FORMULA SHEET

+1 and +2

Physics, Chemistry & Math

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COMPENDIUM – MATHEMATICS

I. ALGEBRA

1. Partial fractions

The resolution into partial fractions depends on the following rules:

(i) If the degree of the numerator of the given fraction is equal to or greater than that of the denominator, divide the numerator by the denominator until a remainder is obtained which is of lower degree than the denominator.

(ii) To every linear factor like $(ax + b)$ in the denominator, there corresponds a partial fraction of the form $\frac{A}{ax + b}$.

(iii) To every repeated factor like $(ax + b)^2$ in the denominator, there corresponds two partial fractions of the form $\frac{A}{ax + b} + \frac{B}{(ax + b)^2}$. Similarly for factors like $(ax + b)^3$, we have three partial fractions $\frac{A}{ax + b} + \frac{B}{(ax + b)^2} + \frac{C}{(ax + b)^3}$ and so on.

(iv) To every non-factorizable quadratic expression like $ax^2 + bx + c$ in the denominator, there corresponds a partial fraction of the form $\frac{Ax + B}{ax^2 + bx + c}$

(v) To every repeated quadratic factor like $(ax^2 + bx + c)^2$, there corresponds two partial fractions of the form $\frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{(ax^2 + bx + c)^2}$

(vi) **Thumb rule:** If in a partial fraction, all the denominator are of first degree, thumb rule can be applied for finding the components.

i.e., if the partial fraction is

$$\frac{A}{ax + b} + \frac{C}{cx + d} + \frac{E}{ex + f} = \frac{f(x)}{(ax + b)(cx + d)(ex + f)} \qquad \dots (1)$$

then A = the value at $x = -\frac{b}{a}$, of the function on the RHS, after omitting the corresponding denominator $ax + b$. Similarly for finding C, omit the denominator $cx + d$ in the RHS function and put $x = -\frac{d}{c}$, and so on.

Definition: The absolute value of a, written as $|a|$ is the positive numerical value of the real number a and is called the **modulus** of a real number.

(a) $|a| = a$ if a is positive

$= -a$ if a is negative

(b) $|a + b| \le |a| + |b|$

(c) $|a - b| \ge \big||a| - |b|\big|$

(d) $|ab| = |a| \cdot |b|$

(e) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $(b \ne 0)$

(f) If x, a, ℓ are 3 numbers such that $|x - a| < \ell$, then $(a - \ell) < x < (a + \ell)$.

(g) If a and b are real, $|a| > |b|$ if and only if $a^2 > b^2$.

2. Inequalities

(a) $a > b$, if a and b are real and $(a - b)$ is positive.

(b) $a < b$, if a and b are real and $(a - b)$ is negative.

(c) If c is real and $a > b$, then

(i) $a + c > b + c$ **(ii)** $a - c > b - c$

(d) If m is real and $a > b$, then

(i) $am > bm$ if $m > 0$

(ii) $am < bm$ if $m < 0$

(iii) $a^m > b^m$ if $m > 0$

(iv) $a^{-m} > b^{-m}$ if $m < 0$

(e) **(i)** If $0 < \frac{a}{b} < 1$, then $\frac{a}{b} < \frac{a + x}{b + x} < 1$ for any $x > 0$

(ii) If $\frac{a}{b} > 1$, then $\frac{a}{b} > \frac{a + x}{b + x} > 1$ for any $x > 0$

(f) **(i)** $a^2 + b^2 \ge 2ab$

(ii) $x + \frac{1}{x} \ge 2$ $(x > 0)$

(iii) $\left(a_1^2 + a_2^2 + \ldots + a_n^2\right)\left(b_1^2 + b_2^2 + \ldots + b_n^2\right) \ge \left(a_1b_1 + a_2b_2 + \ldots + a_nb_n\right)^2$

3. Complex numbers

i denotes a quantity such that $i^2 = -1$. Hence $i = \sqrt{-1}$.

An ordered pair of real numbers (a, b) united symbolically in the form a + ib is called a **complex number**. A complex number consists of two parts a and b, a is called the real part and b is called the imaginary part.

Note: When $b = 0$, $a + ib = a$

When $a = 0$, $a + ib = ib$

$\therefore$ the set of all real numbers and the set of all purely imaginary numbers are subsets of the set of complex numbers.

If $A = a + ib$, $a - ib$ is called its **conjugate** and is denoted by $\overline{A}$.

Properties

(a) If $a + ib = 0$, then $a = 0$ and $b = 0$.

(b) If $a + ib = c + id$, then $a = c$ and $b = d$.

(c) The sum, difference, product or quotient (the denominator is not zero) of two complex numbers is a complex number.

3.1. Modulus amplitude form of a complex number

If $a + ib$ is expressed in the form $r(\cos\theta + i\sin\theta)$, r is called the **modulus** denoted by $|a + ib|$ and θ is called the **amplitude** or argument denoted by $\arg(a + ib)$.

$$|a + ib| = r = \sqrt{a^2 + b^2}$$

θ is given by the equation $\cos\theta = \dfrac{a}{\sqrt{a^2 + b^2}}$, $\sin\theta = \dfrac{b}{\sqrt{a^2 + b^2}}$

The value of θ such that $-\pi < \theta \le \pi$ is called the principal value of the amplitude or argument and the general value of the argument is $2k\pi + \theta$, where k is any integer.

3.2 Properties of modulus and amplitude

(a) A complex quantity and its conjugate have the same modulus.

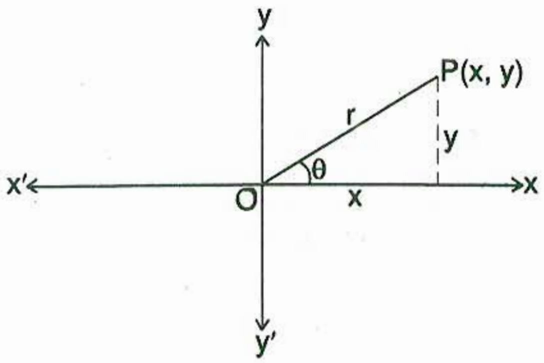
(b) The modulus of the product of two complex quantities is the product of their moduli and the amplitude of the product is the sum of their amplitudes.

(c) The modulus of the quotient of two complex quantities is the quotient of their moduli and the amplitude of the quotient is the difference of their amplitudes.

3.3 Geometrical representation – Argand diagram

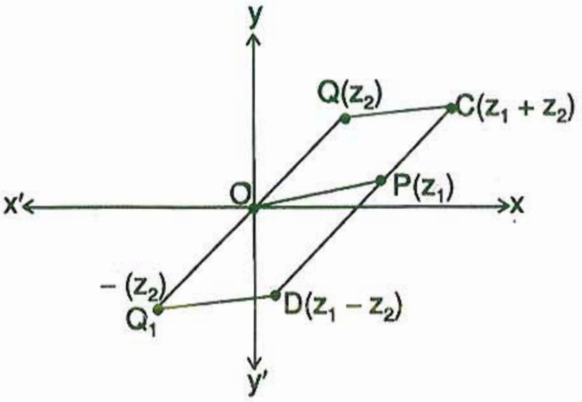
Let xox′, yoy′ be rectangular axes with the usual sign conventions. Then the point P whose Cartesian coordinates are (x, y) represents the complex number $z\ (= x + iy)$. This representation of complex numbers as points in a plane is called the Argand diagram.

xox′ is called the **real** axis and yoy′ is called the **imaginary** axis and the plane containing xox′ and y′oy is called the complex or z plane. The polar coordinates of P gives the modulus and amplitude of the complex quantity.



(a) Addition and subtraction of two complex numbers

Let P and Q represent the complex numbers z_1 (= $x_1 + iy_1$) and z_2 (= $x_2 + iy_2$) respectively in the Argand diagram. Complete the parallelogram OPCQ. Then C represents the number $z_1 + z_2$.



Produce QO to Q_1 such that OQ_1 = QO. Complete the parallelogram OQ_1DP.

Then D represents the number $z_1 - z_2$.

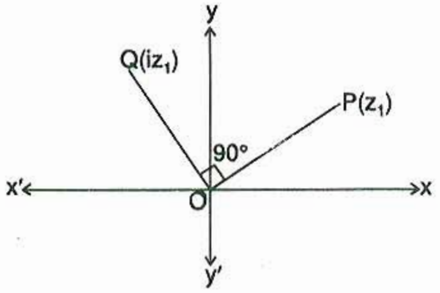
(i) $|z_1 - z_2|$ is the distance between the points represented by z_1 and z_2.

(ii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

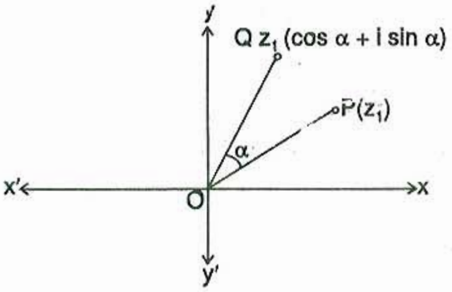
(iii) $|z_1 + z_2| \le |z_1| + |z_2|$

(iv) $|z_1 - z_2| \ge |z_1| \sim |z_2|$

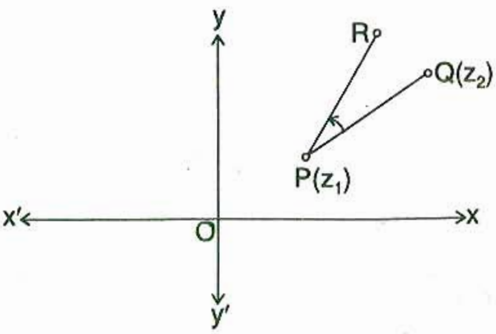
(b) (1) Rotate OP through 90° in the anticlockwise direction. Then P(z_1) will occupy the point Q which represents iz_1



(2) Rotate OP through an angle α in the anticlockwise direction then P(z_1) will occupy the point Q represented by the point, $z_1 (\cos \alpha + i \sin \alpha)$

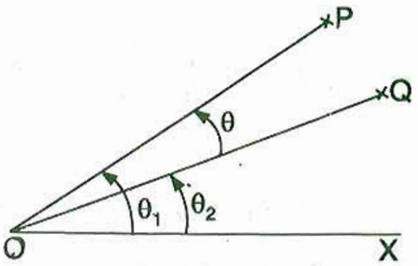


(3) If PQ is rotated through an angle α in the anticlockwise direction, Q will occupy the point R which represents $z_1 + (z_2 - z_1)(\cos \alpha + i \sin \alpha)$



(4) If S be any point other than Q on PR where PS ≠ PQ, then S can be represented by a complex number $z_1 + (z_2 - z_1)(\cos\alpha + i\sin\alpha)\dfrac{PS}{PQ}$

3.4 (i) If P and Q represent the complex numbers z_1 and z_2, $\angle XOP = \theta_1$, $\angle XOQ = \theta_2$ and $\angle QOP = \theta$ where O is the origin.

