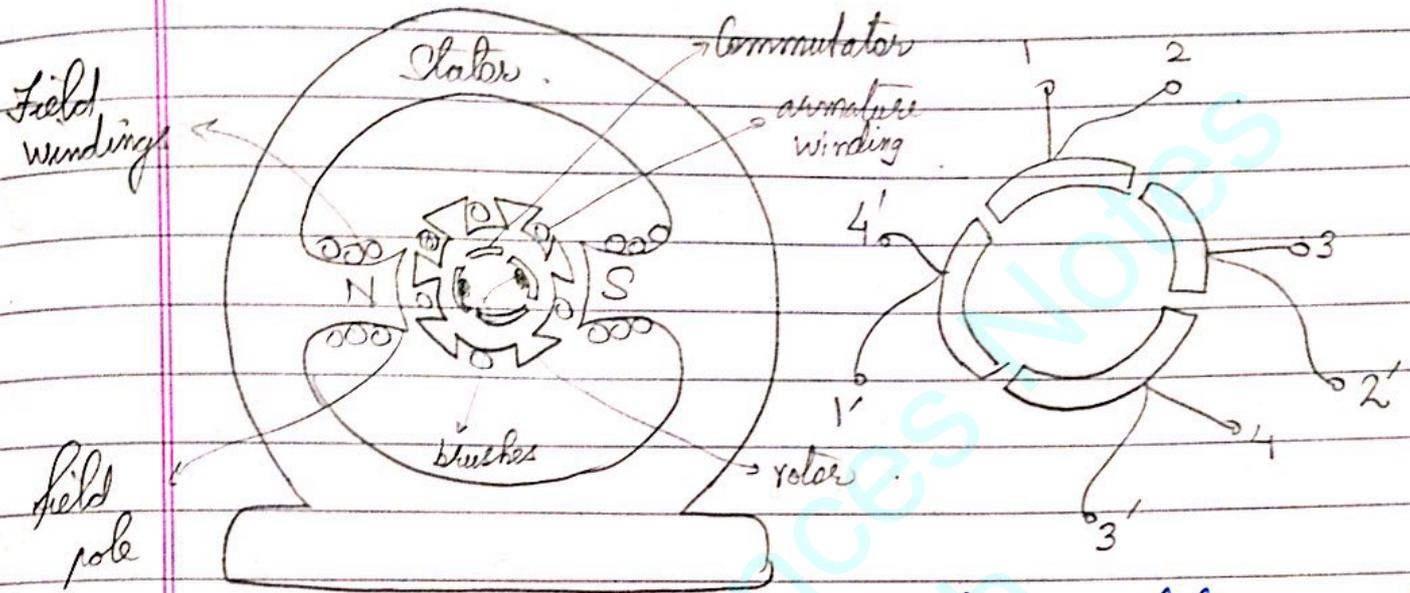


$$V = \begin{cases} -V_{12} = -e = BA\omega \sin \omega t & ; 0 < \omega t < \pi \\ V_{12} = e = -BA\omega \sin \omega t & ; \pi < \omega t < 2\pi \end{cases}$$



: Voltage never takes a -ve value.
V doesn't change polarity.

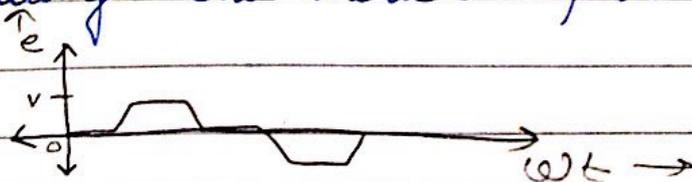
★ D.C. Generator



d.c. generator

coil-commutator connections

- * Generator consists of :-
Stator (Stationary portion) & Rotor (Rotating portion).
- * Field windings: Coils of the stator, in which DC is produced, when magnetic field is applied to it.
- * Rotor consists of :-
Commutator & Armature (conductors ~~are~~ across which emf is produced).
- * Stator has 2 poles → N & S. (Has field windings situated on it).
- * Rotor has iron core - (having slots which houses the armature conductors and a commutator & brushes).
in small air gaps b/w poles & rotor
- * Uniform \vec{B} is produced, when it is passed through field windings. This induces emf in the coil :-



- When armature windings are connected \neq to the commutator (as per connections shown earlier), individual emf gets rectified & all of them add up together.

* Voltage generated by generator, V_g

emf or no load armature voltage \leftarrow

$$V_g = \frac{N}{a} p \phi \frac{n}{60} = k \phi n$$

\rightarrow ND 60a

- $\rightarrow N$: no. of conductors = $\frac{\text{no. of armature coils} \times \text{no. of turns}}{\text{no. of conductors}}$
- $\rightarrow a$: no. of ll paths b/w brushes (path along which current flows)
- $\rightarrow p$: no. of poles
- $\rightarrow \phi$: magnetic flux per pole
- $\rightarrow n$: speed of rotor (rpm)

ex DC generator, $p=2, a=2, A=0.01\text{m}^2, B=1\text{T}$
 $n=1500$

Find V_g $\phi = B \cdot dS = 1.0 \cdot 0.01 = 0.01 \text{ Wb}$

$$V_g = \frac{N}{a} p \phi \frac{n}{60} = \frac{N}{2} \times 2 \times 0.01 \times \frac{n}{60}$$

$(4) \times (12) \times (2) \xrightarrow{12 \text{ turns}} 2 \text{ conductors}$
 $\rightarrow 4 \text{ armature coils}$

drill ex. 15.9 DC generator $p=2, a=2, V_g=16\text{V}, N=4 \times 10 \times 2$
 $\phi = 10\text{mWb}$ $n = ?$

$$n = \frac{V_g \times a \times 60}{N \times p \times \phi} = 1200 \text{ rpm}$$

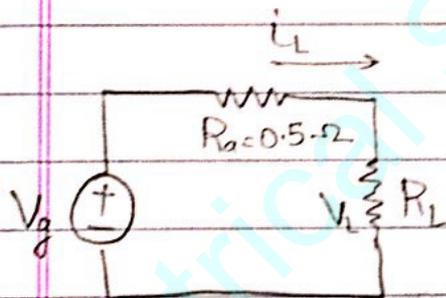
* Generator with a load.

The generated emf, V_g , is voltage across armature terminals (the brushes) when no load is connected to those terminals.

- If \exists some electrical load connected to armature, the resulting load voltage will be diff^t from the no load voltage. Now, it will depend on R_a (resistance) associated with armature due to resistance of the windings and brush contacts.

ex: $P = 2000 \text{ W}$, $V_L = 200 \text{ V}$, $i_L = 40 \text{ A}$, $n = 1200 \text{ rpm}$, $R_a = 0.5 \Omega$
 R_L resistive load.

Find: Full load voltage for generator at 900 rpm



$$-V_g + i_L(R_a) + V_L = 0 \rightarrow \textcircled{1}$$

$$\Rightarrow V_g = V_L + i_L(R_a)$$

$$\Rightarrow = 200 + 40(0.5)$$

$$\Rightarrow V_g = 220 \text{ V}$$

If ϕ constt., $V_g \propto n$

So, if ϕ & $i_L = \text{constt.}$,

$$\text{So, } \frac{V_{g1}}{V_{g2}} = \frac{n_1}{n_2} \Rightarrow V_{g2} = \frac{V_{g1} \times n_2}{n_1} = \frac{220 \times 900}{1200}$$

$$\Rightarrow V_{g2} = 165 \text{ V}$$

From $\textcircled{1}$, $V_L = V_g - i_L(R_a)$

$$\Rightarrow V_L = 165 - (0.5)(40) = 145 \text{ V}$$

di. ex 15.10 $P = 10 \text{ kW}$, $V_L = 200 \text{ V}$, $i = 50 \text{ A}$, $n = 1500 \text{ rpm}$

If $n = 1800 \text{ rpm}$, $V_L = 244 \text{ V}$.

Find: (a) R_a

(b) no load voltage at (i) 1500 rpm

(V_g)

(ii) 1800 rpm .

KVL $-V_g + V_L + i_L R_a = 0$

$$\Rightarrow V_g = V_L + i_L R_a$$

$$\Rightarrow V_g = 200 + 50 R_a$$

Also, $V_g \propto n$

$$\Rightarrow \frac{V_g}{(1800)} = \left(\frac{1800}{1500} \right) \frac{V_g}{(1500)}$$

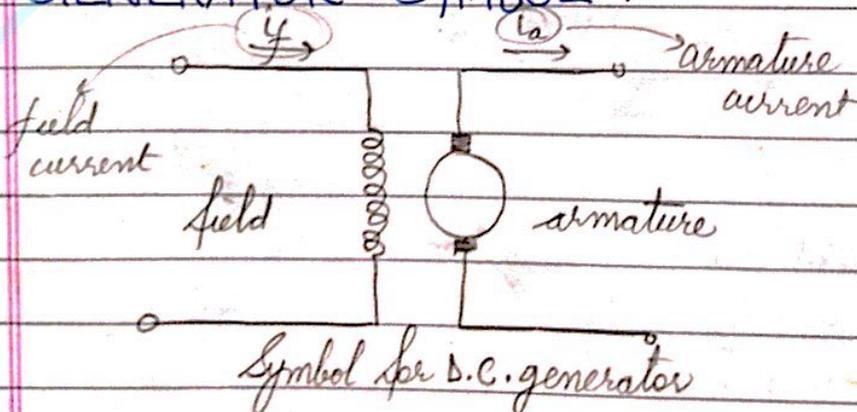
$$\therefore \frac{244}{1800} = -R_a \left(\frac{50}{1800} \right) + \left(\frac{1800}{1500} \right) (50 R_a + 200)$$

$$\Rightarrow R_a = 0.4 \Omega$$

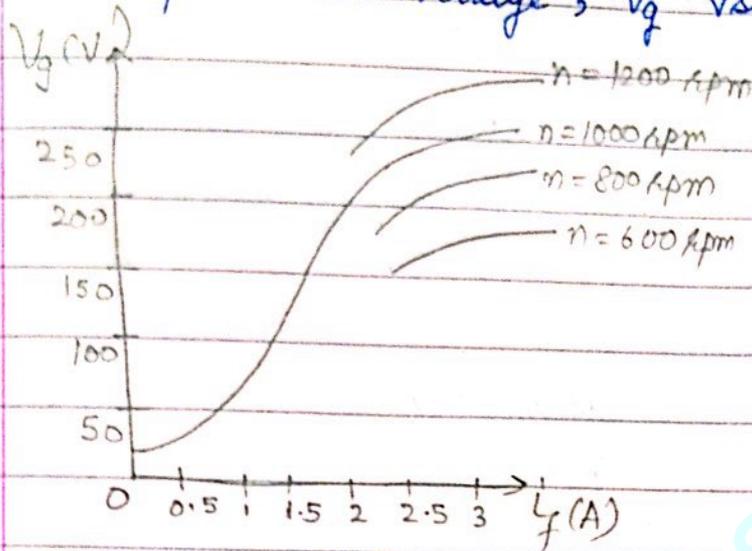
(b) (i) $V_g = 50(0.4) + 200 = 220 \text{ V}$

(ii) $V_g = \left(\frac{1800}{1500} \right) \left(\frac{220}{1500} \right) = 264 \text{ V} \checkmark$

* GENERATOR SYMBOL :



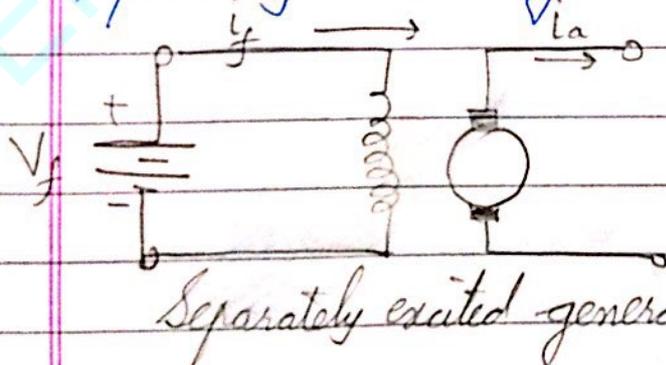
* Plot of no load voltage, V_g vs field current, i_f :



: The curves are called: magnetizⁿ curves of generator

* Generator field excitⁿ

- Field excitⁿ :- When DC voltage is applied to field windings of a D.C. generator, current flows through the windings and sets up a steady \vec{B} . This is called
- This excitⁿ voltage can be produced by generator itself or, it can be applied by an outside source, such as a battery.
- If its supplied by an outside source, it's called separately excited generator, as shown:-

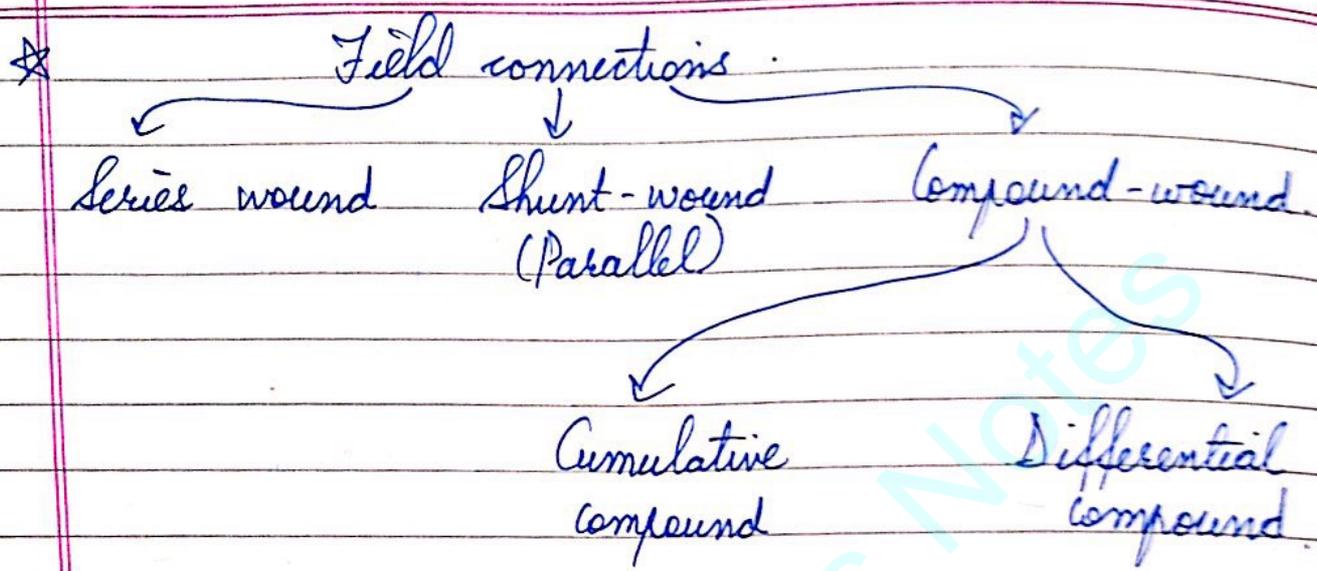


• The generator that supplies its own field excitⁿ is called Self-Excited

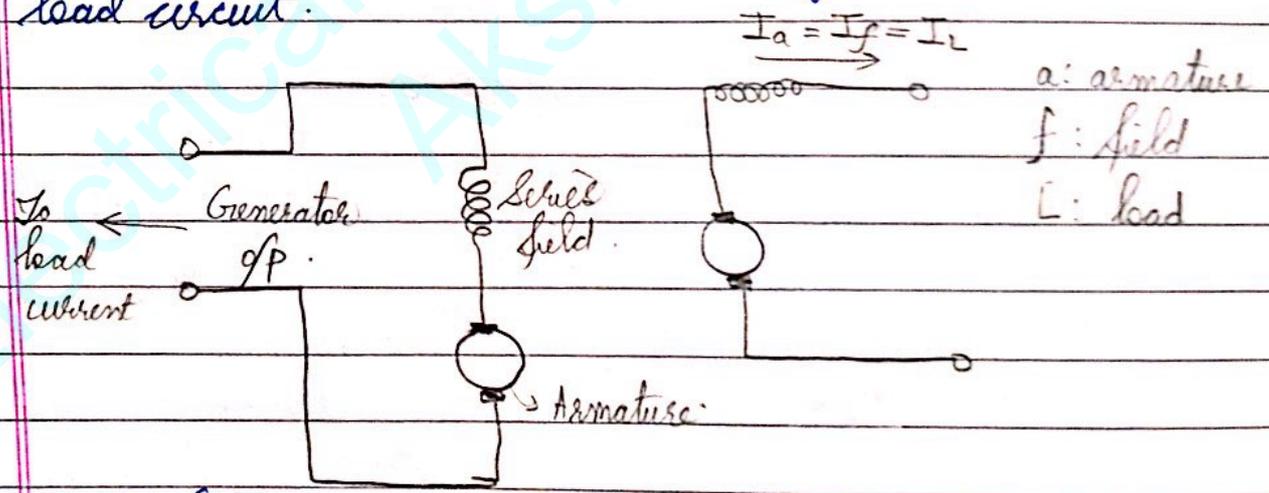
Separately excited generator. Generator.