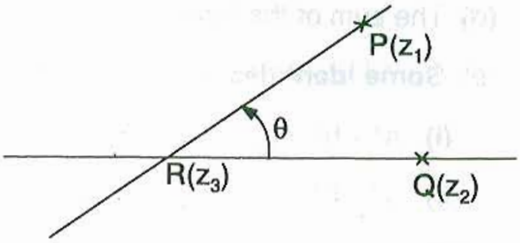


$$\theta = \theta_1 - \theta_2$$

$$= \text{Argument of } z_1 - \text{Argument of } z_2$$

$$= \text{Argument of } \frac{z_1}{z_2}$$

(ii) If O is replaced by $R(z_3)$,



$$\theta = \text{Amplitude of } (z_1 - z_3) - \text{Amplitude of } (z_2 - z_3)$$

$$= \text{Amplitude of } \frac{z_1 - z_3}{z_2 - z_3}$$

(iii) If r is the modulus of $\dfrac{z_1 - z_3}{z_2 - z_3}$,

$\dfrac{z_1 - z_3}{z_2 - z_3} = r(\cos\theta + i\sin\theta) = \left|\dfrac{z_1 - z_3}{z_2 - z_3}\right| e^{i\theta}$, where θ is the angle between RQ and RP.

3.5 De Moivre's theorem

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, if n is an integer and $\cos n\theta + i\sin n\theta$ is one of the values of $(\cos\theta + i\sin\theta)^n$, if n is a fraction.

Corollary: The q values of $(\cos\theta + i\sin\theta)^{\frac{1}{q}}$ are obtained from

$\cos\dfrac{2n\pi + \theta}{q} + i\sin\dfrac{2n\pi + \theta}{q}$ by putting $n = 0, 1, 2, \ldots, (q-1)$.

3.6 Euler's formula

$e^{i\theta} = \cos\theta + i\sin\theta$, $e^{-i\theta} = \cos\theta - i\sin\theta$

From this, $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

3.7 Cube roots of unity

The three cube roots of unity are 1, $\frac{1}{2}(-1+\sqrt{-3})$, $\frac{1}{2}(-1-\sqrt{-3})$ which are the same as 1, $\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)$ and $\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)$.

Properties

(a) Each imaginary cube root of unity is the square of the other.

(b) The product of the two imaginary cube roots of unity is unity. i.e., $\omega \cdot \omega^2 = 1$

(c) The integral power of cube root of unity is 1 or ω or ω^2, where ω is one of the imaginary cube roots of unity.

(d) The sum of the three cube roots of unity is zero. i.e., $1+\omega+\omega^2=0$.

(e) **Some identities involving** ω

(i) $a^3+b^3=(a+b)(a+\omega b)(a+\omega^2 b)$

(ii) $a^3-b^3=(a-b)(a-\omega b)(a-\omega^2 b)$

(iii) $a^4+a^2b^2+b^4=(a-b\omega)(a+b\omega)(a-b\omega^2)(a+b\omega^2)$

(iv) $a^2+b^2+c^2-ab-bc-ca=(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

(v) $a^3+b^3+c^3-3abc=(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

4. Quadratic equations and expressions

4.1. The roots of the quadratic equation

$ax^2+bx+c=0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

4.2 Relation between the roots and coefficients

(a) If α and β be the roots of the equation, then $ax^2+bx+c=a(x-\alpha)(x-\beta)$

(b) Sum of the roots, $\alpha+\beta=-\frac{b}{a}$, product of the roots, $\alpha\beta=\frac{c}{a}$.

4.3 Sign of the roots

(a) If a, b, c are all positive or all negative, the sum of the roots is negative while their product is positive. Hence both the roots are negative.

(b) If a and b are of the same sign and c of the opposite sign, both the sum and the product of the roots are negative. Hence one root is positive and the other root is negative.

(c) If a and c are of the same sign and b of the opposite sign, both the sum and product of the roots are positive. Hence both the roots are positive.

(d) If b and c are of the same sign and a of the opposite sign, the sum of the roots is positive, while their product is negative. Hence one root is positive and the other root is negative.

4.4 Nature of the roots

(a) If $b^2 - 4ac$ be a positive quantity, the roots are real and distinct.

(b) If $b^2 - 4ac$ be a perfect square, the roots are real, rational and unequal.

(c) If $b^2 - 4ac = 0$, the roots are real and equal.

(d) If $b^2 - 4ac$ be a negative quantity, the roots are complex.

Definition: $b^2 - 4ac$ is called the **discriminant** of the equation $ax^2 + bx + c = 0$.

(e) Let $D = b^2 - 4ac$. Hence for real roots $D \geq 0$.

(f) If one root of a quadratic equation is irrational, the other root is its irrational conjugate. (given that a, b, c are rational)

(g) If one root of a quadratic equation is complex, the other root is its complex conjugate. (given that a, b, c are real)

4.5 Symmetrical function of the roots

To get the value of a symmetrical function of the roots, express the function in terms of the sum and the product of the roots and substitute in that expression the values of the sum and the product of the roots in terms of the coefficients of the equation.

4.6 Formation of a quadratic equation whose roots are known

$x^2 - x$ (sum of the roots) + product of the roots = 0

4.7 Common roots

(a) If $ax^2 + bx + c = 0$, $a_1x^2 + b_1x + c_1 = 0$ have a common root, then

$$(ca_1 - c_1a)^2 = (bc_1 - b_1c)(ab_1 - a_1b)$$

The common root is $\dfrac{ca_1 - c_1a}{ab_1 - a_1b}$ or $\dfrac{bc_1 - b_1c}{ca_1 - c_1a}$.

(b) If $ax^2 + bx + c = 0$, $a_1x^2 + b_1x + c_1 = 0$ have both the roots common, then $\dfrac{a}{a_1} = \dfrac{b}{b_1} = \dfrac{c}{c_1}$.

5. Quadratic expressions

$ax^2 + bx + c$ where a, b, c are constants is called a quadratic expression in x.

5.1 Sign of a quadratic expression

(a) When a is positive, $ax^2 + bx + c$ is positive for all real values of x if $b^2 - 4ac$ is negative.

(b) When a is negative, $ax^2 + bx + c$ is negative for all real values of x if $b^2 - 4ac$ is negative.

(c) When a is positive, $ax^2 + bx + c$ is negative when x lies between the roots of the equation $ax^2 + bx + c = 0$.

(d) When a is negative, $ax^2 + bx + c$ is positive when x lies between the roots of the equation $ax^2 + bx + c = 0$.

All these four cases can be combined in a single statement.

The value of the quadratic expression $ax^2 + bx + c$ has always the same sign as a, except when $ax^2 + bx + c = 0$ has real and distinct roots and x lies between them.

5.2 Maximum and minimum values

(a) When a is positive, the minimum value of $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$ and is attained when $x = -\frac{b}{2a}$.

(b) When a is negative, the maximum value of $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$ and is attained when $x = -\frac{b}{2a}$.

5.3 Geometrically, the equation $y = ax^2 + bx + c$ represents a parabola

(a) The curve $y = ax^2 + bx + c$ where $a \neq 0$ opens upwards if a is positive and opens downwards if a is negative.

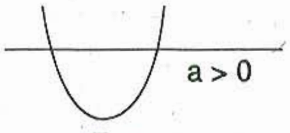


Fig. (i)

(b) The coordinates of the vertex of the parabola are

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) \text{ i.e., } \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$$

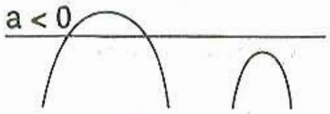
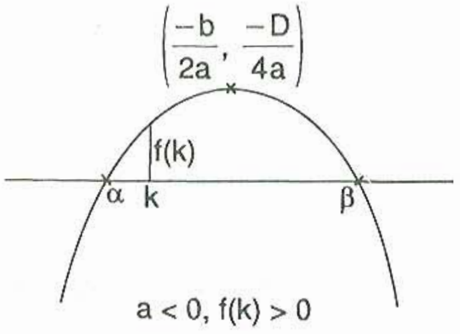
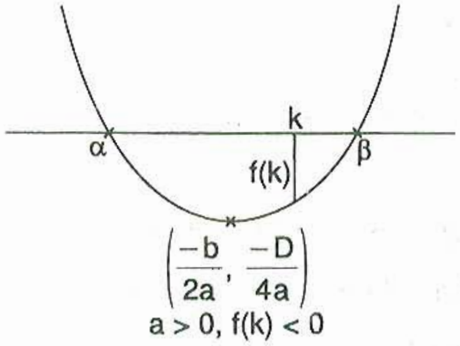


Fig. (ii)

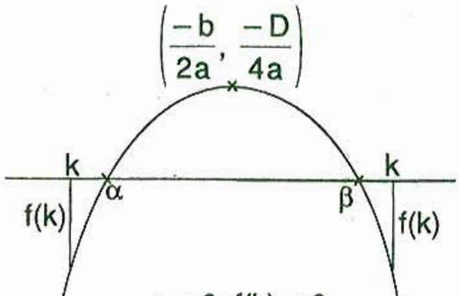
5.4 Position of roots of a quadratic equation

(a) If a real number k lies between the roots of a quadratic equation $f(x) = ax^2 + bx + c = 0$, then $D > 0$, f(k) and 'a' are of opposite signs and therefore $af(k) < 0$ (Refer to **Figures**).

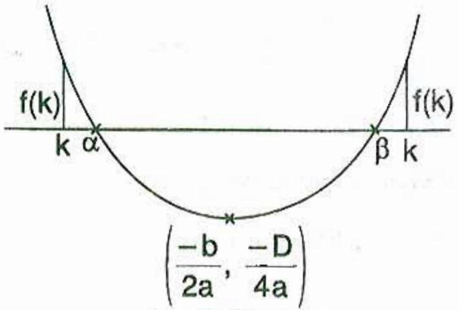




(b) If the two roots of the quadratic equation $f(x) = ax^2 + bx + c = 0$ are real and distinct and a real number k does not lie between the roots, then $D > 0$ and $af(k) > 0$.



$a < 0, f(k) < 0$



$a > 0, f(k) > 0$

(c) If a real number k is less than both the real and distinct roots of a quadratic equation $f(x) = ax^2 + bx + c = 0$, then

(i) $D > 0$ **(ii)** $af(k) > 0$ **(iii)** $k < -\frac{b}{2a}$

(d) If a real number k is greater than both the real roots of the equation, then

(i) $D > 0$ **(ii)** $af(k) > 0$ **(iii)** $k > -\frac{b}{2a}$

Note: If the equation is squared at any stage, extraneous roots may occur. Hence determine whether the solutions satisfy the given equation or not.