* Classificⁿ of Generators :

Self excited generators are classified according to the type of field connectiⁿ they use.



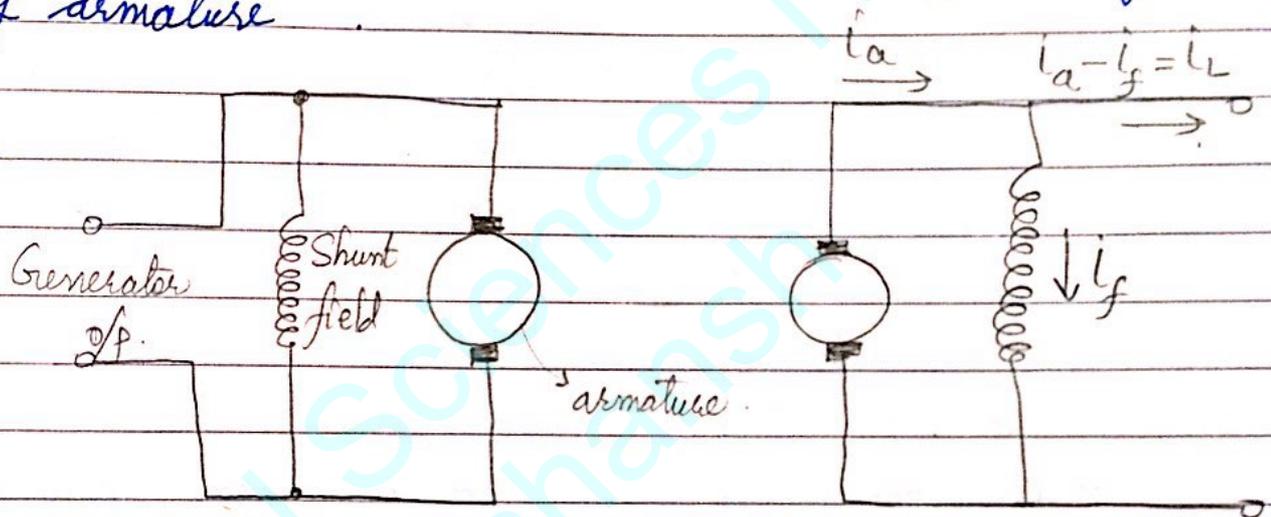
- ★ Series wound generator or series connected generator
- In series wound generator, field windings are connected in series with the armature.
 - Current that flows in the armature flows through ext. circuit and through field windings.
 - The external circuit connected to generator is called load circuit.



- ★ The field current (i_f) = armature current (i_a)
 - ↳ Field winding : very few turns
 - : relatively low resistance.

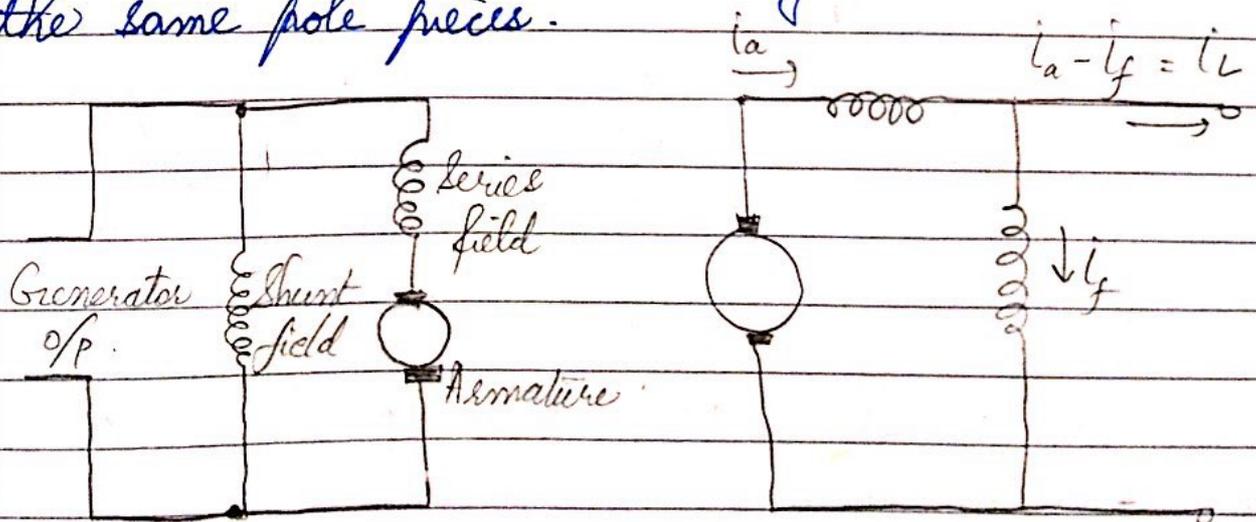
§ * Shunt-Wound generators

- In shunt wound generators, the field coils consist of many turns of small wire & relatively high field resistance.
- They are connected in \parallel with the load. In other words, they are connected across the o/p voltage of armature.



§ * Compound-Wound Generator

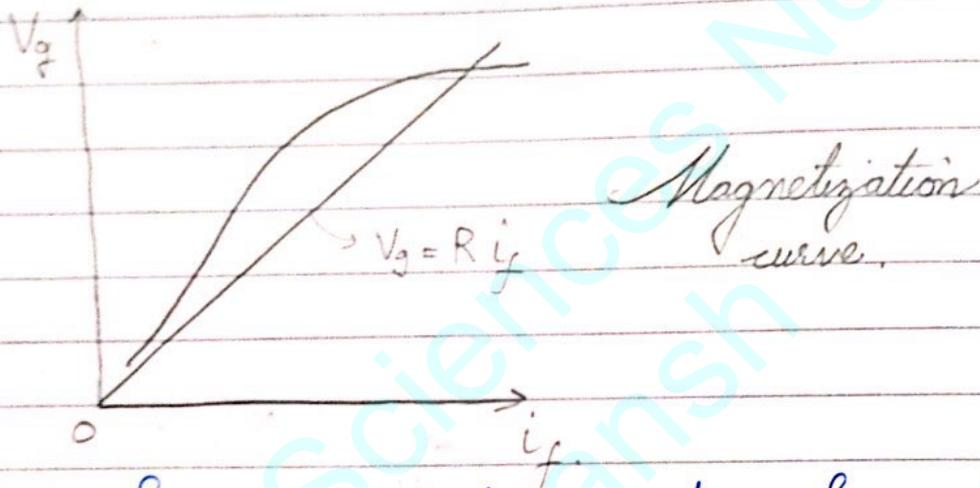
- Compound wound generators have a series field winding, in addⁿ to a shunt field winding.
- The shunt and series windings are wound on the same pole pieces.



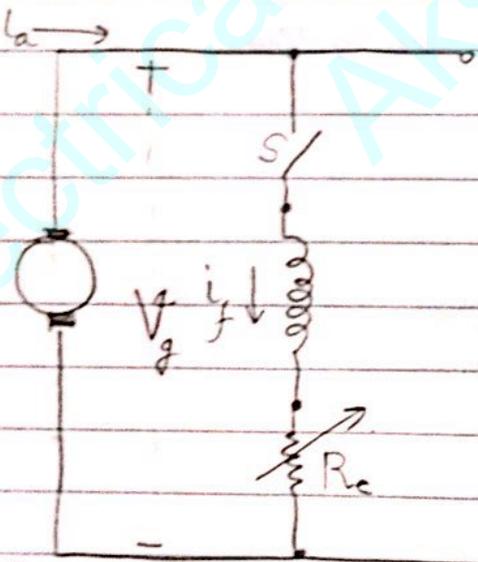
* Generator Build Up:

(How generated voltage \uparrow)

For a separately excited generator turning at n rpm, the value of field current determines the value of generated voltage and following magnetizⁿ curve can be used to find it:



- Consider shunt connected generator where rheostat & switch are included in the circuit.



Shunt connected generator

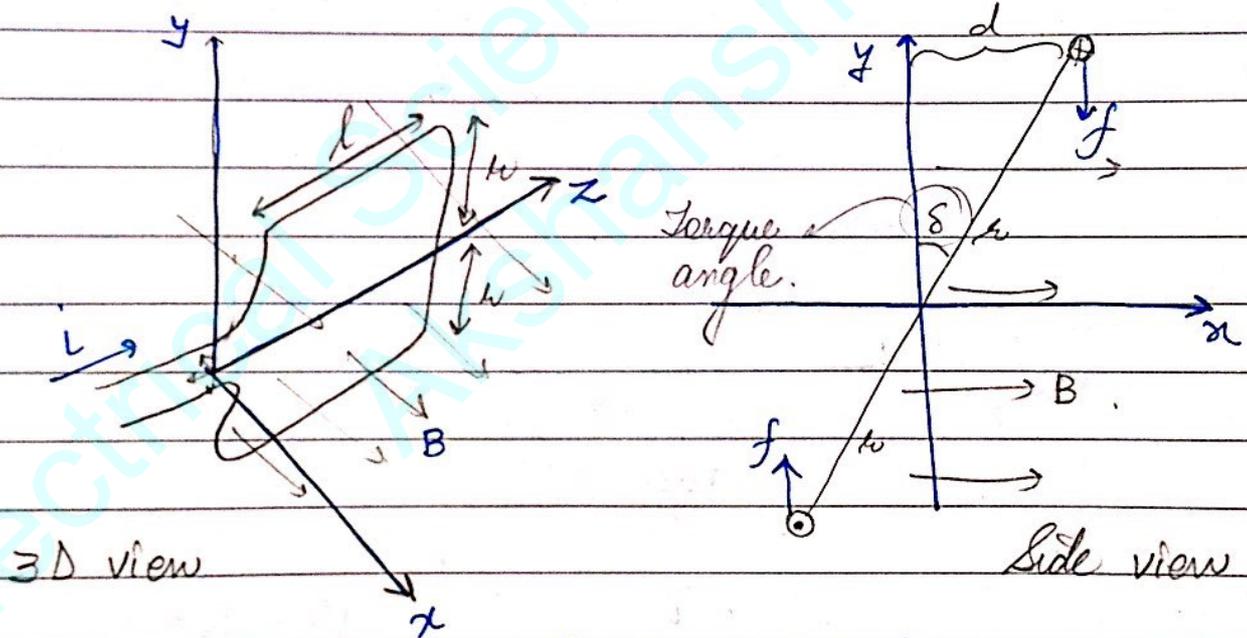
- when switch is open $i_f = 0$; small generated emf due to residual magnetism.

- when switch is closed voltage would be applied across series combinⁿ of field winding & the control rheostat \Downarrow

Produces a non zero field current & hence, \uparrow the flux, thereby, resulting in \uparrow in generated emf.

- This repetitive process of increasing generated voltage and field current is called GENERATOR BUILD UP.
- The ~~on~~ Ω pt. on ~~graph~~ graph shown previously shows \rightarrow generated buildup has ~~ceased~~ ceased.

★ DC Motors



- Consider a conducting loop through which, current i is flowing.
- Slip rings and brushes are present (not shown here)
- The end view of the loop is shown in fig (Side view), where, current in the upper conductor is directed into the page. (shown by \otimes).
- The current in lower conductor is directed out of the page (shown by \odot).

By Fleming's left hand rule (or RHTR),
force = $i(l \times B)$ occurs

in upper dirⁿ: for lower conductor

in lower dirⁿ: for upper conductor

line of action is not same. So, τ is developed.

$$\tau = 2 \times F \times d = Fd = (i l B) (r \sin \delta)$$

δ : angle formed by y-z plane & plane of loop called as TORQUE ANGLE or POWER ANGLE.

for N turns & 2 conductors

$$\tau = 2N (i l B) (r \sin \delta) = NBA i \sin \delta$$

$A = 2rl$: Area of coil

τ_{max} when $\delta = 90^\circ, \sin \delta = 1$.

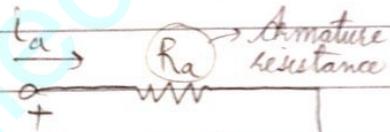
τ rotates the loop in 2 dirⁿ

Principle of DC motor

current produced in dirⁿ opp. to applied current

emf induced

flux per pole, meter (armature) speed



Now, $V_g = K \Phi \omega$; $K = \frac{Np}{60a}$

For a DC motor, by KVL,

$$V_L = R_a I_a + V_g$$

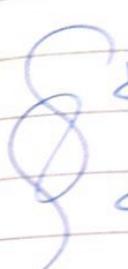
$$\text{or } V_L = R_a I_a + K \Phi \omega$$

$$\Rightarrow \omega = \frac{V_L - I_a R_a}{K \Phi}$$

Armature model for DC motor

Speed eqⁿ for DC motor

$\rightarrow K = \frac{Np}{60a}$



* T developed by DC motor ^{is} determined by i_a & B & hence ϕ .

$$\therefore T = K' \phi i_a$$

\rightarrow constt, determined by construction of motor

★ IDEAL AMPLIFIER

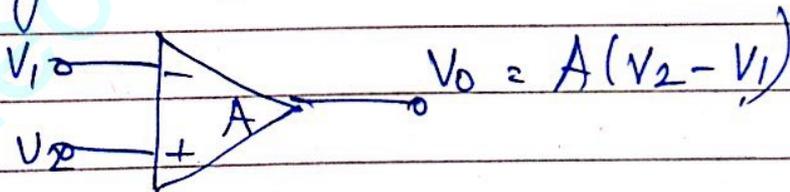
★ IDEAL voltage Amplifier:

\rightarrow A device with 2 i/p voltages V_1 & V_2 & 1 o/p voltage
Relⁿ b/w i/p & o/p voltage is

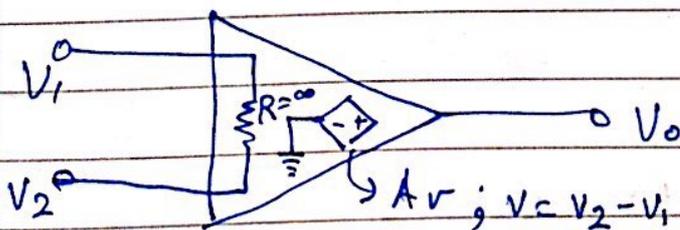
$$V_o = A(V_2 - V_1) \quad \text{or} \quad A = \frac{V_o}{V_2 - V_1}$$

A : gain of amplifier \leftarrow

★ Symbol for an ideal amplifier with gain A is:



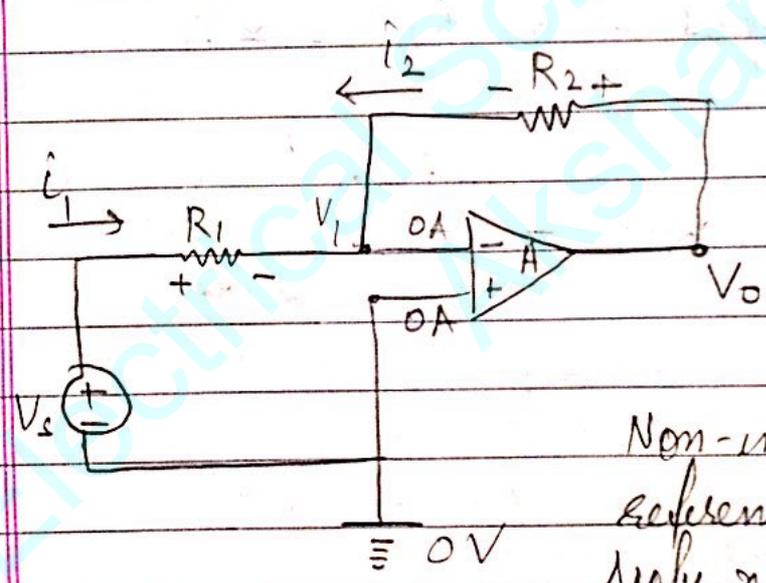
& in details, its internal circuitry is:



- * The resistor, $R \rightarrow \infty \Omega$ (input resistance) for any ideal voltage amplifier.
- * When such an amplifier is connected to any circuit, no current will go into the i/p terminal. However, \exists current going into or coming out of o/p terminal.
- * The i/p terminal labelled '-' is referred to as inverting i/p & input terminal labelled '+' is non-inverting i/p.

ex. 2.7 Given: $R_1 = 1 \text{ k}\Omega$
 $R_2 = 10 \text{ k}\Omega$
 $A = 100,000$
 $V_s = 1 \text{ V}$.

Find: V_o, V_1, i_1, i_2 ,
 Power absorbed by each resistor, independent voltage source & ideal amplifier (i.e., dependent voltage source)



Non-inverting i/p V_2 is at reference potential, hence, $V_2 = 0$.
 Apply nodal analysis at V_1
 $i_1 + i_2 = 0$

Ohm's law,

$$\frac{V_s - V_1}{R_1} + \frac{V_o - V_1}{R_2} = 0 \Rightarrow \frac{R_2 V_s - V_1 R_2 + V_o R_1 - V_1 R_1}{R_1 R_2} = 0$$

$$\Rightarrow R_2 V_s = (R_1 + R_2) V_1 - R_1 V_o \rightarrow (1)$$

$$\text{Now, } V_o = A(V_2 - V_1)$$

$$\therefore V_2 = 0.$$

$$\therefore V_o = -A V_1$$

$$\boxed{V_1 = -\frac{V_o}{A}} \quad \text{or} \quad A = -\frac{V_o}{V_1} \quad \rightarrow (2)$$

(2) in (1)

$$\Rightarrow R_2 V_s = -(R_1 + R_2) V_o - R_1 V_o$$

$$\Rightarrow R_2 V_s = - \left[\frac{1}{A} (R_1 + R_2) + R_1 \right] V_o$$

$$\Rightarrow \boxed{V_o = -\frac{R_2 V_s}{R_1 + \frac{1}{A} (R_1 + R_2)}} \quad \rightarrow (3)$$

Put values of R_1, R_2, V_s & A

$$V_o = \frac{-10(1)}{1 + \frac{1}{100,000} (1+10)} = -10V$$

$$\& V_1 = -\frac{V_o}{A} = \frac{10}{100,000} = 0.1 \text{ mV}$$

$$I_1 = \frac{V_s - V_1}{R_1} = \frac{1 - 0.1}{1} = 0.9 \text{ mA}$$

$$I_2 = \frac{V_o - V_1}{R_2} = \frac{-10 - 0.1}{10} = -1 \text{ mA}$$

$$P_{R_1} = i^2 R_1, \quad P_{R_2} = i^2 R_2; \quad P_s = V_s(i_1), \quad P_A = V_o i_2$$

$$\text{From (3), } V_o = -\frac{R_2 V_s}{R_1 + \frac{1}{A} (R_1 + R_2)}$$

↳ If A is very large ($A \rightarrow \infty$)

$$\text{then, } V_o = -\frac{R_2 V_s}{R_1} \quad \text{or} \quad \boxed{\frac{V_o}{V_s} = -\frac{R_2}{R_1}} \quad \& \quad V_1 = -\frac{V_o}{A}$$

$$\Rightarrow V_1 = 0V \quad (\text{Gain } \infty)$$

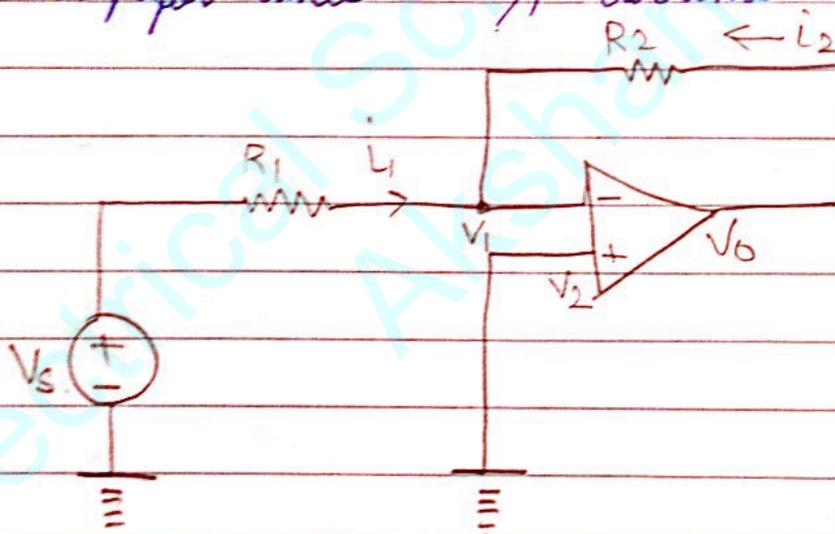
This is called -ve feedback, i.e., portion of op is fed back into i_p through R_2 .

* The Operational Amplifier (OP-AMP).

- An ideal amplifier with gain ($A \rightarrow \infty$) is known as operational amplifier or OP-AMP.
- In OP-AMP, \therefore gain is infinite we must have a feedback resistor b/w op & i_p terminals (either inverting or non inverting).

i.e., when $A \rightarrow \infty$, $V_1 = 0$.

- Amplifier takes no i_p currents.



Nodal analysis at V_1 ,

$$i_1 + i_2 = 0$$

$$\therefore \frac{V_s - V_1}{R_1} + \frac{V_o - V_1}{R_2} = 0$$

Now, $V_1 = 0$.

$$\Rightarrow \frac{V_s}{R_1} = -\frac{V_o}{R_2} \Rightarrow V_o = -\frac{R_2}{R_1} (V_s)$$

$$\text{or } \frac{V_o}{V_s} = -\frac{R_2}{R_1} = \text{Gain of amplifier}$$

∴ This circuit is called inverting amplifier.

- We have seen $V_1 = 0$ & V_2 is at $0V$, \Rightarrow no differential b/w the i/p. voltages V_1 & V_2 (Hence $V_1 = V_2$).

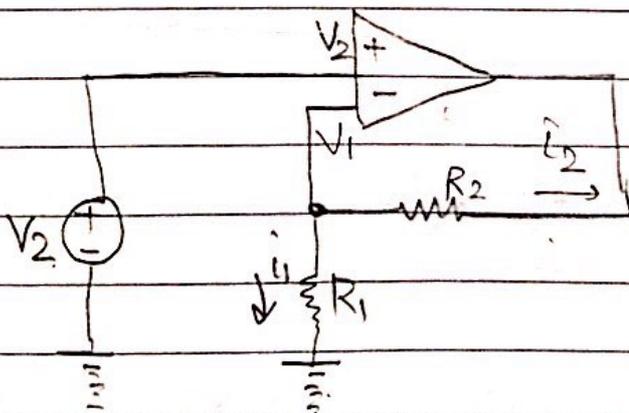
∴ This leads to max gain: $\frac{V_o}{V_s} = -\frac{R_2}{R_1}$

Imp * Hence, cond^{ns} for Inverting amplifier :-

1. Amplifier takes no i/p currents.
2. Gain is max.
3. -ve feedback is present.
4. Non inverting i/p is connected to ground.
5. $V_1 \sim V_2 \sim 0$.
6. Inverting i/p serves as signal.

* Cond^{ns} for Non-inverting amplifier.

1. Inverting i/p is connected to ground.
2. Non-inverting i/p serves as signal.
3. When feedback is given (when R_2 is connected b/w V_1 & V_o), then, $V_1 \sim V_2 \sim 0$ or $V_1 = V_2$ i.e. $A \rightarrow \infty$
4. Amplifier takes no i/p current.



KCL at V_1 :

$$i_1 + i_2 = 0 \quad \&$$

$$\frac{V_1}{R_1} + \frac{V_1 - V_o}{R_2} = 0$$

Now, $V_1 = V_2$

$$\Rightarrow \frac{V_2}{R_1} + \frac{V_2 - V_o}{R_2} = 0$$

$$\Rightarrow V_o = R_2 V_1 + R_1 V_2$$

$$\Rightarrow V_o = \left(\frac{R_2 + R_1}{R_1} \right) V_2 \quad (V_1 \approx V_2)$$

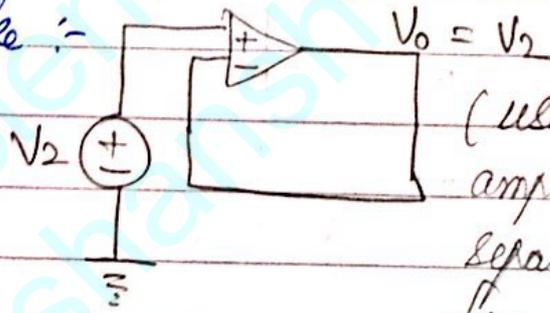
$$\Rightarrow V_o = \left(1 + \frac{R_2}{R_1} \right) V_2$$

$$\Rightarrow \boxed{\frac{V_o}{V_2} = 1 + \frac{R_2}{R_1}} \quad (\text{OVERALL GAIN})$$

↳ Non-inverting amplifier

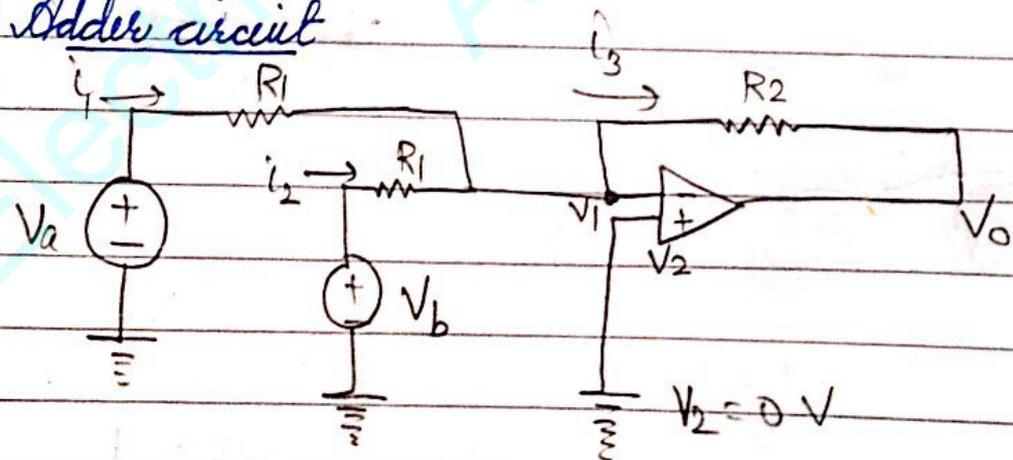
↳ If $R_2 = 0 \Omega$, then, $V_o = V_2$ ✓

↳ When this condⁿ is applied, the resulting OP-Amp is called VOLTAGE FOLLOWER & looks like :-



(used as a buffer amplifier - to separate one circuit from another)

Ex. 2.9 Adder circuit



Here, we would obtain o/p voltage V_o in terms of V_a & V_b .
 ∵ \exists a -ve feedback path, $V_1 = V_2 = 0$

KCL at inverting pt

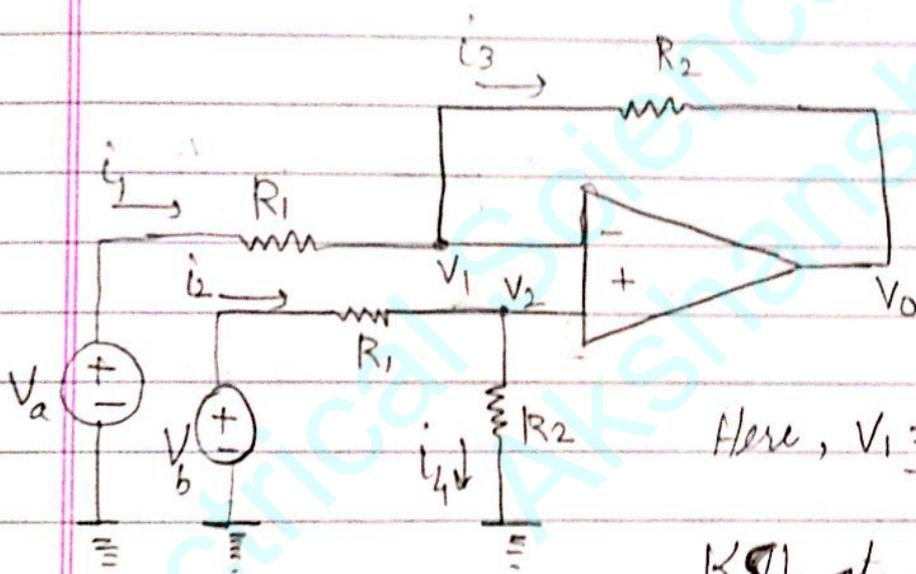
$$i_1 + i_2 = i_3 \Rightarrow \frac{V_a}{R_1} + \frac{V_b}{R_1} - \frac{(V_1 - V_o)}{R_2} = 0$$

$$\Rightarrow \frac{V_a}{R_1} + \frac{V_b}{R_1} = \frac{V_1 - V_o}{R_2}$$

$$\Rightarrow \boxed{V_o = -\frac{R_2}{R_1} (V_a + V_b)}$$

→ This circuit is called adder circuit or summer
 → Used by DT's for audio mixer.