6. Progressions

Definition: If the terms of a series successively increase or decrease by a constant quantity, the series is called an **arithmetic progression.** The constant quantity is called the **common difference.**

6.1 The general form of an arithmetic progression denoted by A.P. is a, a + d, a + 2d, ...

(a) The nth term of the A.P. is $a + (n - 1)d$.

(b) Sum of n terms of the A.P. is $\frac{n}{2}[2a + (n - 1)d]$ or $\frac{n}{2}(\ell + a)$, where ℓ is the nth term.

6.2 **(a)** $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

(b) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

6.3 **Definition:** If the terms of a series increase or decrease by a common ratio, the series is called a **geometric progression.**

6.4 The general form of a geometric progression denoted by G.P. is a, ar, ar^2, ...

(a) The nth term of the G.P. is ar^{n-1}

(b) Sum of n terms of the G.P. = $\frac{a(1-r^n)}{1-r}$ where $r < 1$

(c) Sum of an infinite geometric progression of first term a and common ratio r ($|r| < 1$) is $\frac{a}{1-r}$.

6.5 Definition: A set of terms is said to be in **harmonic progression** when their reciprocals are in arithmetic progression. The general form of a harmonic progression is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots.$

The nth term of the series is $\frac{1}{a+(n-1)d}$.

6.6 Means

Let A be the arithmetic mean, G be the geometric mean and H the harmonic mean between two positive quantities a and b, then

(a) $A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$

(b) A, G, H are in G.P. i.e., $G^2 = HA$

(c) $A \geq G \geq H$

(d) Arithmetic mean of n quantities $a_1, a_2, \ldots, a_n$ is $\frac{a_1 + a_2 + \ldots + a_n}{n}$ and their geometric mean is $\left(a_1 a_2 \ldots a_n\right)^{\frac{1}{n}}$.

(e) $\frac{a_1 + a_2 + \ldots + a_n}{n} \geq \left(a_1 a_2 \ldots a_n\right)^{\frac{1}{n}}$ and the equality sign is got when $a_1 = a_2 = \ldots = a_n$.

(f) The harmonic mean of $a_1, a_2, \ldots, a_n$ = Reciprocal of the A.M. of $\frac{1}{a_1}, \frac{1}{a_2}, \ldots \frac{1}{a_n}$

7. Permutation and Combination

7.1 Permutation: A permutation is an arrangement. The number of ways of arranging n given things taken r at a time is denoted by the symbol ${}^nP_r$.

(i) The number of permutations of n different objects taken r at a time without repetition is ${}^nP_r = n(n-1)\ldots(n-r+1) = \frac{\lfloor\underline{n}}{\lfloor\underline{n-r}}$

(ii) ${}^nP_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

(iii) Particularly ${}^nP_0 = 1$; ${}^nP_1 = n$; ${}^nP_n = \lfloor\underline{n}$

7.2 If there are n objects of which p are alike and of one sort, q are alike and of a second sort, r are alike and of third sort and so on, the number of permutations is $\frac{\lfloor\underline{n}}{\lfloor\underline{p}\lfloor\underline{q}\lfloor\underline{r}..}$.

7.3 The number of permutations of n different objects taken r at a time with repetition is n^r.

7.4 n unlike objects can be arranged in a circle in $\lfloor\underline{n-1}$ ways.

However, if no difference is made between clockwise and anticlockwise arrangements, the number of arrangements is $\frac{1}{2}\lfloor\underline{n-1}$.

7.5 **(a)** The total number of permutations of n unlike things taken r at a time in which a particular thing always occurs is $r \cdot {}^{(n-1)}P_{r-1}$.

(b) The number of permutations of n unlike things in which a particular thing will not occur is ${}^{(n-1)}P_r$.

7.6 Combination

A selection of r objects out of n different objects without reference to the order in which the objects stand in it, is called the combination of n things taken r at a time and the number of combinations is denoted by ${}^nC_r$.

(a) **(i)** $${}^nC_r = \frac{{}^nP_r}{\lfloor\underline{r}} = \frac{\lfloor\underline{n}}{\lfloor\underline{n-r}\,\lfloor\underline{r}}$$

(ii) ${}^nC_r = {}^nC_{n-r}$

(iii) ${}^nC_r = {}^{n-1}C_{r-1} + {}^{(n-1)}C_r$

(iv) Particularly, ${}^nC_0 = {}^nC_n = 1;\ {}^nC_1 = {}^nC_{n-1} = n$

(b) The number of combinations of n things taken r at a time in which p particular things will always occur is ${}^{(n-p)}C_{r-p}$.

(c) The number of combinations of n things taken r at a time in which p particular things will not occur is ${}^{(n-p)}C_r$.

7.7 If all the objects are different, the total number of combinations of n objects taken any number at a time is $2^n - 1$.

7.8 If p are alike objects of one sort, q are alike of second sort, r are alike of third sort and so on, the number of combinations of n objects taken any number at a time is
$(p + 1)(q + 1)(r + 1) \ldots - 1$.

7.9 The number of combinations of n things taken r at a time when repetitions are allowed is ${}^{(n+r-1)}C_r$.

(a) The number of ways of dividing $(m + n + p + \ldots)$ unlike objects into q unequal groups containing m, n, p, ... objects and so on is $\dfrac{\lfloor\underline{m+n+p+\ldots}}{\lfloor\underline{m}\,\lfloor\underline{n}\,\lfloor\underline{p}\ldots}$

(b) If $m = n = p = \ldots$ q terms (i.e., if each group contains m objects), the number of divisions is $\dfrac{\lfloor\underline{mq}}{\lfloor\underline{q}\left(\lfloor\underline{m}\right)^q}$

7.10 ${}^nC_r$ is greatest when $r = \frac{n}{2}$ when n is even and when $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ when n is odd.

7.11 Product of r consecutive integers is exactly divisible by $\underline{\lfloor r}$.

8. Binomial theorem

8.1 $(x + a)^n = x^n + {}^nC_1\, x^{n-1} a + {}^nC_2\, x^{n-2} a^2 + \ldots + a^n$, where n is a positive integer. $(r + 1)$th term is called the general term and is usually denoted by u_{r+1}.

$\therefore\ u_{r+1} = {}^nC_r\, x^{n-r} a^r$

(a) The expansion contains $(n + 1)$ terms.

(b) The coefficients of terms equidistant from the beginning and the end of the expansion are equal.

(c) $(1 + x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

It is convenient to write the expansion in the form $C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$.

(d) Series involving Binomial coefficients

(i) $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$

(ii) $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$

(iii) $C_0 + 2\cdot C_1 + 3\cdot C_2 + \ldots + (n + 1)\cdot C_n = 2^n + n\cdot 2^{n-1}$

(iv) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$

(v) $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{\underline{\lfloor 2n}}{\left(\underline{\lfloor n}\right)^2}$

8.2 $(1 + x)^n = 1 + nx + \frac{n(n-1)}{\underline{\lfloor 2}} x^2 + \frac{n(n-1)(n-2)}{\underline{\lfloor 3}} x^3 + \ldots$ when $|x| < 1$.

When n is not a positive integer, the number of terms in the expansion is infinite.

Expansions

$(1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots + (-1)^n x^n + \ldots$

9. Determinant

9.1 Solutions of linear simultaneous equations by using determinants – Cramer's rule

Let the equations be

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$, then

$$\frac{x}{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}} = \frac{z}{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

9.2 Consistency of equations

The following equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$

are consistent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$.

9.3 Homogeneous linear equations

The equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ have infinite solutions if $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ and no solutions (inconsistent) if $\Delta \neq 0$

9.4 Matrices

A set of numbers arranged in a rectangle of m rows and n columns is called a matrix.

Thus $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ is a matrix of order $m \times n$ and it is written as $A\ [a_{ij}]$.

If $m = n$, it is called a square matrix of order n.

Addition of two matrices A and B is defined if and only if A and B are of the same order $m \times n$.

In this case their sum $A + B$ is simply the matrix got by adding the corresponding elements of the matrices A and B.

So matrix addition is associative and commutative.

If A is a matrix of order $m \times n$ and B is of order $n \times p$, then AB is defined as follows:

$$(AB)_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} \text{ where } A = [a_{ij}] \text{ and } B = [b_{jk}]$$

where $i = 1, 2, 3, \dots, m$; $j = 1, 2, \dots, n$ and $k = 1, 2, 3, \dots, p$.

If AB is defined, it does not follow BA is defined.

Even if AB and BA are defined, then AB and BA need not be the same.

Matrix multiplication is associative but not commutative.

If A is a square matrix of order n and determinant A i.e., $|A| \neq 0$, then the inverse of A exists and $AA^{-1} = A^{-1}A = I_n$.

where $I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$ (the unit matrix of order n) $(AB)^{-1} = B^{-1}A^{-1}$.

If $A = [a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A and it is denoted by A^T or A^t or A'. Note that A^T is of order $n \times m$.

Properties of Transpose

(i) $\left(A^T\right)^T = A$

(ii) $(A+B)^T = A^T + B^T$

(iii) $(kA)^T = kA^T$ (k being a scalar)

(iv) $(AB)^T = B^TA^T$

A square matrix $A = [a_{ij}]$ is called a symmetric matrix if $a_{ij} = a_{ji}$, $\forall$ i, j. It is called skew symmetric if $a_{ij} = -a_{ji}$ $\forall$ i, j.

Note that the elements in the leading diagonal of a square matrix are all zero. Thus $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 6 \end{bmatrix}$ is symmetric while $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ is skew symmetric.

In general, a square matrix A is symmetric if $A^T = A$ and skew symmetric if $A^T = -A$.

II. PROBABILITY

1. Let A and B be any two events. Then A or B happening is denoted by $A \cup B$ (sometimes this is denoted by $A + B$) and A and B happening at the same time is denoted by $A \cap B$ (sometimes it is denoted by AB).
2. If A and B are any two events, then

   (i) $P(A) = P(A \cap B) + P(A \cap \overline{B})$

   (ii) $P(B) = P(A \cap B) + P(\overline{A} \cap B)$

   (iii) $P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$

(iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(v) $P(A \cap B) = 1 - P(\overline{A} \cup \overline{B})$

(vi) $P(A \cup B) = 1 - P(\overline{A} \cap \overline{B})$

3. (a) **Mutually exclusive events:** Two or more events are said to be **mutually exclusive** if the events cannot occur together i.e., the occurrence of any one of them precludes the occurrence of the other and hence we speak of the probability of occurrence of one or other of a set of mutually exclusive events.