ex. 2.10 Difference Amplifier or Differential Amplifier



Find :- o/p voltage (V_o) in terms of V_a & V_b .

$$\text{Here, } \underline{V_1 = V_2 = V}$$

KCL at inverting i/p,

$$i_1 = i_3$$

$$\Rightarrow \frac{V_a - V_1}{R_1} = \frac{V_1 - V_o}{R_2}$$

$$\Rightarrow \frac{V_a - V}{R_1} = \frac{V - V_o}{R_2}$$

$$\Rightarrow V_o = V + \frac{R_2}{R_1} (V - V_a) \quad \text{--- (1)}$$

KCL at non inverting i/p,

$$i_2 = i_4$$

$$\Rightarrow \frac{V_b - V_2}{R_1} = \frac{V_2}{R_2} \Rightarrow \frac{V_b - V}{R_1} = \frac{V}{R_2} \Rightarrow V = V_b \frac{R_2}{R_1 + R_2}$$

$$\Rightarrow V_o = \frac{R_2}{R_1} (V_b - V_a)$$

★ Differential amplifier

drill ex 2.8

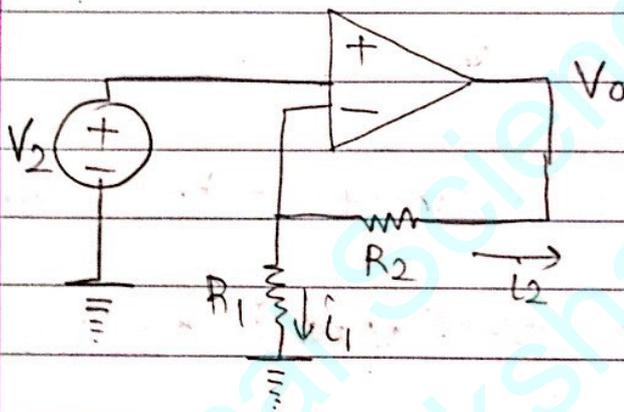
$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 9 \text{ k}\Omega$$

$$V_2 = 1 \text{ V}$$

Find: ^(a) V_o, i_1, i_2

(b) Power absorbed by each resistor, the independent voltage source and op-amp.



KCL at inverting pt.

$$i_1 = i_2$$

$$\Rightarrow \frac{1}{1000} = \frac{1 - V_o}{9000}$$

$$\Rightarrow V_o = 10 \text{ V}$$

$$i_1 = \frac{1}{1000} = 1 \text{ mA}$$

$$i_2 = \frac{-9}{9000} = -1 \text{ mA}$$

$$P_1 = 10^{-3} \text{ W}$$

$$P_2 = +9 \times 10^{-3} \text{ W}$$

$$P_s = V_s(i_s) = 0 \text{ W}$$

$$P_A = V_o i_2 = -10 \text{ mW}$$

drill 2.9 Op-amp

adder.

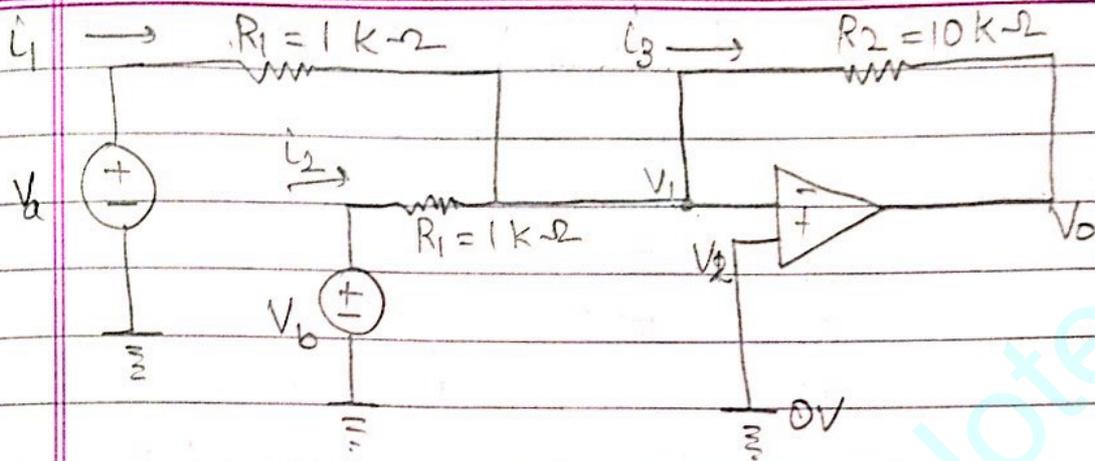
$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 10 \text{ k}\Omega$$

$$V_a = -0.2 \cos(2000\pi t) \text{ V}$$

$$V_b = 0.3 \cos(4000\pi t) \text{ V}$$

Find: V_o, i_1, i_2, i_3

KCL

$$i_1 + i_2 = i_3$$

$$\Rightarrow \frac{V_a}{R_1} + \frac{V_b}{R_1} = \frac{V_1 - V_o}{R_2}$$

$$V_1 = 0 \text{ (adder)}$$

$$\Rightarrow V_o = -\frac{R_2}{R_1} (0.3 \cos(4000\pi t) - 0.2 \cos(2000\pi t))$$

$$= \frac{R_2}{R_1} (0.2 \cos(2000\pi t) - 0.3 \cos(4000\pi t))$$

$$V_o = 2 \cos(2000\pi t) - 3 \cos(4000\pi t)$$

$$i_1 = (-2 \times 10^{-4}) \cos(2000\pi t) \text{ A}$$

$$i_2 = (3 \times 10^{-4}) \cos(4000\pi t) \text{ A}$$

$$i_3 = 0.3 \cos(4000\pi t) - 0.2 \cos(2000\pi t) \text{ mA}$$

Drill 2.10 Differential amplifier

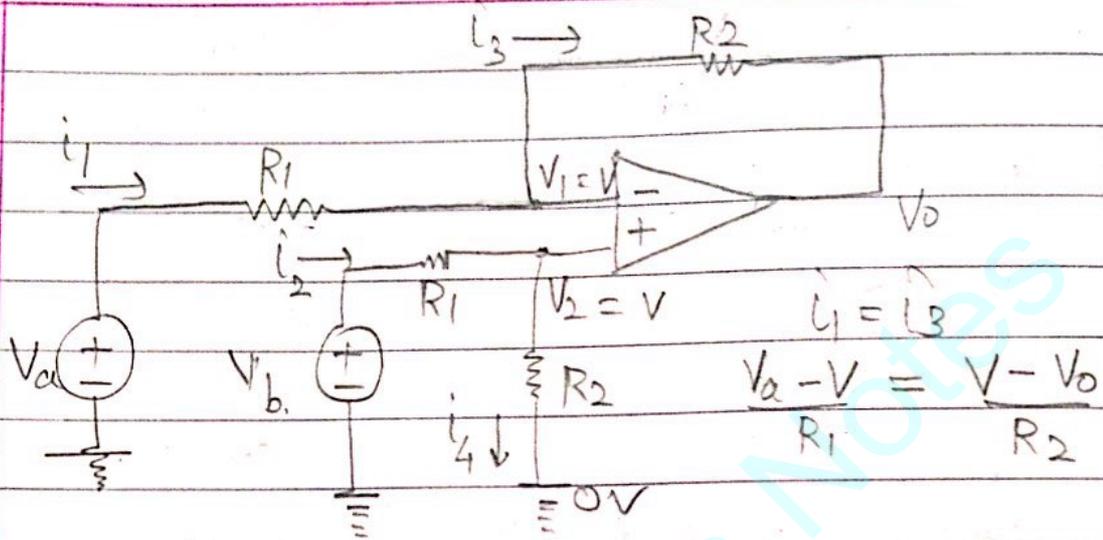
$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 9 \text{ k}\Omega$$

$$V_a = -0.2 \cos(2000\pi t)$$

$$V_b = 0.3 \cos(4000\pi t)$$

Find: $V_o, V_1, i_1, i_2, i_3, i_4$



$$V_o = 9(0.3 \cos(4000\pi t)) + 0.2 \cos(2000\pi t)$$

$$\frac{V}{R_2} + \frac{V}{R_1} = \frac{V_a}{R_1} + \frac{V_o}{R_2}$$

$$\Rightarrow \frac{V}{9 \times 10^3} + \frac{V}{10^3} = \frac{V_a}{10^3} + \frac{V_o}{9 \times 10^3}$$

$$\Rightarrow 10V = 9V_o + V_o$$

$$\Rightarrow V = 0.27 \cos(4000\pi t)$$

Now

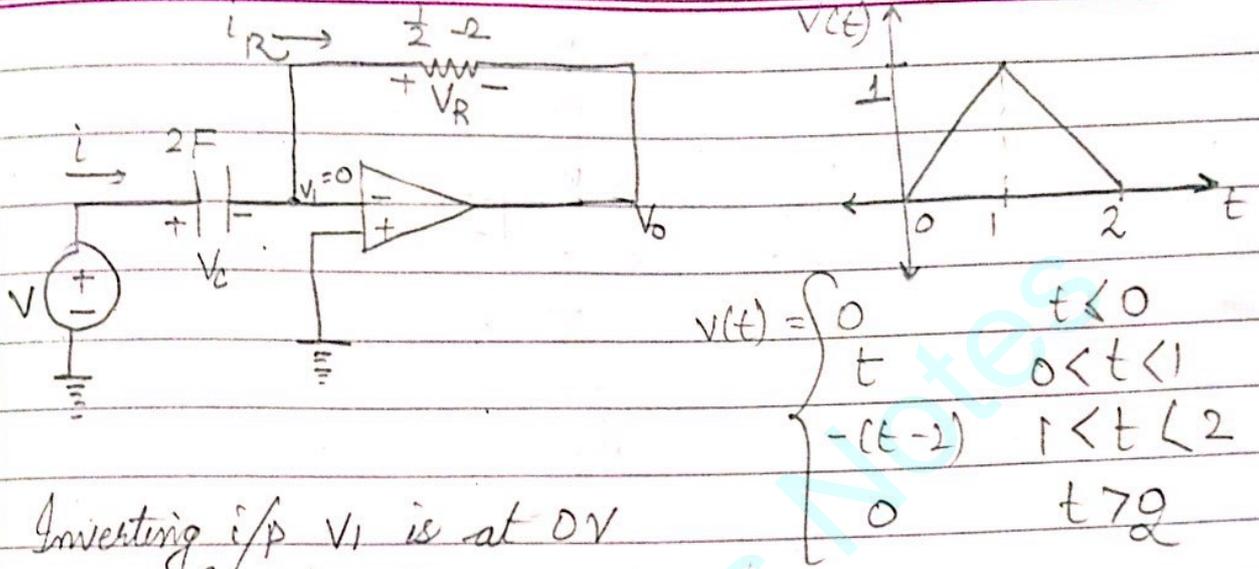
$$i_1 = i_3 = \frac{V_a - V}{R_1} = \frac{-0.2 \cos(2000\pi t) - 0.27 \cos(4000\pi t)}{10^3}$$

$$\text{or } \frac{V_a - V_o}{R_1 + R_2} = \frac{-0.2 \cos(2000\pi t) - 0.27 \cos(4000\pi t)}{10^3 + 9 \times 10^3}$$

$$i_2 = i_4 = \frac{V_b - 0}{R_1 + R_2} = \frac{V_b}{10} = \frac{0.3 \cos(4000\pi t)}{10}$$

$$\Rightarrow i_2 = i_4 = 0.03 \cos(4000\pi t) \text{ A}$$

ex. 3.4 Find: V_o in terms of v
op-amp circuit



$$V(t) = \begin{cases} 0 & t \leq 0 \\ t & 0 < t < 1 \\ -(t-2) & 1 < t < 2 \\ 0 & t \geq 2 \end{cases}$$

Inverting i/p V_1 is at 0V
So, $V_o = V$

$$\therefore i = C \frac{dV_o}{dt} = 2 \frac{dV}{dt}$$

\therefore The inputs of op-amp didn't draw any current,

$$\text{So, } V_R = \frac{1}{2} i = \frac{1}{2} (2 \frac{dV}{dt}) = \frac{dV}{dt}$$

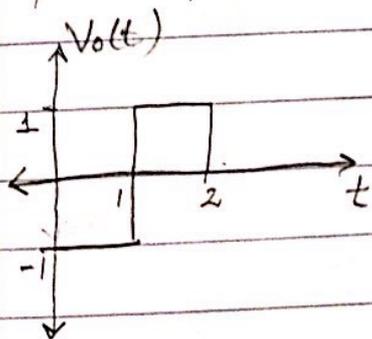
By KVL, $V_o + V_R = 0$

$$\Rightarrow V_o = -V_R$$

$$\Rightarrow V_o = -\frac{dV}{dt}$$

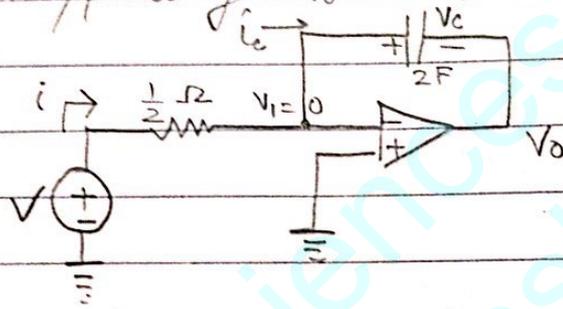
\rightarrow op voltage is derivative of i/p voltage.
 \rightarrow circuit called as differentiator.

o/p voltage:



$$V_o(t) = -\frac{dV}{dt} = \begin{cases} 0 & ; t \leq 0 \\ \frac{d(t)}{dt} = -1 & ; 0 < t < 1 \\ +1 & ; 1 < t < 2 \\ 0 & ; t \geq 2 \end{cases}$$

ex 3.7 Find :- op voltage V_o in terms of ip voltage V
 op-amp



$$\therefore V_1 = 0V; i = 2V$$

\therefore ip terminals don't draw any current,

$$i_c = i = 2V$$

$$\therefore V_o = \frac{1}{C} \int_{-\infty}^t i_c(t) dt = \frac{1}{2} \int_{-\infty}^t 2V(t) dt = \int_{-\infty}^t V(t) dt$$

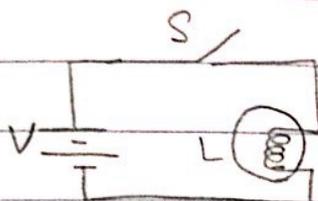
$$\underline{\text{KVL}} \quad V_o + V_c = 0 \Rightarrow V_o = -V_c$$

$$\therefore V_o(t) = - \int_{-\infty}^t V(t) dt$$

Chapter - 11

DIGITAL LOGIC CIRCUITS.