$P(A \cap B) = 0$ if A and B are mutually exclusive.

Hence $P(A \cup B) = P(A) + P(B)$, by (iv)

(b) **Independent events:** Two or more events are said to be **independent** if the occurrence of one event in no way affects the occurrence of the other.

If A and B are independent, $P(A \cap B) = P(A) \cdot P(B)$

4. **Conditional probability:** The probability of occurrence of event B depending on the occurrence of event A is conditional probability of B with respect to A and is written as P(B/A).

The probability of occurrence of event A depending on the occurrence of event B is conditional probability of A with respect to B and is written as P(A/B).

$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

(i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

(ii) $P(\overline{B}/\overline{A}) = \dfrac{P(\overline{B} \cap \overline{A})}{P(\overline{A})}$

(iii) If A and B are independent, then $P(B/A) = P(B)$ and $P(A/B) = P(A)$.

$\therefore P(A \cap B) = P(A)\,P(B)$

5. **Baye's rule:** If $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events and E is any event such that $P(E_i) > 0$, then

$$P(E_i/E) = \frac{P(E_i) \cdot P(E/E_i)}{\sum_{k=1}^{n} P(E_k) \cdot P(E/E_k)} \text{ for } 1 \le i \le n$$

III. TRIGONOMETRY

1. Trigonometrical ratios

Let P be the point (x, y) in a plane in rectangular coordinates and let P be (r, θ) in polar coordinates with the x-axis as the initial line and O as pole.

Then $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ $\Rightarrow \tan\theta = \frac{y}{x}$

$\text{cosec}\,\theta = \frac{r}{y}$, $\sec\theta = \frac{r}{x}$ $\Rightarrow \cot\theta = \frac{x}{y}$

1.1 The signs of the trigonometrical ratios for angles in the four quadrants are given below.

Quadrant	**Angle**	**sine/cosecant**	**cosine/secant**	**Tangent/co-tangent**
I	0° – 90°	+	+	+
II	90° – 180°	+	–	–
III	180° – 270°	–	–	+
IV	270° – 360°	–	+	–

1.2 Basic identities

(a) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

(b) $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \text{cosec}^2\theta$

1.3 If the value of one trigonometrical ratio is given, the values of the other trigonometrical ratios can be calculated, by taking two sides of right-angled triangle in such a way that the given trigonometrical ratio has the given value and calculating the third side of the triangle.

1.4 Trigonometrical ratios of some well known angles

	0°	30°	45°	60°	90°
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Values of cosec θ, sec θ and cot θ when θ = 0°, 30°, 45°, 60°, 90° can be found from the above **Table**.

1.5 Radian

The angle subtended at the centre of any circle by an arc equal in length to the radius of the circle is a radian. π radians = 180°

1.6 Ratios for the supplementary and complementary angles.

Angle	$-\theta$	$\frac{\pi}{2} \pm \theta$	$\pi \pm \theta$	$\frac{3\pi}{2} \pm \theta$	$2n\pi \pm \theta$
sine	$-\sin\theta$	$\cos\theta$	$\mp\sin\theta$	$-\cos\theta$	$\pm\sin\theta$
cosine	$\cos\theta$	$\mp\sin\theta$	$-\cos\theta$	$\pm\sin\theta$	$\cos\theta$
tangent	$-\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$

2. Functions of compound angles

Addition formulae

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

In particular, $\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\tan 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$\sin 3A = 3 \sin A - 4 \sin^3 A$

$\cos 3A = 4 \cos^3 A - 3 \cos A$

$\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

3. Transformations of sums into products and vice versa

$\sin A + \sin B = 2 \sin \dfrac{A + B}{2} \cos \dfrac{A - B}{2}$

$\sin A - \sin B = 2 \cos \dfrac{A + B}{2} \sin \dfrac{A - B}{2}$

$\cos A + \cos B = 2 \cos \dfrac{A + B}{2} \cos \dfrac{A - B}{2}$

$\cos A - \cos B = 2 \sin \dfrac{A + B}{2} \sin \dfrac{B - A}{2}$

$\sin A \sin B = \dfrac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\cos A \cos B = \dfrac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \cos B = \dfrac{1}{2} [\sin (A + B) + \sin (A - B)]$

4. Identities relating to the angles of a triangle

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(ii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(iii) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

(iv) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(vi) $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

(vii) $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1$

(viii) $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2}$

5. Relations between the sides and angles of a triangle

(a) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (sine formula) where R is the circumradius.

(b) $c^2 = a^2 + b^2 - 2ab \cos C$

$b^2 = c^2 + a^2 - 2ca \cos B$

$a^2 = b^2 + c^2 - 2bc \cos A$ (cosine formula)

(c) $a = c \cos B + b \cos C$

$b = a \cos C + c \cos A$

$c = b \cos A + a \cos B$ (Projection formula)

Half angle formulae

(d) $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$

$\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(e) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$

$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(f) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

where $2s = a + b + c$

(g) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$ (Napier's formula)

6. (a) Area of the triangle

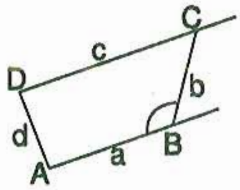
(i) $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$

(ii) $\Delta = \frac{abc}{4R}$

(iii) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

(b) (i) Area of cyclic quadrilateral of sides a, b, c, d is given by

$\sqrt{s(s-a)(s-b)(s-c)(s-d)}$ where $s = \frac{a+b+c+d}{2}$.



(ii) $\cos B = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$

(c) Area of a regular polygon of n sides, with side $a = mr^2\tan\frac{\pi}{n}R^2 = \frac{m}{2}R^2\sin\frac{2\pi}{n}$

where r and R respectively the inradius and circumradius of the shown polygon respectively.

7. Inradius (r) and exradii (r_1, r_2, r_3)

(i) $r = \dfrac{\Delta}{s}$, $r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$, $r_3 = \dfrac{\Delta}{s-c}$

(ii) $r = (s-a)\tan\dfrac{A}{2}$

$= (s-b)\tan\dfrac{B}{2}$

$= (s-c)\tan\dfrac{C}{2}$

(iii) $r_1 = s\tan\dfrac{A}{2}$

$r_2 = s\tan\dfrac{B}{2}$

$r_3 = s\tan\dfrac{C}{2}$

(iv) $r = 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

(v) $r_1 = 4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

(vi) $r_2 = 4R\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$

(vii) $r_3 = 4R\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2}$

(viii) $r_1 + r_2 + r_3 - r = 4R$

8. General solutions of equations

If θ radians is one of the solutions of the equation

(i) $\sin\theta = c\ (-1 \le c \le 1)$, the general solution is $n\pi + (-1)^n\theta$

(ii) $\cos\theta = c\ (-1 \le c \le 1)$, the general solution is $2n\pi \pm \theta$

(iii) $\tan\theta = c$, the general solution is $n\pi + \theta$

where n is any positive or negative integer or zero and the corresponding expressions for solutions in degrees can be got from these. Generally θ is taken to be the smallest numerical solution of the equation.

9. Inverse circular functions

(a) (i) $\theta = \sin^{-1} x$ or $\theta = \text{arc} \sin x$ denotes the angle whose sine is x and the numerically smallest angle satisfying the relation $x = \sin \theta$ is chosen as the principal value.

$$-1 \le x \le 1 \text{ and } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

(ii) $\theta = \cos^{-1} x$ or $\theta = \text{arc} \cos x$ denotes the angle whose cosine is x.

$$-1 \le x \le 1 \text{ and } 0 \le \theta \le \pi$$

(iii) $\theta = \tan^{-1} x$ or $\theta = \text{arc} \tan x$ denotes the smallest angle whose tangent is x.

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

(b) Relations between inverse functions

(i) $$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$$

(ii) $$\text{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\sin^{-1} x = \text{cosec}^{-1}\left(\frac{1}{x}\right), \text{ for suitable } x \ne 0$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right), \text{ for suitable } x \ne 0$$

$$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$$

$$\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right), \text{ for suitable } x \ne 0$$

(iii) When x and y are positive,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy < 1 \text{ and}$$

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy > 1$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

(iv) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

$\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

$\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right]$

(v) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(vi) **(1)** $\sin(\sin^{-1} x) = x$ if $-1 \le x \le 1$ and

$\sin^{-1}(\sin\theta) = \theta$ if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(2) $\cos(\cos^{-1} x) = x$ if $-1 \le x \le 1$ and

$\cos^{-1}(\cos\theta) = \theta$ if $0 \le \theta \le \pi$

(3) $\tan(\tan^{-1} x) = x$ if $-\infty < x < \infty$ and

$\tan^{-1}(\tan\theta) = \theta$ if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(4) $\cot(\cot^{-1} x) = x$ if $-\infty < x < \infty$ and

$\cot^{-1}(\cot\theta) = \theta$ if $0 < \theta < \pi$

(5) $\sec(\sec^{-1} x) = x$ if $|x| \ge 1$ and

$\sec^{-1}(\sec\theta) = \theta$ if $0 \le \theta \le \pi$ and $\theta \ne \frac{\pi}{2}$

(6) $\text{cosec}(\text{cosec}^{-1} x) = x$ if $|x| \ge 1$ and

$\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$ if $\theta \ne 0$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

IV. ANALYTICAL GEOMETRY

1. Points

(a) Let A, B, C be respectively the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(i) The centroid of the triangle ABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(ii) The incentre of the triangle ABC is $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

where $BC = a$, $CA = b$ and $AB = c$.

(iii) The area of the triangle ABC is the numerical value of

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\text{or } \frac{1}{2}[y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2)]$$

$$\text{or } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(b) Conditions for three points to be collinear

(i) The area of the triangle formed by three points is zero or

(ii) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) Find the distances between the points taken two at a time. If the distance between a pair of points is equal to the sum or difference between the other two pairs of points, then the three points lie on a line.

2. Locus

When a point moves in accordance with a geometric law, its path is called a locus. The equation to the locus is merely the equation connecting the x and y coordinates of every point on the curve.

3. (a) Standard forms of equations to a straight line

(i) $ax + by + c = 0$ (General form)

(ii) The line making an angle θ with the positive direction Ox and an intercept c on the y-axis, has equation

$$y = (\tan\theta)x + c$$

If $\tan\theta = m$, the equation is $y = mx + c$ (Slope form)

(iii) The line making intercepts a and b on the x and y axes is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ (Intercept form)}$$

(iv) If the perpendicular from the origin to the line is of length p and makes an angle α with the positive direction of the x-axis, the equation of the line is

$$x\cos\alpha + y\sin\alpha = p \text{ (Normal form)}$$

(b) Other forms

(i) Lines parallel to the coordinate axes

x = constant (parallel to y-axis)

y = constant (parallel to x-axis)

(ii) Line through the origin $y = mx$

(iii) Line through (x_1, y_1) with slope m

$$y - y_1 = m(x - x_1)$$

(iv) Line passing through the points (x_1, y_1) and (x_2, y_2)

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

Slope of this line is $\dfrac{y_1 - y_2}{x_1 - x_2}$

(v) The line passing through a fixed point (x_1, y_1) and having inclination θ with the positive direction of x-axis is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

where r is the algebraic distance of any point (x, y) from the given point. The coordinates of any point (x, y) on this line are given by

$$x = x_1 + r\cos\theta,\ y = y_1 + r\sin\theta.$$

(vi) The slope of the line $ax + by + c = 0$ is $-\dfrac{a}{b}$.

4. Position of a point with respect to a line

Two points (x_1, y_1) and (x_2, y_2) lie on the same side (or opposite sides) of the straight line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign (or of opposite signs).

5. (a) Length of the perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

(b) Sign of the perpendicular

When the equation of the straight line is written with the constant term as negative, the perpendicular from a point P to a straight line is positive or negative according as P and the origin lie on the same side of the line or on the opposite sides.

6. Two or more lines

(a) The coordinates of the point of intersection of two straight lines are obtained by solving the equations of the two lines, simultaneously.

(b) If θ is the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$, then $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1m_2}$

The angle θ is the acute or the obtuse angle according as $\left(\dfrac{m_1 - m_2}{1 + m_1m_2}\right)$ is positive or negative.

(i) The condition that $y = m_1x + c_1$ and $y = m_2x + c_2$ to be parallel is $m_1 = m_2$. The equation of the line parallel to $ax + by + c = 0$ is $ax + by + d = 0$.

(ii) The condition that $y = m_1x + c_1$ and $y = m_2x + c_2$ to be perpendicular to one another is $m_1 m_2 = -1$.

The equation of the line perpendicular to the line $ax + by + c = 0$ is $bx - ay + d = 0$.

(c) If the two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represent the same line, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

(d) The equation to any line passing through the point of intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ can be written as

$$a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0 \qquad \ldots (1)$$

where k is a constant. For various real values of k, equation (1) represents a line through the point of intersection of the given lines.

(e) Bisectors of the angle between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have equations $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

(i) Let the equations be written with constant terms positive. i.e., $c_1 > 0, c_2 > 0$.

If $a_1a_2 + b_1b_2$ is positive, then the origin lies in the obtuse angle between the lines. If $a_1a_2 + b_1b_2$ is negative, then the origin lies in the acute angle between the lines.

(ii) The bisector of the angle in which the origin is situated is given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$ and the other bisector is given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

(f) The three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, or if constants k_1, k_2, k_3 can be found such that $k_1(a_1x + b_1y + c_1) + k_2(a_2x + b_2y + c_2) + k_3(a_3x + b_3y + c_3) = 0$ or the coordinates of the point of intersection of two lines satisfy the equation of the third line.

7. Circles

(a) Equation to the circle

(i) The equation to the circle having its centre at O(h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. Any point P on this circle is $(h + r\cos\theta, k + r\sin\theta)$, where θ is the angle made by OP with the x-axis.

(ii) If the centre is the origin and radius is r, the equation of the circle is $x^2 + y^2 = r^2$.

Any point on this circle can be taken as $(r\cos\theta, r\sin\theta)$, where θ is the parameter of the point.

(iii) The general equation to the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.

Its centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$.

(iv) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a circle if $a = b$ and $h = 0$. To find the centre and radius of this circle, reduce it to the form

$$x^2 + y^2 + \frac{2g}{a}x + \frac{2f}{a}y + \frac{c}{a} = 0$$

Centre is $\left(-\frac{g}{a}, -\frac{f}{a}\right)$ and radius is $\frac{\sqrt{g^2 + f^2 - ca}}{a}$

(v) Equation to the circle described on the line joining the points (x_1, y_1) and (x_2, y_2) as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

(b) A circle can be made to satisfy any three geometrical conditions such as

(i) passing through three given points

(ii) passing through two given points and touching a given line

(iii) passing through a given point and touching two given lines

(iv) touching three given lines

(v) passing through two given points and having its centre on a given line.

8.The length of the tangent from the point

(i) (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

(ii) The point (x_1, y_1) lies outside, on or inside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \gtreqless 0$$

9. Tangent to the circle

(a) **(i)** Equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

(ii) Condition for the line $y = mx + c$ to touch the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$.

(b) The equation of a tangent to the circle $x^2 + y^2 = a^2$ is $y = mx \pm a\sqrt{1 + m^2}$.

10. Two or more circles

(a) Let d be the distance between the centres of two circles with radii r_1, r_2.

(i) If $d < r_1 - r_2$, one circle lies completely inside the other circle.

(ii) If $d > r_1 + r_2$, the two circles do not intersect.

(iii) If $r_1 - r_2 < d < r_1 + r_2$, the two circles intersect.

(iv) If $d = r_1 + r_2$, the two circles touch one another externally and the point of contact divides the line joining the centres internally in the ratio $r_1 : r_2$.

(v) If $d = r_1 \sim r_2$, the two circles touch one another internally and the point of contact divides the line joining the centres externally in the ratio $r_1 : r_2$.

(b) Let $S = x^2 + y^2 + 2gx + 2fy + c = 0$ and $S' = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ be the equations of two circles. Then the two circles will cut each other orthogonally if $2gg_1 + 2ff_1 = c + c_1$.

(c) **(i)** If $S = 0$ and $S' = 0$ are the equations of two circles, then $S - S' = 0$ is the equation of the radical axis. However, if the two circles $S = 0$, $S' = 0$ intersect their radical axis coincides with the common chord. Therefore $S - S' = 0$ is the equation of common chord of the intersecting circles.

(ii) Equation of a circle passing through the points of intersection of the circles $S = 0$ and $S' = 0$ can be written as $S + kS' = 0$, where k is a real number.

(iii) The radical axes of three circles taken in pairs are concurrent and the point of concurrence is the **radical centre** of the three circles.

11. Parabola

(a) Standard equation is $y^2 = 4ax$

x-axis is axis of the parabola

y-axis is tangent at the vertex

Vertex is A(0, 0)

Focus is S(a, 0)

Directrix is $x + a = 0$

(b) Parametric form of representation of a point P on $y^2 = 4ax$ is $P(at^2, 2at)$.

(c) At $P(at^2, 2at)$ on $y^2 = 4ax$

Slope of the tangent is $\frac{1}{t}$

Equation of the tangent at P is $x - yt + at^2 = 0$

Slope of the normal is $-t$

Equation of the normal at P is $y + tx - 2at - at^3 = 0$.

(d) If P is $\left(at_1^2, 2at_1\right)$ and Q is $\left(at_2^2, 2at_2\right)$, then slope of the chord PQ is $2/(t_1 + t_2)$.

Equation to the chord PQ is $2x - y(t_1 + t_2) + 2at_1t_2 = 0$.

(e) If the tangent at $P\left(at_1^2, 2at_1\right)$ and at $Q\left(at_2^2, 2at_2\right)$ are perpendicular, then $t_1t_2 = -1$. In this case, the tangents intersect on the directrix.

(f) If PQ is a chord passing through the focus, then $t_1t_2 = -1$.

(g) If the normal at t_1 meets the parabola, again at t_2, then $t_2 = -t_1 - \frac{2}{t_1}$.

(h) If the point P on $y^2 = 4ax$ is taken in the Cartesian form, then

(i) equation to the tangent at (x_1, y_1) is $yy_1 - 2a(x + x_1) = 0$,

(ii) equation to normal at (x_1, y_1) is $xy_1 + 2ay - (x_1y_1 + 2ay_1) = 0$.

(i) If the line $y = mx + c$ touches the parabola $y^2 = 4ax$, the condition is $c = \frac{a}{m}$.

$\therefore\ y = mx + \frac{a}{m}$ is a tangent to the parabola for all values of m.

(j) The chord of contact of tangents to the parabola $y^2 = 4ax$ from the point (x_1, y_1) is $yy_1 - 2a(x + x_1) = 0$ i.e., $T = 0$.

(k) The chord of the parabola $y^2 = 4ax$ whose midpoint is (x_1, y_1) is $T = S_1$ i.e., $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$.

(l) A line which bisects a system of parallel chords of a parabola is called a diameter of the parabola. If m is the slope of the parallel chords, $y = \frac{2a}{m}$ is the equation of the diameter.

12. Ellipse

(a) Standard equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

x-axis is major axis, length 2a

y-axis is minor axis, length 2b

and $b^2 = a^2(1 - e^2)$ ($e < 1$ is the eccentricity of the ellipse).

(b) There are two foci, one S(ae, 0) and the other S′(− ae, 0) respectively, the corresponding directrices are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$.

(c) If P be any point on the ellipse, then

(i) $SP + S'P = 2a$

(ii) $SP \cdot S'P = CD^2$ where CD is the semi-diameter parallel to the tangent at P.

(d) Parametric form of representation of a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is P(a cos θ, b sin θ).

(e) At P(a cos θ, b sin θ) on $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$,

(i) Equation to the tangent is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta - 1 = 0$.

(ii) Equation to the normal is $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$.

(iii) If CD be a semi-diameter parallel to the tangent at P, then D is (− a sin θ, b cos θ) and $CD^2 = a^2\sin^2\theta + b^2\cos^2\theta$.

(f) The locus of points of intersection of perpendicular tangents of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ is the circle (called the director circle) $x^2 + y^2 = a^2 + b^2$.

(g) The circle $x^2 + y^2 = a^2$ is called the auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and it is the locus of the foot of the perpendicular from the centre of the ellipse to its tangents.

13. Hyperbola

(a) Standard equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

x-axis is transverse axis, length 2a

y-axis is conjugate axis, length 2b

and $b^2 = a^2 (e^2 - 1)$ ($e > 1$ is the eccentricity of the hyperbola).

(b) There are two foci, one S(ae, 0) and the other S′(–ae, 0) respectively, the corresponding directrices are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$.

(c) If P be any point on the hyperbola, then $|SP - S'P| = 2a$.

(d) Parametric form of representation of a point P on the hyperbola is $(a \sec\theta, b \tan\theta)$.

(e) At $P(a \sec\theta, b \tan\theta)$ on $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$, equation to the tangent at ‘θ’ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta - 1 = 0$.

(f) The locus of point of intersection of perpendicular tangents of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ is the circle (called the director circle) $x^2 + y^2 = a^2 - b^2$.

(g) The combined equation of the two asymptotes is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

(h) The angle between the asymptotes is given by $2\sec^{-1}(e)$.

(i) Equation to the conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$.

(j) The hyperbola is said to be rectangular if a = b, the equation to the rectangular hyperbola is $x^2 - y^2 = a^2$.