- can be expressed by 0 &/or 1.



When S is closed, $S = 1$,

L is ON, $L = 1$.

When S is open, $S = 0$

L is OFF, $L = 0$

$\therefore S = L$.

Truth table for L & S

	(a)	(b)	
S	$L (= S)$	S	\bar{L}
0	1	0	1
1	1	1	0

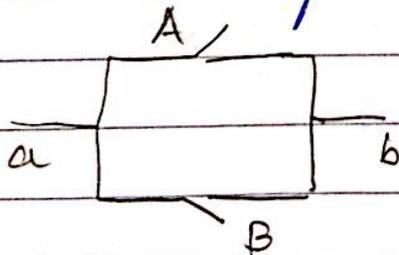
\bar{L} : complement of L .
 $\bar{L} = \bar{S}$

* Switching circuits:

Circuit shown above where values of L depend upon S , &, it is ON or OFF accordingly.

* OR CIRCUIT (GATE):

- Consider 2 switches A & B connected in ||. Battery & light bulb are present, but, not shown.



OR

A	B	$L = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

★ AND GATE

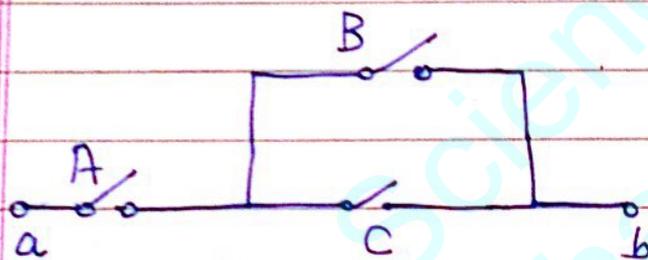


A	B	$L = A \cdot B$
0	0	0
1	0	0
0	1	0
1	1	1

0 \equiv open

1 \equiv closed

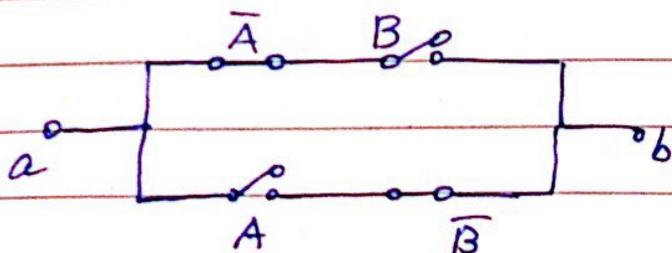
★ 3 switch combinations



$A \cdot (B + C)$
and or

	A	B	C	$L = A(B + C)$
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

★ Concept of Complement



\bar{A}, \bar{B} : complement of A & B

Exclusive OR operⁿ.

Truth table	A	B	$L = \bar{A}B + A\bar{B}$
	1	0	1
	0	1	1
	0	0	0
	1	1	0

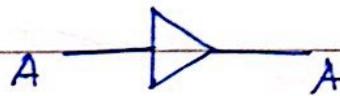
→ Binary oper^{ns} such as AND, OR, exclusive OR & complement are called logical oper^{ns} & circuits that perform these oper^{ns} are called Logic gates.

LOGIC GATES

1. BUFFER :

A digital logic gate whose i/p is a binary variable (0 or 1) and whose o/p is also A.

Symbol :-

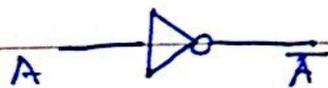


A	L
0	0
1	1

2. INVERTER or NOT GATE :-

i/p :- A o/p :- \bar{A} .

Symbol :-

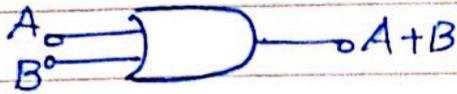


A	$L = \bar{A}$
0	1
1	0

3. OR GATE :

i/p :- A and B (or more) o/p :- $L = A + B$ (+ more)

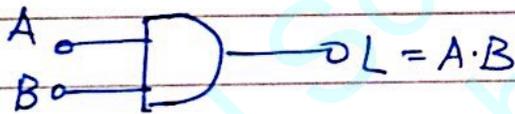
Symbol :-



A	B	$L = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

4. AND GATE

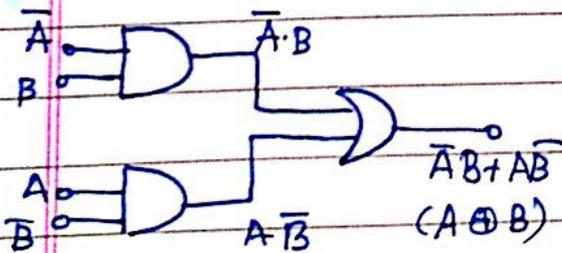
i/p :- A & B (~~&~~ or more) o/p :- $L = A \cdot B$ (or more)



A	B	$L = A \cdot B$
1	0	0
0	1	0
0	0	0
1	1	1

5. EXCLUSIVE OR GATE

i/p :- A & B o/p :- $A \oplus B = \bar{A}B + A\bar{B}$

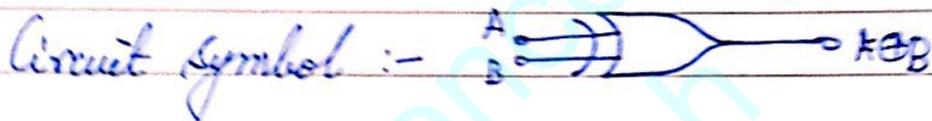
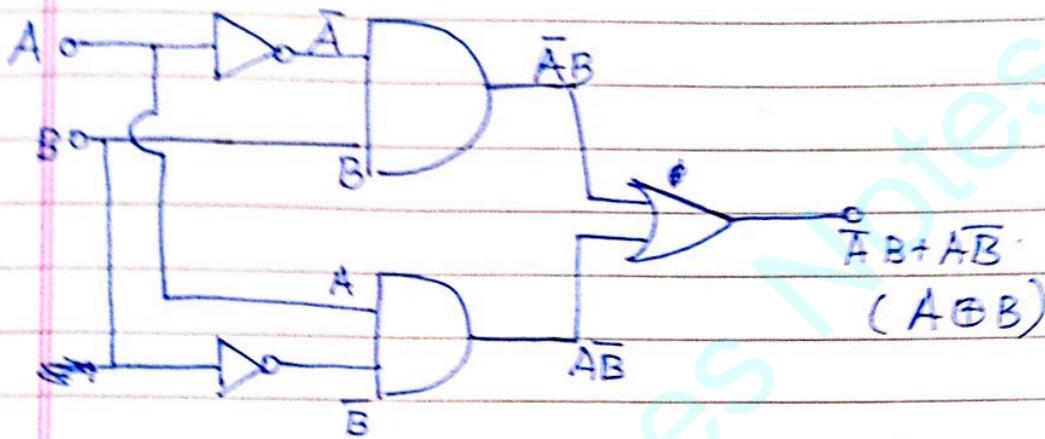


A	B	$L = \bar{A} \oplus B$
1	0	1
0	1	1
0	0	0
1	1	0

Symbol :-

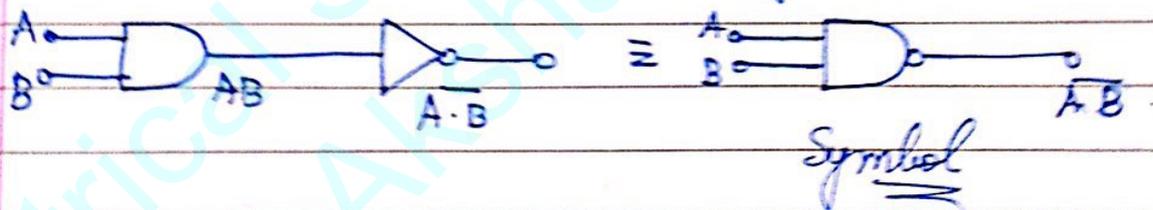
* Exclusive or gate can be written by using AND & OR gates.

Another way for making exclusive OR circuit :-



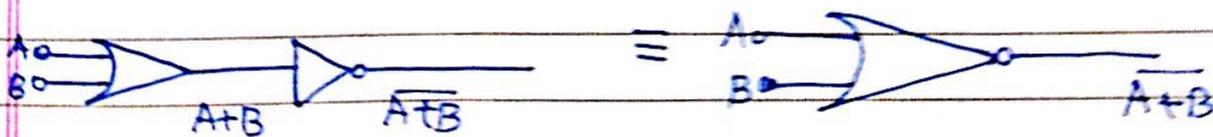
6. NAND GATE

Combinⁿ of AND & NOT gate



A	B	L = $\overline{A \cdot B}$
1	0	1
0	1	1
0	0	1
1	1	0

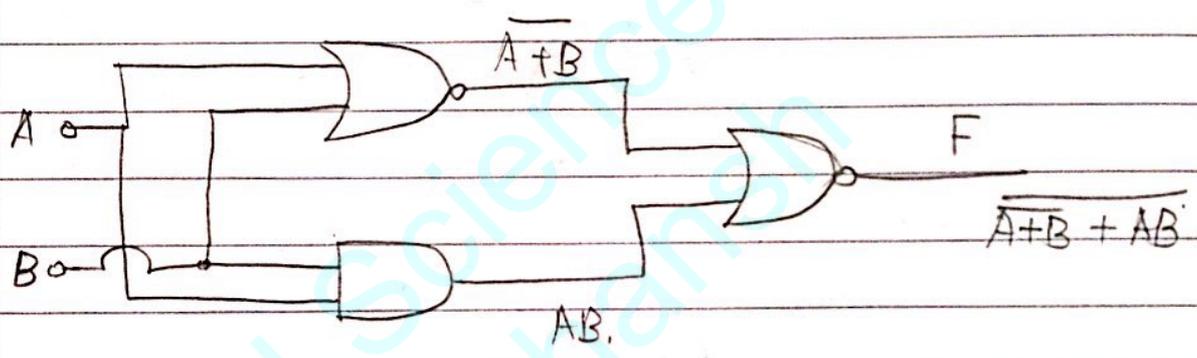
7. NOR gate :- Combinⁿ of OR & NOT gate



the truth table.

A	B	$L = \overline{A+B}$
1	0	0
0	1	0
0	0	1
1	1	0

Q. Consider a logic circuit,
 $F = \overline{\overline{A+B} + AB}$



A	B	$\overline{A+B}$	AB	$\overline{\overline{A+B} + AB}$
1	0	0	0	1
0	1	0	0	1
0	0	1	0	0
1	1	0	1	0

★ RULES for BOOLEAN ALGEBRA

- | | |
|----------------------------|------------------------|
| $A \cdot 1 = A$ | $A + 0 = A$ |
| $A \cdot A = A$ | $A + A = A$ |
| $A \cdot 0 = 0$ | $A + 1 = 1$ |
| $A \cdot \overline{A} = 0$ | $A + \overline{A} = 1$ |

Involution $\overline{\overline{A}} = A$
 Commutative $AB = BA$ & $A+B = B+A$

Associative : $A(BC) = (AB)C$ & $A+(B+C) = (A+B)+C$

Distributive :- $A(B+C) = AB+AC$ & $A+BC = (A+B)(A+C)$

★★

Absorption :- $A+AB = A$

$$(A \cdot 1 + A \cdot B = A(1+B) = A \cdot 1 = A)$$

$$A(A+B) = A$$

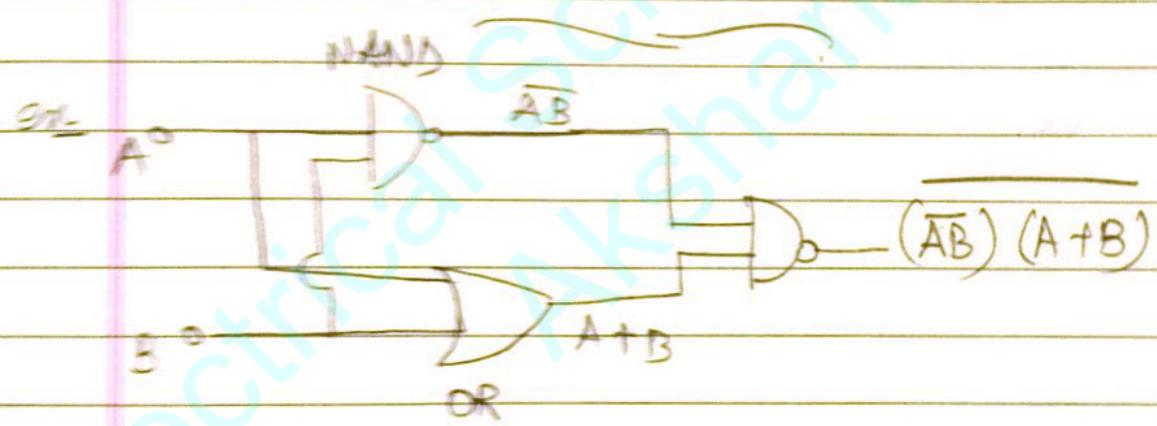
$$(AA + AB = A + AB = A)$$

$$A + \bar{A}B = A + B$$

$$((A + \bar{A})(A + B) = 1(A + B) = A + B)$$

$$A(\bar{A} + B) = AB$$

$$(A\bar{A} + AB = 0 + AB = AB)$$



A	B	$\bar{A}B$	$A+B$	$(\bar{A}B)(A+B)$
1	0	1	1	0
0	1	1	1	0
0	0	1	0	1
1	1	0	1	1

* De-Morgan's theorem :

$$\overline{A+B} = \bar{A} \cdot \bar{B} \quad \& \quad \overline{AB} = \bar{A} + \bar{B}$$

or TPT : $\overline{\overline{A+B} + AB} = \bar{A}B + A\bar{B}$

LHS

$$\overline{\overline{A+B} + AB} = \overline{\overline{A+B} \cdot \overline{AB}}$$

$$= (A+B) \overline{AB}$$

$$= (A+B) [\bar{A} + \bar{B}]$$

$$= A\bar{A} + \bar{A}B + A\bar{B} + B\bar{B}$$

$$= \bar{A}B + A\bar{B}$$

$$= \text{RHS} \quad \text{H.P.}$$

or TPT : $\overline{(\bar{A}B)(A+B)} = \bar{A}\bar{B} + AB$

$$\text{LHS} = \overline{(\bar{A}B)(A+B)} = \overline{\bar{A}B} + \overline{A+B}$$

$$= AB + \bar{A} \cdot \bar{B} = \text{RHS} \quad \text{H.P.}$$

* Duality :

2 expressions are called duals if you interchange

AND & OR

0 & 1

ex :- AB and $A+B$

$$A+0=A \quad \& \quad A \cdot 1=A$$

$$B \cdot 0=0 \quad \& \quad B+1=1$$

$$A(BC)=(AB)C \quad \& \quad A+(B+C)=(A+B)+C$$

★ The change that occurred in laws.

- $A(BC) = (AB)C$ & $A+(B+C) = (A+B)+C$
- Associative Law
- $A(B+C) = AB+AC$ & $A+BC = (A+B)(A+C)$
- Distributive Law
- $A+AB = A$ & $A(A+B) = A$
- Absorption Law
- $A+\bar{A}B = A+B$ & $A(\bar{A}+B) = AB$
- Absorption Law
- $\overline{A+B} = \bar{A}\bar{B}$ & $\overline{\bar{A}B} = \bar{A}+\bar{B}$
- De Morgan's Law

★ Dual of exclusive OR

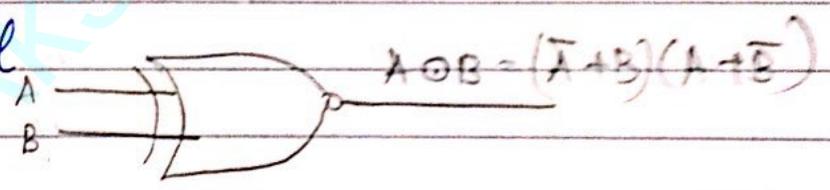
$$A \oplus B = \bar{A}B + A\bar{B}$$

becomes exclusive NOR & denoted by

$$A \odot B = (\bar{A}B + A\bar{B})$$

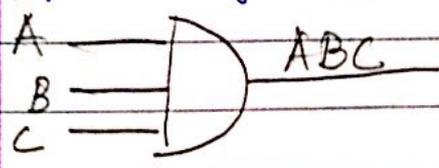
★ Exclusive NOR

↳ circuit symbol

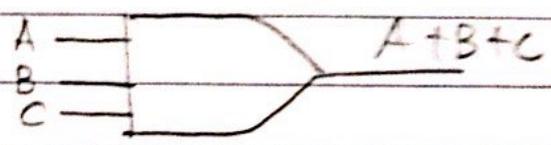


★ Multiple Input Gates

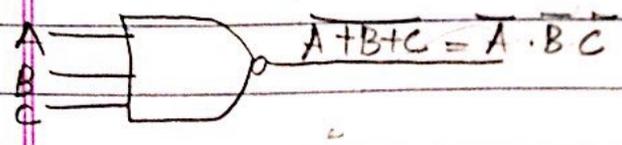
• 3 i/p AND gate for 3 i/p₂



• 3 i/p OR gate for 3 i/p₂

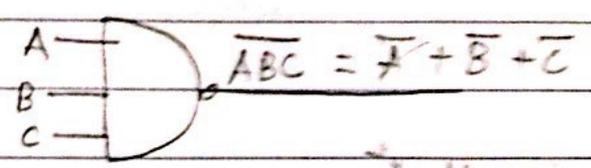


• 3 i/p NOR gate for 3 i/p₂



De Morgan's

• 3 i/p NAND gate for 3 i/p₂



De Morgan's

Q Find dual of $(\bar{A} + \bar{B})(A + B)$

$$(\bar{A} + \bar{B})(A + B) = \bar{A}\bar{B} + AB$$

A	B	\bar{A}	\bar{B}	AB	$\bar{A}\bar{B} + AB$
1	0	0	1	0	0
0	1	1	0	0	0
0	0	1	1	0	1
1	1	0	0	1	1

★ Sum of Products

- A given fⁿ F has n variables and the corresponding truth table has 2ⁿ rows.

- Suppose,

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

F has 4 terms called MINTERMS which are combined by OR (sum) operⁿ. F is said to be the sum of 4 minterms.

- Every minterm consists of 3 variables, each being complemented or uncomplemented which are combined by the AND (product) operⁿ.
- Because AND operⁿ is represented as multiplicⁿ, each of the 4 minterms is said to be product of 3 variables.
- For these reasons, the expression for the Boolean algebra fⁿ F is said to be in the form of sum of products.
- For a boolean fⁿ of 3 variables, $\exists 2^3 = 8$ minterms - 1 for each row.

Truth table for Boolean $f^n F$.

A	B	C	F	Minterms	Minterms for variables A, B, C
0	0	0	0	0	$m_0 \quad \bar{A} \bar{B} \bar{C}$
0	0	1	1	1	$m_1 \quad \bar{A} \bar{B} C$
0	1	0	1	1	$m_2 \quad \bar{A} B \bar{C}$
0	1	1	0	0	$m_3 \quad \bar{A} B C$
1	0	0	1	1	$m_4 \quad A \bar{B} \bar{C}$
1	0	1	0	0	$m_5 \quad A \bar{B} C$
1	1	0	0	0	$m_6 \quad A B \bar{C}$
1	1	1	1	1	$m_7 \quad A B C$

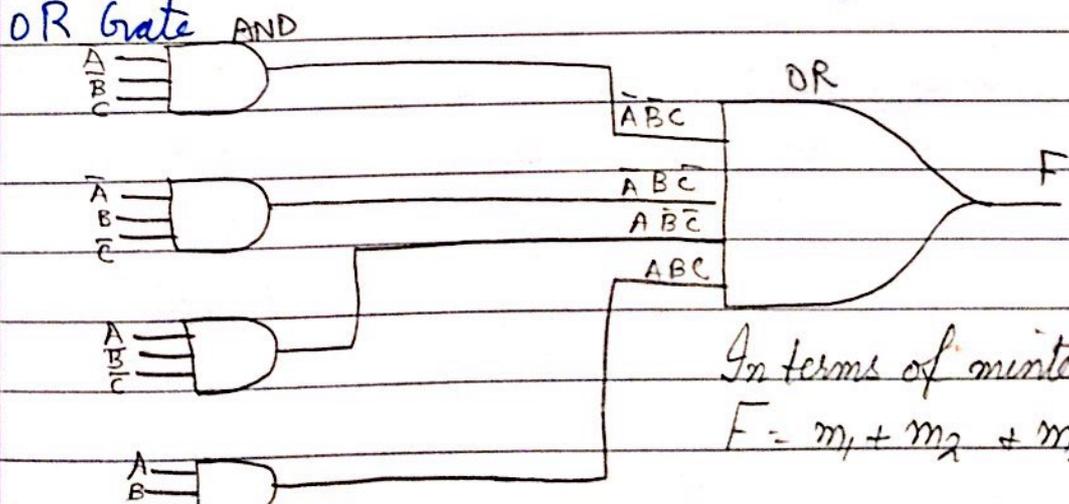
Now, the expression for a Boolean $f^n F$ is obtained by summing the minterms that correspond to rows in which $F = 1$ in the column labelled F.

$$F = m_1 + m_2 + m_4 + m_7$$

$$= \bar{A} \bar{B} C + \bar{A} B \bar{C} + A \bar{B} \bar{C} + A B C$$

- A logic circuit whose op is described by this Boolean f^n :
 $F = \bar{A} \bar{B} C + \bar{A} B \bar{C} + A \bar{B} \bar{C} + A B C$
 is like shown below.

We will have 4 AND Gates with 3 i/p, & one 4 i/p OR Gate



In terms of minterms ;
 $F = m_1 + m_2 + m_4 + m_7$

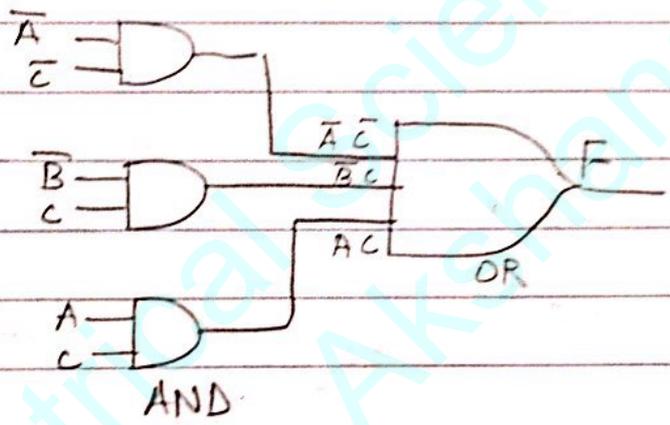
Q1 Express the Boolean fn.

$$F = \bar{A}\bar{C} + (A+B)C = \bar{A}\bar{C} + BC + AC \rightarrow \text{①}$$

as a sum of minterms

X ① by 1. Use $A + \bar{A} = 1$.

$$\begin{aligned} &= \bar{A}\bar{C} \cdot 1 + AC \cdot 1 + BC \cdot 1 \\ &= \bar{A} [B + \bar{B}] \bar{C} + A [B + \bar{B}] C + [A + \bar{A}] BC \\ &= \bar{A} B \bar{C} + \bar{A} \bar{B} \bar{C} + ABC + A\bar{B}C + \bar{A} B C + \bar{A} \bar{B} C \\ &= \bar{A} B \bar{C} + \bar{A} \bar{B} \bar{C} + ABC + A\bar{B}C + \bar{A} \bar{B} C \\ &= m_2 + m_0 + m_7 + m_1 + m_5 \\ &= m_0 + m_1 + m_2 + m_5 + m_7 \checkmark \end{aligned}$$



Q2 Express the boolean fn: $F = \bar{A}\bar{B} + B\bar{C}$ as a sum of minterms.

$$F = \bar{A}\bar{B} + B\bar{C} \rightarrow \text{①}$$

X ① by 1.

$$\begin{aligned} \Rightarrow F &= \bar{A}\bar{B} \cdot 1 + 1 \cdot B\bar{C} \\ &= \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})B\bar{C} \\ &= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C} \\ &= m_1 + m_0 + m_6 + m_2 \\ &= m_0 + m_1 + m_2 + m_6 \end{aligned}$$

Q Express $F = A \cdot \bar{B} + B\bar{C} + AB\bar{C} + \bar{A}C$ as a sum of minterms

× (1) by 1, some terms

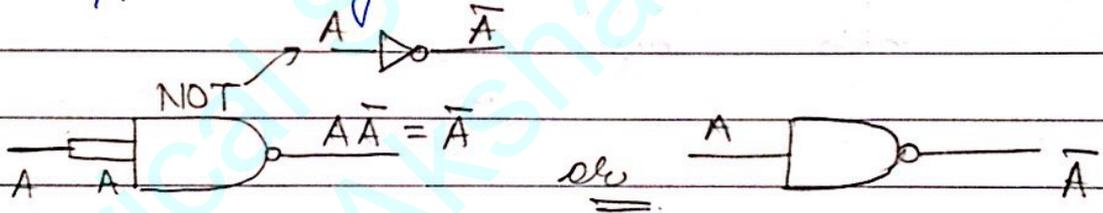
$$\begin{aligned} \Rightarrow F &= A\bar{B}(C+\bar{C}) + (A+\bar{A})B\bar{C} + AB\bar{C} + \bar{A}B(\bar{B}+B)C \\ &= A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C \\ &\quad + \bar{A}BC + \bar{A}\bar{B}C \end{aligned}$$

$$= m_5 + m_4 + m_6 + m_2 + m_3 + m_1$$

$$F = m_1 + m_2 + m_3 + m_4 + m_5 + m_6$$

* NAND GATE REALIZATIONS :

- Suppose: same i/p is applied to both i/p_s of a 2 i/p NAND gates.



- The o/p of a NAND gate is always a complement. Hence, the gate is an inverter, i.e., NOT gate.

- This is "NAND gate realization" of a NOT gate.

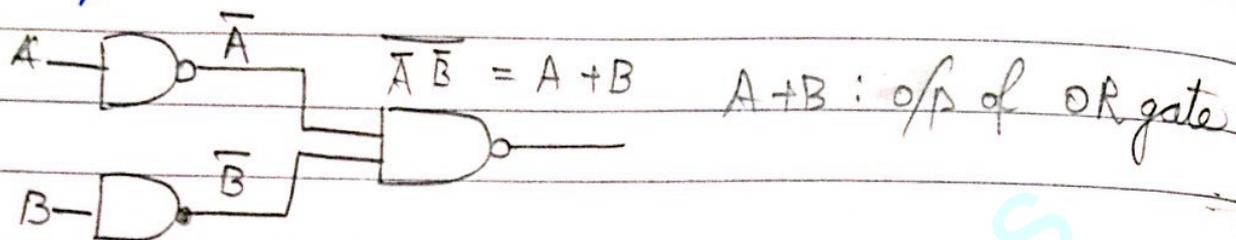
* Let's see how NAND gates can be connected to form an OR gate & AND gate.

By De Morgan's law:

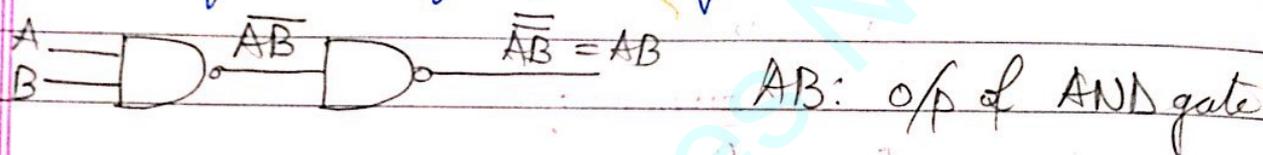
$$A + B = \overline{\overline{A + B}} = \overline{\bar{A} \bar{B}}$$

This expression can be realized by a logic circuit which is an OR Gate. We know, $AB = \overline{\bar{A}\bar{B}}$. So, this can be realized by a circuit which is an AND gate.

2 ip, A & B.



* NAND gate realiz^{ns} of AND gate:



- \therefore any Boolean fⁿ can be implemented with AND, OR and NOT gates, it can also be implemented using only NAND gates.
- This is also because AND, OR & NOT gates can be constructed from NAND gates.

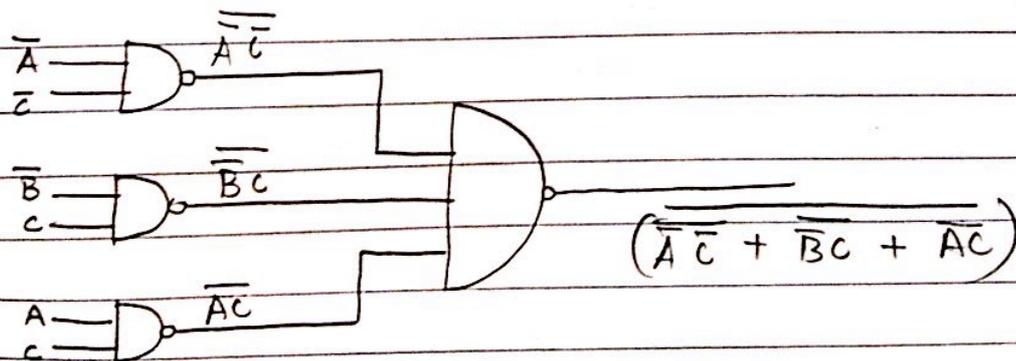
ex Use De-Morgan's theorem & determine NAND gate realizⁿ of

$$F = \overline{A} \overline{C} + \overline{B} C + AC.$$

$$F = \overline{\overline{\overline{A} \overline{C}} + \overline{\overline{\overline{B} C}} + \overline{\overline{AC}}}$$

$$= (\overline{A} \overline{C}) (\overline{B} C) (\overline{AC})$$

This can be shown by a 2 level NAND circuit.