14. Rectangular hyperbola

(a) Standard equation is $xy = c^2$ (axes are the asymptotes)

(b) Parametric form of representation of a point P on $xy = c^2$ is $\left(ct, \frac{c}{t}\right)$

(c) At $P\left(ct, \frac{c}{t}\right)$ on $xy = c^2$

Slope of the tangent is $-\frac{1}{t^2}$

Equation to the tangent is $x + yt^2 - 2ct = 0$

Slope of the normal is t^2

Equation to the normal is $xt^3 - yt + c - ct^4 = 0$

(d) Eccentricity e of the rectangular hyperbola is $e = \sqrt{2}$

V. DIFFERENTIAL CALCULUS

1. Functions

Definition: When two variable quantities x and y are so related that the value of y depends on the value of x, then y is said to be a **function** of x. The variable quantity x is called the **independent variable** and the variable y whose value depends on that given to x, is referred to as the **dependent variable.**

1.1. Classification of functions

(a) If for every given value of x, there corresponds one and only one value of y, y is called a function of x.

(b) **Implicit and explicit functions:** If the functional relation between two variables x and y is expressed in the form $y = f(x)$, y is called an **explicit function** of x.

Eg: $y = ax^2 + bx + c$, $y = 2 \sin 3x$, $y = 3e^{2x}$, $y = -4 \log x$.

If the relation between two variables x and y is expressed in the form $f(x, y) = 0$, y is called an **implicit function** of x.

Eg: $y \sin x + y^2x = 3$, $bx^2 + axy + cy^2 + 2 = 0$.

(c) **Polynomial of x**

An expression of the form $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$ where n is a positive integer is called a polynomial of degree n. Here $a_0, a_1, ..., a_n$ are constants.

(d) **Rational function** is the quotient of two polynomials.

(e) **Algebraic function** of x, y is a function of integral powers of y and polynomials in x.

Eg: $a_0y^n P_1(x) + a_1y^{n-1} P_2(x) + ... + P_n(x) = 0$

(f) A function, which is not algebraic, is called transcendental. Functions involving trigonometrical functions $\sin x$, $\cos x$, ..., inverse trigonometrical functions $\sin^{-1} x$, $\cos^{-1} x$, ..., exponential functions, logarithmic functions are called **transcendental functions.**

1.2. Limits

f(x) tends to a limit l as $x \longrightarrow a$; Given any positive number ε, however small it may be, if we can find a positive number δ such that $|f(x) - l| < \varepsilon$ for all x satisfying $|x - a| < \delta$, then l is the limit of f(x).

The limit of f(x) as $x \longrightarrow a$ and the value of f(x) at $x = a$, are quite distinct. They may not be the same.

(a) Laws on limits

If u and v are functions of x,

(i) $\lim_{x \to a} (u + v) = \lim_{x \to a} u + \lim_{x \to a} v$

(ii) $\lim_{x \to a} (u - v) = \lim_{x \to a} u - \lim_{x \to a} v$

(iii) $\lim_{x \to a} (cu) = c \lim_{x \to a} u$ where c is a constant.

(iv) $\lim_{x \to a} (uv) = \lim_{x \to a} u \lim_{x \to a} v$

(v) $\lim_{x \to a} \left(\frac{u}{v}\right) = \frac{\lim_{x \to a} u}{\lim_{x \to a} v}$ provided $\lim_{x \to a} v \neq 0$

(vi) $\lim_{x \to a} (u)^v = \left(\lim_{x \to a} u\right)^{\lim_{x \to a} v}$

(b) Some important limits

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1, \lim_{x \to 0} \frac{\tan x}{x} = 1$

(iii) $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e, \lim_{x \to 0} (1 + x)^{1/x} = e$

(iv) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

1.3. Continuous and discontinuous functions

Definition: The function f(x) is continuous at the point x = a, if given any positive number ε, we can find a positive number δ, such that

$|f(x) - f(a)| < \varepsilon$, where $|x - a| < \delta$

(a) It follows that f(x) is continuous at a if and only if $f(x) \longrightarrow f(a)$ as $x \longrightarrow a$

i.e., $\lim_{x \to a - 0} f(x) = \lim_{x \to a + 0} f(x) = f(a)$

(b) If f(x) and $\phi(x)$ are continuous at x = a, then

(i) $f(x) \pm \phi(x)$

(ii) $f(x) \cdot \phi(x)$

(iii) $\frac{f(x)}{\phi(x)}$ when $\phi(a) \neq 0$ are also continuous at $x = a$.

(c) If f(x) be a continuous function in the closed interval (a, b) with f(a) and f(b) having opposite signs, then f(x) is zero for at least one value of x lying between a and b.

(d) A function which is not continuous at $x = a$ is said to be discontinuous at $x = a$.

2. Differentiation

Definition: The differential coefficient or the derivative f′(x) of f(x) at the point x is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If y is a function of x, then $\frac{dy}{dx} = \lim_{x \to 0} \frac{\Delta y}{\Delta x}$ where Δy is the increment in y corresponding to a small increment Δx in x.

(a) The function f(x) is said to be differentiable at the point x if f′(x) exists as a unique finite limit.

(b) A function which is differentiable at a point a is continuous at a, but a function may be continuous at a and yet not be differentiable at a.

2.1. Some general theorems on differentiation

(a)

Function		**Derivative**
C (constant)		0
Cu, where u is a function of x		$C\frac{du}{dx}$
u + v, where u and v are functions of x		$\frac{du}{dx} + \frac{dv}{dx}$
u – v	– do –	$\frac{du}{dx} - \frac{dv}{dx}$
uv	– do –	$u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{u}{v}$	– do –	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(b) Function of a function $\frac{du}{dx} = \frac{du}{dt} \cdot \frac{dt}{dx}$, where t is a variable

(c) $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

(d) If x and y are expressed in terms of third variable t, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

(e) logarithmic differentiation

To differentiate $y = uvw \ldots$, and $y = u^v$ take logarithms and then differentiate.

(f) Derivatives of some well known functions.

Function	**Derivative**
x^n (n is a rational number)	nx^{n-1}
e^x	e^x
$\log x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\text{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$

3. Geometrical meaning of derivative

(a) If the tangent at $x = a$ to the curve $y = f(x)$ makes an angle θ with the x-axis, then $\tan\theta$ = the value of $\frac{dy}{dx}$ at $x = a$ i.e., $f'(a)$.

(b) The equation of the tangent at (x_1, y_1) on the curve $y = f(x)$ is

$y - y_1 = \left(\frac{dy}{dx}\right)_1 (x - x_1)$, where $\left(\frac{dy}{dx}\right)_1$ is the value of $\frac{dy}{dx}$ at (x_1, y_1).

(c) The equation of the **normal** to the curve $y = f(x)$ at (x_1, y_1) on the curve is

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_1}(x - x_1)$$

(d) Angle between two curves $y = f(x)$ and $y = \phi(x)$. Let (x_1, y_1) be a point of intersection of the two curves and $m_1 = \left(\frac{dy}{dx}\right)_1$ for $y = f(x)$ and $m_2 = \left(\frac{dy}{dx}\right)_1$ for $y = \phi(x)$.

If θ is the acute angle between the curves, then $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1m_2}$.

The two curves intersect orthogonally if $m_1m_2 = -1$.

(e) Let P(x, y) be a point on the curve y = f(x). Then

(i) the length of the tangent at P = $\dfrac{y\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}}{\dfrac{dy}{dx}}$

(ii) the length of normal at P = $y\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

(iii) the length of subtangent = $\dfrac{y}{\dfrac{dy}{dx}}$

(iv) the length of subnormal = $y\dfrac{dy}{dx}$

4. Derivative as a rate measurer

$\dfrac{dy}{dx}$ is the rate of change of y with respect to x.

(a) If s is the distance travelled in time t by a particle, then the velocity of the particle at time t is $\dfrac{ds}{dt}$ and acceleration is $\dfrac{d^2s}{dt^2}$.

(b) If θ is the angle described by a line revolving about a fixed point on it, then the angular velocity of the line at time t is $\dfrac{d\theta}{dt}$ and the angular acceleration is $\dfrac{d^2\theta}{dt^2}$.

5. Monotonic functions

f(x) is an increasing function at x = a when f′(a) is positive and a decreasing function when f′(a) is negative. A function f(x) is called a **monotonic increasing** function in an interval if it is an increasing function in that interval and is called a **monotonic decreasing** function in an interval if it is a decreasing function in that interval.

6. Maxima and minima

(a) f(x) attains a maximum at x = a if f′(a) = 0 and f″(a) is negative and the maximum value is f(a).

(b) f(x) attains a minimum at x = a if f′(a) = 0 and f″(a) is positive and the minimum value is f(a).

(c) Rules for finding the maximum and minimum values of a function f(x).

(i) Find $f'(x)$ and $f''(x)$

(ii) Find the roots of the equation $f'(x) = 0$. Let the roots be $a_1, a_2, \ldots$

(iii) Substitute the values $a_1, a_2, \ldots$ in $f''(x)$ and determine the sign of $f''(a_1), f''(a_2), \ldots$

If $f''(a_1) = -$ ve, f(x) is a maximum at $x = a_1$ and the maximum value is $f(a_1)$.

If $f''(a_1) = +$ ve, f(x) is a minimum at $x = a_1$ and the minimum value is $f(a_1)$.

If $f''(a_1) = 0$, nothing can be said at this stage.

7. Rolle's theorem

If a function f(x) is differentiable in an interval (a, b) and also f(a) = f(b), then there exists at least one value x_1 of x in the interval (a, b) such that $f'(x_1) = 0$.

8. Mean value theorem

If a function f(x) is differentiable in an interval (a, b), then there exists at least one value of x_1 in the interval (a, b) such that $f'(x_1) = \dfrac{f(b) - f(a)}{b - a}$

(a) Another form: If a function f(x) is differentiable in an interval (a, a + h), then $f(a + h) = f(a) + hf'(a + \theta h)$, where $0 < \theta < 1$

(b) Indeterminate forms: $\dfrac{0}{0}, \dfrac{\infty}{\infty}, 0 \times \infty, \infty - \infty, 0^0, 1^\infty, \infty^0$

9. L' Hospital's rule

(i) $\dfrac{0}{0}$ form: $\lim\limits_{x \to a} \dfrac{f(x)}{\phi(x)} = \dfrac{f'(a)}{\phi'(a)}$ when $f(a) = 0, \phi(a) = 0$

$$= \frac{f''(a)}{\phi''(a)} \text{ when } f'(a) = 0,\ \phi'(a) = 0 \text{ and so on}$$

(ii) $\dfrac{\infty}{\infty}$ form: $\lim\limits_{x \to a} \dfrac{f(x)}{\phi(x)} = \dfrac{f'(a)}{\phi'(a)}$ when $f(a) = \infty,\ \phi(a) = \infty$

$$= \frac{f''(a)}{\phi''(a)} \text{ when } f'(a) = \infty,\ \phi'(a) = \infty \text{ and so on}$$

(iii) Forms $0 \times \infty$ and $\infty - \infty$ can be reduced to the form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$

(iv) Forms $0^0, \infty^0, 1^\infty$ can be reduced to the forms $\dfrac{0}{0}$ or to $\dfrac{\infty}{\infty}$ by taking logarithms.

VI. INTEGRAL CALCULUS

1. Integration

If a function is given, its derivative can be found. Let us consider the reverse process i.e., to find a function F(x) whose derivative is f(x). This reverse process is called **integration** and a function thus found is called the integral of f(x) with respect to x. In symbols it is written $\int f(x)\,dx$.

Hence if $\frac{d}{dx}F(x) = f(x)$, then $F(x) = \int f(x)\,dx$.

If one solution F(x) is known, then the general solution is F(x) + C where C is an arbitrary constant. F(x) + C is called the indefinite integral of f(x) and C is called the **constant of integration**.

$\int_a^b f(x)\,dx = F(b) - F(a)$, where $\int f(x)\,dx = F(x)$, b is called the upper limit and a is called the lower limit of the **definite integral** $\int_a^b f(x)\,dx$.

2. Two results on integration

(a) $\int cu\,dx = c\int u\,dx$, where c is a constant and u a function of x.

(b) $\int (u + v - w)\,dx = \int u\,dx + \int v\,dx - \int w\,dx$, where u, v, w are functions of x.

3. Methods of integration

(a) **Method by substitution:** In this method the independent variable is changed into another variable by proper substitution and the integral is brought to one of the standard forms.

$$\int f(x)\,dx = \int f[\phi(t)]\frac{dx}{dt}, \text{ where } x = \phi(t)$$

(b) **Some standard substitutions**

	Integral	Substitution
(i)	$\int f(ax+b)\,dx$	$t = ax + b$
(ii)	$\int x^{n-1} f(x^n)\,dx$	$t = x^n$
(iii)	$\int \frac{f'(x)}{f(x)}\,dx$	$t = f(x)$
(iv)	$\int [f(x)]^n f'(x)\,dx$	$t = f(x)$
(v)	$\int F\{f(x)\} f'(x)\,dx$	$t = f(x)$

(a) Integration by parts

$\int u\,dv = uv - \int v\,du$, where u and v are functions of x. Choose u and v so that $\int v\,du$ can easily be integrated.

Proper choice of the second function:

(i) If the integrand is a product of positive integral power of x and either a trigonometric or exponential function whose integral is easily known, take the latter as the second function.

(ii) If the integral contains a single logarithmic or single inverse trigonometric function, take unity as the second function.

(iii) If the integrals of both the functions are known, the function which is easy to integrate is taken as the second function.

(iv) Sometimes, integration by parts is applied more than once.

(v) In certain cases, integration by parts will lead to a simple equation involving the integral. Solve the equation and determine the integral.

(b) Integration of rational algebraic equations

(i) $\int \frac{dx}{ax^2 + bx + c}$, where $ax^2 + bx + c$ cannot be split into rational linear factors. Express $ax^2 + bx + c$ in the form $a\{(x + p)^2 \pm q^2\}$ and put $x + p = t$. This will reduce the integral to one of the forms

$$\int \frac{dx}{x^2 \pm a^2}$$

(ii) $\int \frac{px + q}{ax^2 + bx + c}\,dx$

Put $px + q = A$ (derivative of denominator) $+ B = A(2ax + b) + B$

$$\text{Then the integral } = A\int \frac{2ax + b}{ax^2 + bx + c}dx + B\int \frac{dx}{ax^2 + bx + c}$$

$$= A\log(ax^2 + bx + c) + B\int \frac{dx}{ax^2 + bx + c}$$

This integral can be found by (i)

(iii) $\int \frac{\text{Any polynomial of degree 2 or more in x}}{ax^2 + bx + c}dx$

Divide the numerator by the denominator and express it as

quotient $+ \frac{px + q}{ax^2 + bx + c}$

Then the integral is split into 2 integrals which can be integrated.

(iv) $\displaystyle\int \frac{\text{A polynomial in x}}{\text{Another polynomial in x}}dx$

If the denominator can be resolved into rational factors of the first or second degree, the method of partial fractions is to be used.

(v) Special cases

Sometimes the following substitutions may be helpful.

If integrand involves $a^2 - x^2$ put $x = a \sin\theta$ or $a \cos\theta$; if it involves $a^2 + x^2$ put $x = a \tan\theta$ and if it involves $x^2 - a^2$ put $x = a \sec\theta$.

(vi) $\displaystyle\int \frac{ax^2 + b}{x^4 + cx^2 + 1}dx$

Put $ax^2 + b = A(x^2 + 1) + B(x^2 - 1)$

The integral is $\displaystyle A\int \frac{(x^2+1)}{x^4 + cx^2 + 1}dx + B\int \frac{(x^2-1)}{x^4+cx^2+1}dx$

$$= A\int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + c + 2}dx + B\int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + c - 2}dx$$

To integrate the first integral put $t = x - \frac{1}{x}$ and for the second integral put $t = x + \frac{1}{x}$.

(e) Integration of $\int \sin^n x \cos^m x\, dx$, where m and n are positive integers.

(i) If n be odd and m be even, put $t = \cos x$

(ii) If n be even and m be odd, put $t = \sin x$

(iii) If m and n are both odd, put $t = \cos x$ or $\sin x$

(iv) If both m and n are even and not very large, reduce them by the following trigonometrical identities:

$\sin x \cos x = \frac{1}{2}\sin 2x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2}$ and then integrate term by term.

(v) Sometimes we may have m + n as an even negative integer. In that case put $t = \tan x$.

(f) **(i)** For the integrals of the following functions

$\frac{1}{a + b\cos x}, \frac{1}{a + b\sin x}, \frac{1}{a\cos x + b\sin x + c}$ put $\tan\frac{x}{2} = t$.

(ii) For the integrals of the following functions

$$\frac{1}{a + b\cos^2 x}, \frac{1}{a + b\sin^2 x}, \frac{1}{a\cos^2 x + b\sin^2 x + c} \text{ put } \tan x = t.$$

(g) Integration of irrational functions

(i) $\int f\left\{x, (ax+b)^{\frac{p}{q}}\right\} dx$ · Put $ax + b = t^q$

The integrand will reduce to a rational function of t.

(ii) Sometimes the rationalisation of the denominator leads to integrals which can be integrated.

(iii) $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$ can be reduced to

$\int \frac{dx}{\sqrt{a}\sqrt{(x+p)^2 \pm q^2}}$ or to $\int \frac{dx}{\sqrt{-a}\sqrt{q^2 - (x+p)^2}}$ according as a is positive or negative. Then put $t = x + p$

(iv) $\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$

Express $px + q = A(2ax + b) + B$

Then the integral reduces to $A\int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx + B\int \frac{dx}{\sqrt{ax^2 + bx + c}}$

Both the integrals can be integrated.

(v) $\int \frac{dx}{(x - k)\sqrt{ax^2 + bx + c}}$

Put $x - k = \frac{1}{t}$

(vi) $\int \frac{dx}{(Ax^2 + B)\sqrt{Cx^2 + D}}$ Put $x = \frac{1}{t}$

(vii) $\int \frac{dx}{\sqrt{(x - \alpha)(\beta - x)}}$ Put $x = \alpha\cos^2\theta + \beta\sin^2\theta$ or $\beta\cos^2\theta + \alpha\sin^2\theta$

(viii) If integrand involves

$\sqrt{a^2 - x^2}$ put $x = a\sin\theta$ or $a\cos\theta$

$\sqrt{a^2 + x^2}$ put $x = a\tan\theta$

$\sqrt{x^2 - a^2}$ put $x = a\sec\theta$

$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$ put $x^2 = a^2\cos 2\theta$

4. Properties of definite integrals

(i) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(ii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(iii) $\int_{-a}^{+a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(x)$ is an even function [i.e., $f(x) = f(-x)$]

$= 0$ if $f(x)$ is an odd function [i.e., $f(x) = -f(-x)$]

(iv) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(v) $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(2a-x) = f(x)$

$= 0$ if $f(2a-x) = -f(x)$

(vi) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

5. Reduction formulae

(i) If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,dx$ or $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$ where n is a positive integer,

then $I_n = \frac{n-1}{n} I_{n-2}$

The integral reduces to either I_0 or to I_1

$I_0 = \frac{\pi}{2}$ and $I_1 = 1$

(ii) If $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\,dx$ (m, n are positive integers),

then $I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$ or $\frac{m-1}{m+n} I_{m-2,n}$

If m is odd, the integral reduces to $\int_0^{\frac{\pi}{2}} \sin x \cos^n x \, dx$ which is $\frac{1}{n+1}$.

If n is odd, the integral reduces to $\int_0^{\frac{\pi}{2}} \sin^m x \cos x \, dx$ which is $\frac{1}{m+1}$.

If m and n are even, the integral reduces to $\int_0^{\frac{\pi}{2}} \sin^m x \, dx$ or $\int_0^{\frac{\pi}{2}} \cos^n x \, dx$ and

$$\int_0^{\frac{\pi}{2}} \sin^m x \, dx = \int_0^{\frac{\pi}{2}} \cos^m x \, dx = \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{1}{2} \frac{\pi}{2} \text{ (m even)}$$

$$= \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{2}{3} \text{ (m odd)}$$

Leibnitz rule: $\frac{d}{dx} \int_{f(x)}^{g(x)} F(t) \, dt = g'(x) F(g(x)) - f'(x) F(f(x))$

6. Limit of a series as a definite integral

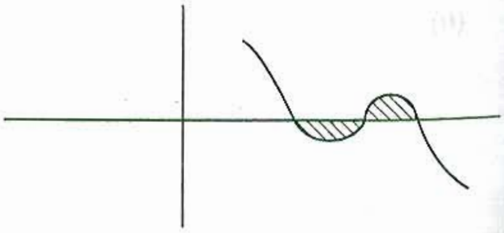
If a series can be put in the form $\frac{1}{n} \sum_{r=0}^{r=n-1} f\left(\frac{r}{n}\right)$ or $\frac{1}{n} \sum_{r=1}^{r=n} f\left(\frac{r}{n}\right)$, then its limit as $n \to \infty$ is $\int_0^1 f(x) \, dx$.

7. Areas and volumes

(i) The area bounded by the curve y = f(x), the x-axis and ordinates x = a and x = b is $\int_a^b y \, dx$.