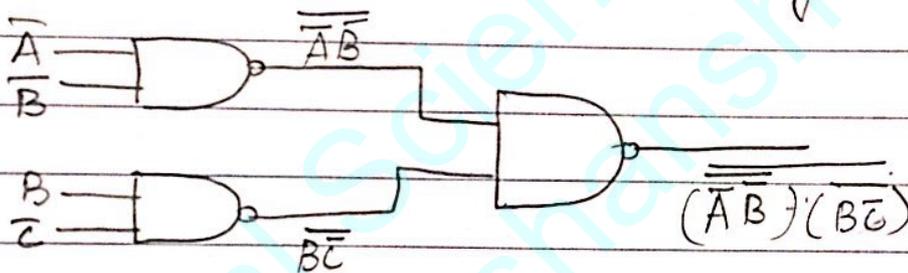
This is NAND gate implementⁿ of F

Reason → * NAND gate : also called Universal Gate.
 * Any arbitrary fn can be implemented by a logic circuit consisting of NAND gates.

11.11 ex 2 level realizⁿ of $F = \overline{A}B + B\overline{C}$ using NAND gates.
SI Involutions

$$F = \overline{\overline{A}B + B\overline{C}}$$

$$= \overline{\overline{A}B} \overline{B\overline{C}} \quad (\text{De Morgan's Thm})$$



★ Product of Sums

1.

- Given a truth table, the Boolean fn, F, can be expressed as a Sⁿ which is a sum of minterms.
- An alternate equivalent can also be obtained.
- F as a sum of minterms is

$$F = \overline{A}BC + A\overline{B}C + A\overline{B}\overline{C} + ABC.$$

$$\text{Complement of } F = \overline{F} = A\overline{B}C + A\overline{B}\overline{C} + \overline{A}BC + \overline{A}\overline{B}C.$$

Truth table.

A	B	C	F	\bar{F}		
0	0	0	0	1	m_0	$\bar{A}\bar{B}\bar{C}$
0	0	1	1	0	m_1	$\bar{A}\bar{B}C$
0	1	0	1	0	m_2	$\bar{A}B\bar{C}$
0	1	1	0	1	m_3	$\bar{A}BC$
1	0	0	1	0	m_4	$A\bar{B}\bar{C}$
1	0	1	0	1	m_5	$A\bar{B}C$
1	1	0	0	1	m_6	$AB\bar{C}$
1	1	1	1	0	m_7	ABC

$$\begin{aligned} \text{Here, } \bar{F} &= m_0 + m_3 + m_5 + m_6 \\ &= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} \end{aligned}$$

→ Sum of product

$$\begin{aligned} \Rightarrow (\bar{F}) &= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} \\ &= (\bar{A}\bar{B}\bar{C} + \bar{A}BC)(A\bar{B}C + AB\bar{C}) \\ &= (\bar{A}\bar{B}\bar{C})(\bar{A}BC)(A\bar{B}C)(AB\bar{C}) \end{aligned}$$

$$\Rightarrow F = (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

→ Product of sum

(By De Morgan's Law)

→ each sum is called maxterm. The maxterms are designated as $M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7$.

→ each maxterm consists of a sum of 3 variables, where each variable is either complemented &/or uncomplemented.

→ given the truth table for a Boolean f^n, F , an alternate expression to the sum of minterms is obtained by taking the product

of minterms that correspond to the rows in which \exists a 0 in the column \Rightarrow labelled as F (or column labelled as $\bar{F} = 1$).

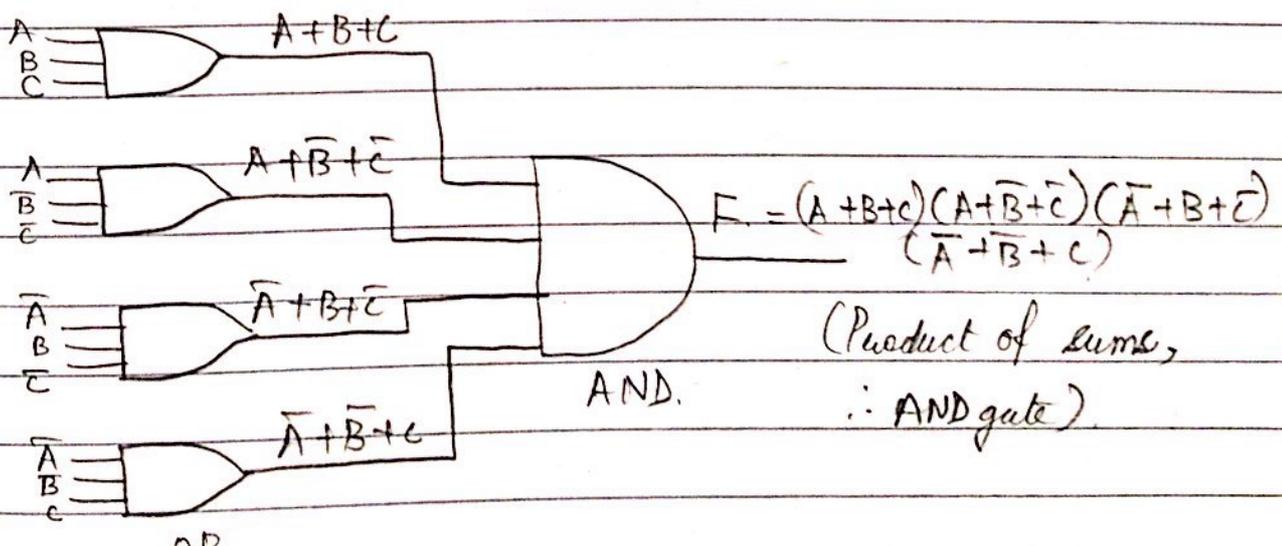
Truth table for F and \bar{F}

A	B	C	F	\bar{F}	Minterms of variables A, B & C.
0	0	0	0	1	$M_0 = A + B + C$
0	0	1	1	0	$M_1 = A + B + \bar{C}$
0	1	0	1	0	$M_2 = \bar{A} + \bar{B} + C$
0	1	1	0	1	$M_3 = A + \bar{B} + \bar{C}$
1	0	0	1	0	$M_4 = \bar{A} + B + C$
1	0	1	0	1	$M_5 = \bar{A} + B + \bar{C}$
1	1	0	0	1	$M_6 = \bar{A} + \bar{B} + C$
1	1	1	1	0	$M_7 = \bar{A} + \bar{B} + \bar{C}$

$$\therefore F = M_0 M_3 M_5 M_6$$

$$\Rightarrow F = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

- The 2nd level logic circuit for this f^n is shown below. Here, \exists 4 OR gates with 3 i/p, each & one 4 i/p AND gate.



Q Express F as a product of maxterms. (simple method)

$$\begin{aligned}
 F &= \bar{A}\bar{B} + B\bar{C} \\
 &= \bar{A}\bar{B}(\bar{C} + C) + (A + \bar{A})B\bar{C} \\
 &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C} \\
 &= m_1 + m_2 + m_6 + m_7
 \end{aligned}$$

(F corresponds to terms which have 1 in F column)

Remaining 4 minterms correspond to \bar{F} (\bar{F} corresponds to terms which have 1 in \bar{F} column).

$$\therefore \bar{F} = m_3 + m_4 + m_5 + m_7$$

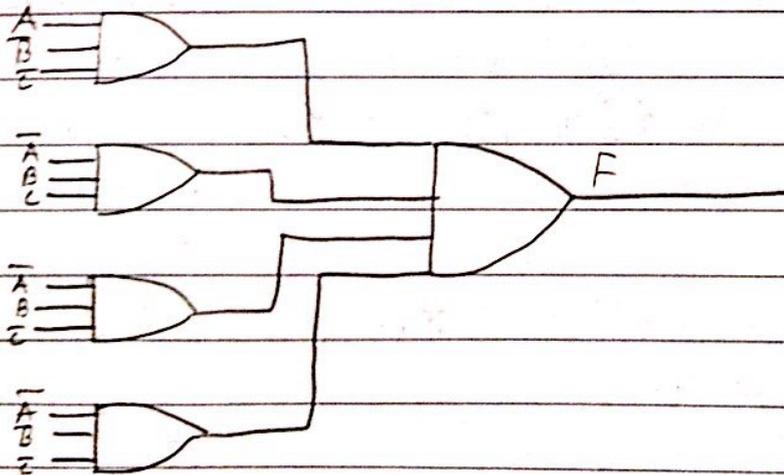
Now,

$$\begin{aligned}
 (\bar{F}) &= \overline{m_3 + m_4 + m_5 + m_7} \\
 &= \bar{m}_3 \cdot \bar{m}_4 \cdot \bar{m}_5 \cdot \bar{m}_7. \text{ (DeMorgan's)}
 \end{aligned}$$

$$\Rightarrow F = M_3 M_4 M_5 M_7$$

* The complement of minterms = Maxterms.

$$\text{eg: } \bar{m}_0 = M_0, \bar{m}_1 = M_1$$



11.12

8 Express F as a product of minterms.

$$F = \bar{A}\bar{C} + (A+B)C$$

M2 (Using Algebraic manipulation)

Distributive law: $A + BC = (A+B)(A+C)$

$$F = \underbrace{(\bar{A}\bar{C})}_A + \underbrace{(A+B)}_B \underbrace{C}_C$$

$$\Rightarrow F = (\bar{A}\bar{C} + A + B)(\bar{A}\bar{C} + C)$$

$$\Rightarrow F = (A + \bar{A}\bar{C} + B)(C + \bar{A}\bar{C}) \rightarrow \textcircled{1}$$

Consider $A + \bar{A}\bar{C}$

$$\times A \text{ by } 1 + \bar{C} \quad (\because 1 + \bar{C} = 1)$$

$$\Rightarrow A(1 + \bar{C}) + \bar{A}\bar{C}$$

$$\Rightarrow A + A\bar{C} + \bar{A}\bar{C}$$

$$= A + \bar{C}(A + \bar{A})$$

$$= A + \bar{C}$$

$$\text{Similarly, } C + \bar{C}A = C + \bar{A}$$

So, eqⁿ $\textcircled{1}$ becomes

$$F = \underbrace{(A + \bar{C} + B)}_{\text{minterm}} \underbrace{(C + \bar{A})}_{B \text{ not there}}$$

$$= (A + \bar{B} + \bar{C})(\bar{A} + 0 + C)$$

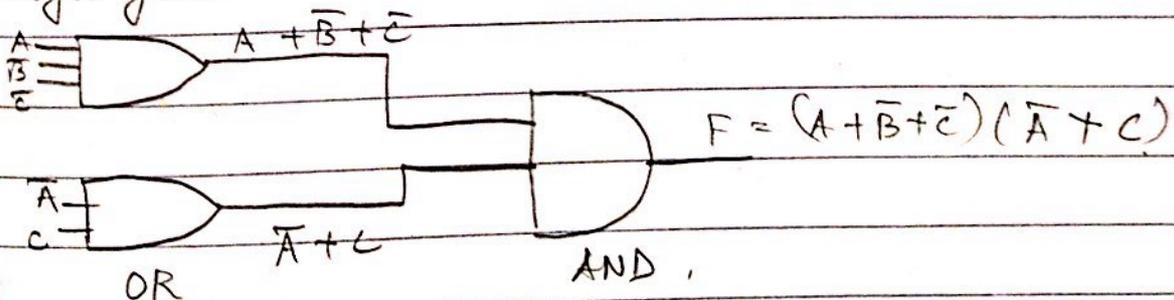
$$= (A + \bar{B} + \bar{C})(\bar{A} + B\bar{B} + C)$$

$$= (A + \bar{B} + \bar{C})(B + C + \bar{A})(\bar{B} + C + A)$$

$$\Rightarrow F = (A + \bar{C} + \bar{B})(\bar{A} + \bar{B} + C)(\bar{A} + B + C)$$

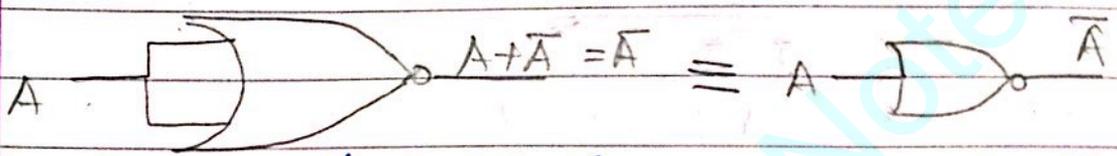
$$\Rightarrow F = M_3 M_4 M_6 \text{ (see table made earlier)}$$

Logic gate:



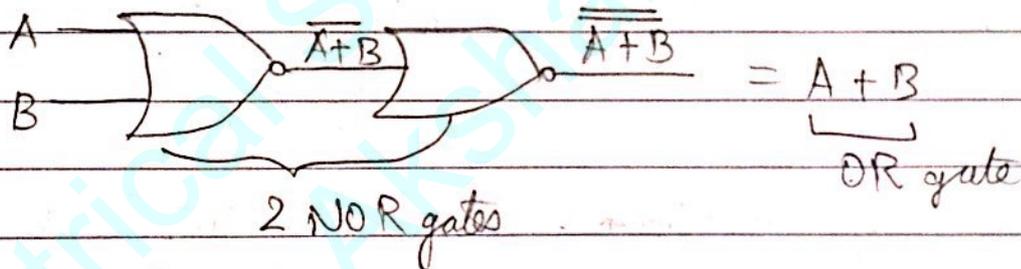
* NOR GATE REALIZATION.

- We show that AND, OR and NOT gates can be constructed from NOR gates (same way as we did for NAND).



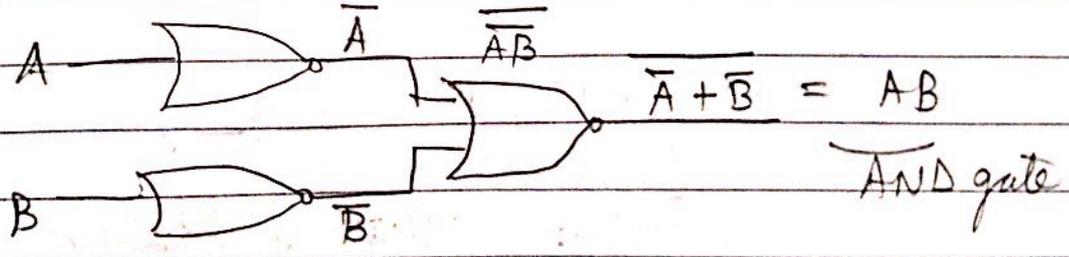
- Connecting both ip of a NOR gate together, we obtain NOT gate.

Now, we know $A + B = \overline{\overline{A + B}}$. So, connection of ~~two~~ NOR gates will give OR gate.



Now, Also, $AB = \overline{\overline{AB}} = \overline{\bar{A} + \bar{B}}$, then, NOR gate circuit realizes an AND gate.

* Just like NAND gate, NOR gate is also a universal gate.



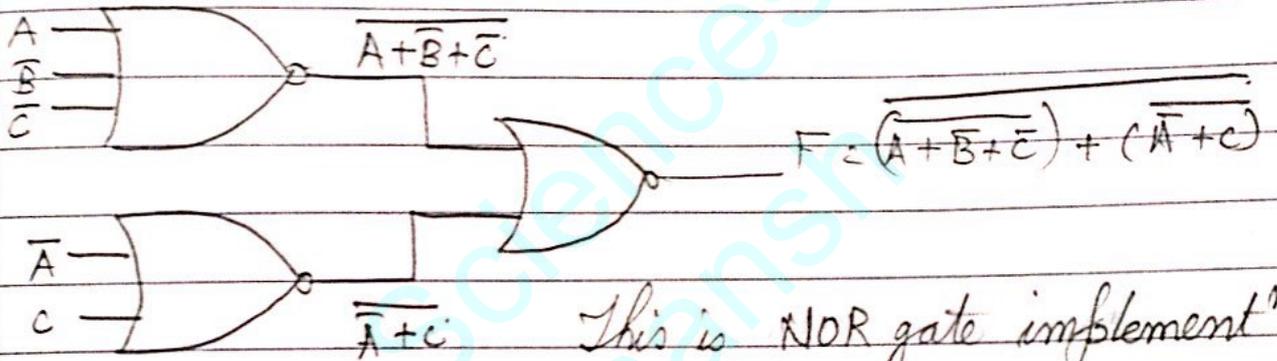
ex 11.3

Q. For f^n $F = (A + \bar{B} + \bar{C})(\bar{A} + C)$

Implement it by a 2-level NOR circuit

$$\begin{aligned} F &= (A + \bar{B} + \bar{C})(\bar{A} + C) \\ &= \overline{\overline{(A + \bar{B} + \bar{C})(\bar{A} + C)}} \\ &= \overline{\overline{(A + \bar{B} + \bar{C})} \cdot \overline{\overline{(\bar{A} + C)}}} \end{aligned}$$

The 2 level NOR circuit is:-



This is NOR gate implementⁿ of F

Drill ex 11.13 Use de-Morgan's thm. to obtain a 2 level

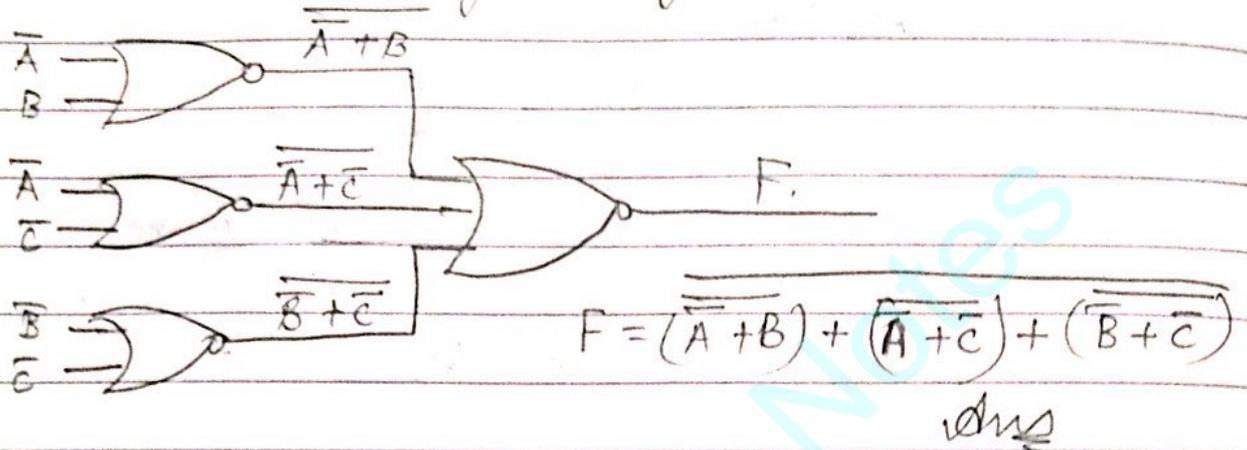
NOR gate realizⁿ of $F = \bar{A}\bar{B} + B\bar{C}$

We know, $(A + BC) = (A + B)(A + C)$

$$\begin{aligned} \Rightarrow F &= \bar{A}\bar{B} + B\bar{C} \\ &= \overline{\overline{\bar{A}\bar{B} + B\bar{C}}} \\ &= \overline{(\bar{A}\bar{B} + B)(\bar{A}\bar{B} + \bar{C})} \\ &= \overline{(\bar{A} + B)(\bar{B} + B)(\bar{A} + \bar{C})(\bar{B} + \bar{C})} \\ &= \overline{(\bar{A} + B)(\bar{A} + \bar{C})(\bar{B} + \bar{C})} \end{aligned}$$

$$\begin{aligned} \Rightarrow \bar{F} &= \overline{(\bar{A} + B)(\bar{A} + \bar{C})(\bar{B} + \bar{C})} \\ &= \overline{(\bar{A} + B) + (\bar{A} + \bar{C}) + (\bar{B} + \bar{C})} \end{aligned}$$

2 level NOR gate realizⁿ :-



Q. 11.49

$$F = AB + BC + AC$$

Use algebraic manipⁿ to express F as sum of minterms & product of sums.

As sum of min terms

$$F = AB + BC + AC$$

$$= AB(C + \bar{C}) + (A + \bar{A})BC + A(B + \bar{B})C$$

$$= ABC + AB\bar{C} + ABC + \bar{A}BC + ABC + A\bar{B}C$$

$$\Rightarrow F = m_7 + m_6 + m_3 + m_5$$

(Take common terms just once)

As product of sums

$$F = AB + BC + AC$$

$$= AB + C(A+B)$$

$$= (AB + C)(AB + A + B)$$

$$= (AB + C)(A(C(B+1)) + B)$$

$$= (AB + C)(A + B)$$

$$= (A+B)(A+C)(A+B) \quad (\because AB+C = (C+A)(C+B))$$

$$= (B+C+A\bar{A})(A+B\bar{B}+C)(A+B+C\bar{C})$$

$$\because A\bar{A} = 0$$

$$= (A+B+C)(\bar{A}+B+C)(A+B+C)(A+B+C) \\ (A+B+C)(A+B+\bar{C})$$

$$= (A+B+C)(\bar{A}+B+C)(A+B+C)(A+B+\bar{C})$$

(Again, $AB+C = (A+\bar{C})(B+C)$)

$$\Rightarrow F = M_0 M_1 M_2 M_4$$

(Take common terms just once)

✓ Ans

Q. 11.50) Use algebraic manipⁿ to express

$$F = \bar{A}\bar{B} + B\bar{C} \text{ as}$$

(i) sum of minterms

(ii) product of maxterms

(i) $F = \bar{A}\bar{B} + B\bar{C}$

$$= \bar{A}\bar{B}(C+\bar{C}) + (A+\bar{A})B\bar{C} \quad (\because A+\bar{A}=1)$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$$

$$= m_1 + m_0 + m_6 + m_2$$

(ii) $F = \bar{A}\bar{B} + B\bar{C}$

$$= (\bar{A}\bar{B} + B)(\bar{A}\bar{B} + \bar{C})$$

$$= (\bar{A}+B)(\bar{B}+\bar{B})(\bar{A}+\bar{C})(\bar{B}+\bar{C})$$

$$= (\bar{A}+B)(\bar{A}+\bar{C})(\bar{B}+\bar{C})$$

$$= (\bar{A}+B+C\bar{C})(\bar{A}+B\bar{B}+\bar{C})(A\bar{A}+\bar{B}+\bar{C})$$

$$= (\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})(A+\bar{B}+\bar{C})$$

$$= M_4 M_5 M_7 M_3$$

Q.11.48 Implement the Boolean fⁿ $F = A + BC = (A+B)(A+C)$ with a 2 level logic circuit employing only NAND gates & (b) NOR gates.

$$F = A + BC = (A+B)(A+C)$$

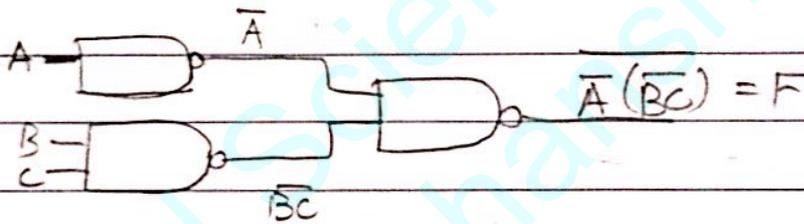
$$\Rightarrow \overline{F} = \overline{A + BC} = \overline{(A+B)(A+C)}$$

$$= \overline{(A+B)} + \overline{(A+C)}$$

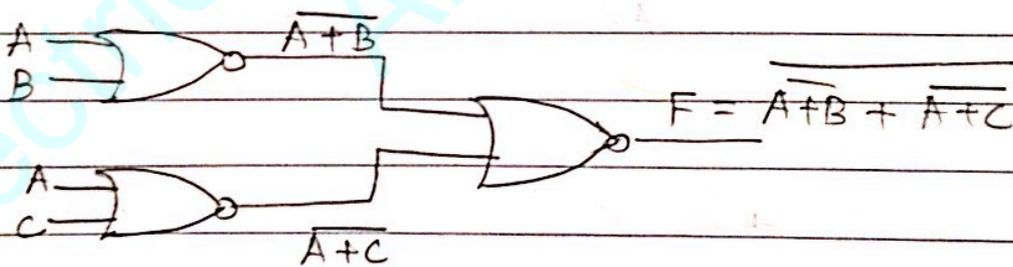
(Using De Morgan's)

$$\overline{A + BC} = (\overline{A})(\overline{BC})$$

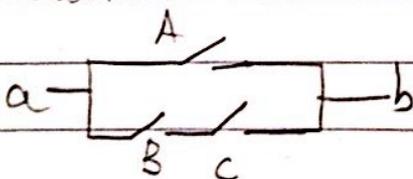
2 level NAND Gate :-



2 level NOR gate :-



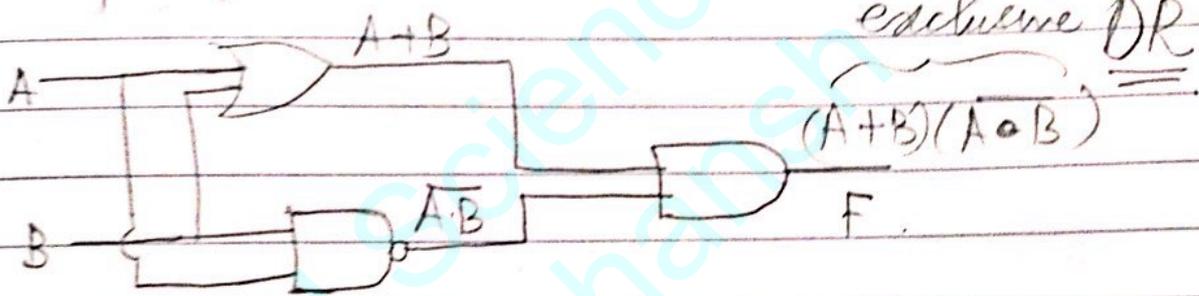
Q.11.20 Determine truth table for connection shown :-



$$L = A + BC$$

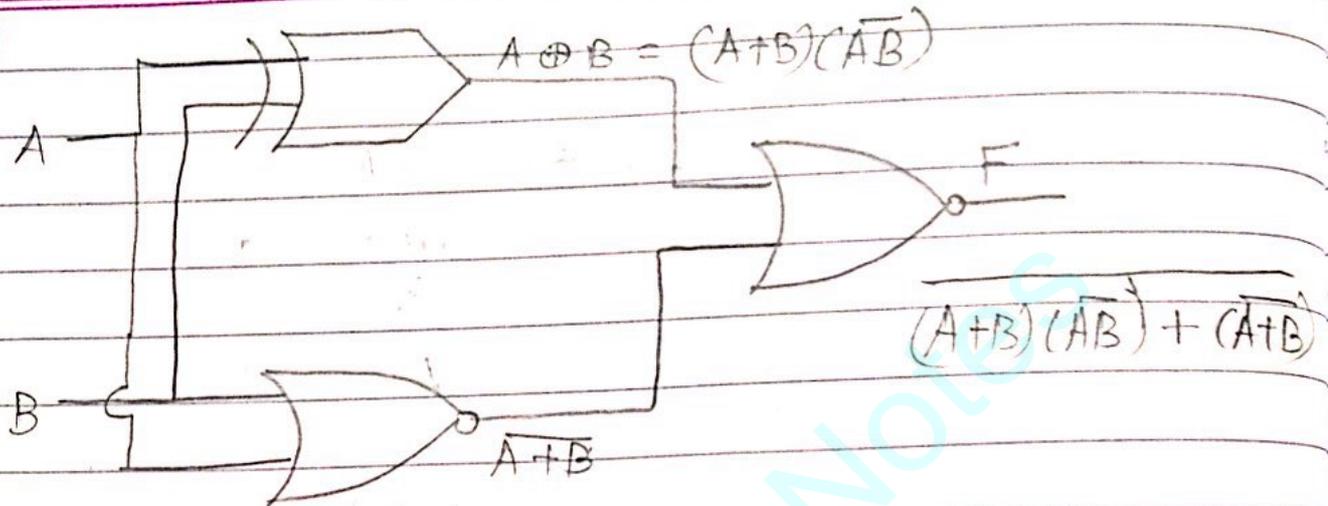
A	B	C	$L = A + BC$
1	0	0	1
0	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	0	0

Q.11.27



$$L = (A+B)(\overline{A \cdot B})$$

A	B	$A+B$	$(\overline{A \cdot B})$	L
1	0	1	1	1
0	1	1	1	1
0	0	0	1	0
1	1	1	0	0



A	B	$\overline{A+B}$	\overline{AB}	$A+B$	$A \oplus B$	F
1	0	0	1	1	1	0
0	1	0	1	1	1	0
0	0	1	1	0	0	0
1	1	0	0	1	0	1

$$F = \left[(\overline{A+B}) + (\overline{AB}) \right] (A+B)$$

Q. Use algebraic manipulations to show:
exclusive NOR operation has this form.

$$A \odot B = (A+\overline{B})(\overline{A}+B) = AB + \overline{A}\overline{B}$$

~~$$\begin{aligned} (A+\overline{B})(\overline{A}+B) &= \overline{(A+\overline{B})(\overline{A}+B)} \\ &= \overline{(A+\overline{B})} + \overline{(\overline{A}+B)} \\ &= \overline{A}\overline{B} + \overline{A}B \end{aligned}$$~~

$$\begin{aligned} (A+\overline{B})(\overline{A}+B) &= A\overline{A} + AB + \overline{A}\overline{B} + B\overline{B} \\ &= AB + \overline{A}\overline{B} \quad (\because A\overline{A} = 0) \end{aligned}$$

$$\text{Also, } V_o = A (V_o - V_i)$$

$$\Rightarrow V_o = -AV_i$$

$$\Rightarrow V_i = -\frac{V_o}{A}$$

$$\Rightarrow V_o = -\frac{V_o}{A} - i_s R$$

$$\Rightarrow V_o \left(1 + \frac{1}{A}\right) = -i_s R$$

$$\Rightarrow V_o = \frac{-A i_s R}{1+A} \quad \underline{\text{Ans}}$$