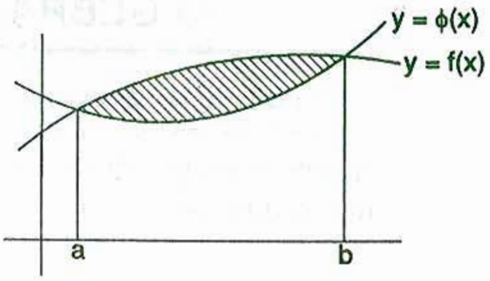
(ii) The area bounded by the curve x = f(y), the y-axis and the lines y = a and y = b is $\int_a^b x \, dy$.

(iii) To find the total area enclosed by a curve which crosses the x-axis at points between two abscissa, draw the diagram and divide the range of integration into appropriate subranges.



(iv) Area between the curves $y = f(x)$ and $y = \phi(x)$. Find the x coordinates of the points of intersection and let them be a and b.

Then the area is $\int_a^b [f(x) - \phi(x)]\,dx$



(v) The volumes of the solid generated by the revolution about the x-axis of the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$, $x = b$ is $\int_a^b \pi y^2\,dx$.

8. Differential equations

(i) Order of a differential equation is the order of the highest derivative occurring in the equation.

(ii) Degree of a differential equation is the power to which the highest order derivative is raised after the equation is cleared of the radicals and fractions.

(iii) (a) Variables separable type (1st order, 1st degree)

$$f(x) + g(y)\frac{dy}{dx} = 0$$

(b) Homogeneous type $\frac{dy}{dx} = \frac{f(x,\ y)}{g(x,\ y)}$, where both f and g are functions of x and y, homogeneous of the same degree.

Substitution: $y = vx$

(iv) Linear differential equation

A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constants is called the linear differential equation of the first order.

The solution of this differential equation is

$$ye^{\int P\,dx} = \int Qe^{\int P\,dx}dx + C$$

$e^{\int P\,dx}$ is called the integrating factor for this equation.

Similarly if P and Q are functions of y or constants, then the solution of $\frac{dx}{dy} + Px = Q$ is

$$xe^{\int P\,dy} = \int Q\,e^{\int P\,dy}dy + C$$

VII. VECTOR ALGEBRA

1. **Scalar and vector:** A quantity which has only magnitude is called **scalar** and a quantity which has both magnitude and direction is a **vector**. A vector is represented by a directed line segment. A vector whose direction is from O to A and whose magnitude is the length OA is denoted by $\overrightarrow{OA}$. The magnitude or module is denoted by $|\overrightarrow{OA}|$. O is called the initial point.

   (a) Multiplication of vector by scalar follows the ordinary rules of Algebra $m(n\vec{a}) = n(m\vec{a}) = mn\vec{a}$.

   (b) $-\vec{a}$ is a vector which has the same magnitude as $\vec{a}$ and direction opposite to that of $\vec{a}$.

   (c) **Position vector:** If O is the origin or the initial point, then $\overrightarrow{OP}$ is called the position vector of P with respect to O.

   (i) The position vector of the point which divides the distance between the two end points of the vectors $\vec{a}$ and $\vec{b}$ in the ratio m : n internally is $\frac{m\vec{b}+n\vec{a}}{m+n}$ and externally is $\frac{m\vec{b}-n\vec{a}}{m-n}$.

   (ii) The position vector of the midpoint of the line joining two points whose position vectors are $\vec{a}, \vec{b}$ is $\frac{1}{2}\left(\vec{a}+\vec{b}\right)$.

   (iii) The position vector of the centroid of the triangle with vertices having position vectors $\vec{a}, \vec{b}, \vec{c}$ is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.

   (iv) The end points of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear if three constants ℓ, m, n (not all zero) can be found such that $\ell\vec{a} + m\vec{b} + n\vec{c} = 0$ where $\ell + m + n = 0$.

   (d) **Components of a vector**

   (i) If $\vec{a}, \vec{b}$ are two non-parallel vectors in a plane, then any vector in that plane can be represented as $\ell\vec{a} + m\vec{b}$, where ℓ and m are scalars.

   (ii) If three constants ℓ, m, n (not all zero) can be found such that $\ell\vec{a} + m\vec{b} + n\vec{c} = 0$, then $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

   (iii) Any vector $\vec{d}$ in space can be represented as $\ell\vec{a} + m\vec{b} + n\vec{c}$, where ℓ, m, n are scalars and $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors.

   (e) **Unit vectors:** Consider a coordinate system in space whose axes are three mutually perpendicular straight lines OX, OY, OZ. The unit vectors in the direction of OX, OY, OZ are respectively denoted by $\vec{i}, \vec{j}, \vec{k}$.

Let P be the point (x, y, z). Then the position vector $\overrightarrow{OP}$ of P denoted by $\vec{r}$ is $x\vec{i} + y\vec{j} + z\vec{k}$.

For any vector $\vec{a}$ represented by $\overrightarrow{PQ}$, the coordinates of P, Q being (x_1, y_1, z_1) and (x_2, y_2, z_2), then

$$\vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$x_2 - x_1, y_2 - y_1, z_2 - z_1$ are called the components of the vector $\vec{a}$ in the directions of the coordinate axes. If the components are denoted by a_1, a_2, a_3,

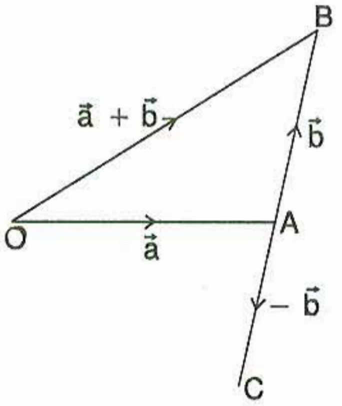
then $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$. We have $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

2. **Equal vectors:** $\vec{a} = \vec{b}$ means that the vectors $\vec{a}$ and $\vec{b}$ are equal in magnitude and parallel in directions irrespective of their initial points.

3. **Addition and subtraction**

If $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{AB} = \vec{b}$, then the vector $\overrightarrow{OB} = \vec{a} + \vec{b}$.

If BA is produced to C such that $\overrightarrow{AC} = -$ b, then $\overrightarrow{OC} = \vec{a} - \vec{b}$.



3.1. Vectors obey commutative, associative and distributive laws.

(i) $m(\vec{a} + \vec{b} - \vec{c}) = m\vec{a} + m\vec{b} - m\vec{c}$

(ii) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(iii) $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + \vec{b} + \vec{c}$

4. **Scalar product** (or dot product) of two vectors $\vec{a}$ and $\vec{b}$ is written as $\vec{a} \cdot \vec{b}$ and is defined as $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$, where $\theta (0 \le \theta \le \pi)$ is the angle between $\vec{a}$ and $\vec{b}$.

(a) Scalar multiplication is commutative and distributive

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ and $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

(b) The orthogonal projection of $\vec{a}$ on a straight line in the direction of $\vec{b}$ is given by

$$P = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

(c) $\vec{i}\cdot\vec{i}=1,\ \vec{j}\cdot\vec{j}=1,\ \vec{k}\cdot\vec{k}=1$

$\vec{i}\cdot\vec{j}=0=\vec{j}\cdot\vec{i}$

$\vec{j}\cdot\vec{k}=0=\vec{k}\cdot\vec{j}$

$\vec{k}\cdot\vec{i}=0=\vec{i}\cdot\vec{k}$

(d) $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$

where $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$ and $\vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$

(e) $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

(f) Two non-zero vectors are orthogonal if and only if $\vec{a}\cdot\vec{b}=0$

(g) If $\vec{a}\cdot\vec{b}=0$, then either $\vec{a}=0$ or $\vec{b}=0$ or $\vec{a}$ and $\vec{b}$ are orthogonal.

(h) **Work done** by a force $\vec{F}$ in a displacement $\vec{d}$ is given by $\vec{F}\cdot\vec{d}$.

5. Vector product (or cross product)

A vector whose magnitude is equal to the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as adjacent sides and whose direction is perpendicular to both $\vec{a}$ and $\vec{b}$ and is such that $\vec{a}$, $\vec{b}$ and that vector form a right-handed screw is called the vector product of $\vec{a}$ and $\vec{b}$ and is denoted by $\vec{a}\times\vec{b}$.

Thus $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\vec{n}$, where θ is the angle between the 2 vectors and $\vec{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{a}$, $\vec{b}$ and $\vec{n}$ form a right-handed triad.

(a) $(k\vec{a})\times\vec{b}=k(\vec{a}\times\vec{b})=\vec{a}\times(k\vec{b})$, where k is a scalar.

(b) Cross multiplication by vectors is not commutative $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

(c) Cross multiplication is distributive $\vec{a}\times(\vec{b}+\vec{c})=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$

(d) $\vec{i}\times\vec{j}=\vec{k}=-\vec{j}\times\vec{i}$

$\vec{j}\times\vec{k}=\vec{i}=-\vec{k}\times\vec{j}$

$\vec{k}\times\vec{i}=\vec{j}=-\vec{i}\times\vec{k}$

$\vec{i}\times\vec{i}=0,\ \vec{j}\times\vec{j}=0,\ \vec{k}\times\vec{k}=0$

(e) $\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3\\ b_1 & b_1 & b_3\end{vmatrix}$ where $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$, $\vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$

(f) Area of a triangle whose vertices have position vectors $\vec{a}, \vec{b}, \vec{c}$ is

$$\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

(g) Condition for three points with position vectors $\vec{a}, \vec{b}, \vec{c}$ to be collinear is

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$$

(h) Moment of a force $\vec{F}$ about a point P is $\vec{r} \times \vec{F}$, where $\vec{r}$ is the vector of any point on the line of action of the force $\vec{F}$ from P.

6. Scalar triple product

If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3 k$, $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3 k$, $\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$ are three vectors, then the dot product of $\vec{a}$ and $\vec{b} \times \vec{c}$ i.e., $\vec{a} \cdot (\vec{b} \times \vec{c})$ is called the scalar triple product of $\vec{a}$, $\vec{b}$ and $\vec{c}$ and is denoted by $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ and its value is $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(a) The absolute value of the scalar triple product $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is the volume of the parallelepiped with $\vec{a}, \vec{b}, \vec{c}$ as coterminous edges.

- **(i)** If two of the vectors in a scalar triple product are equal, then the value of the triple product is zero.
- **(ii)** If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.
- **(iii)** If three vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.

(b) **(i)** $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$

The positions of dot and cross can be interchanged without affecting the value of the scalar triple product.

(ii) If two vectors are interchanged in a scalar triple product, the sign is changed

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = -\left[\vec{b}\ \vec{a}\ \vec{c}\right] = -\left[\vec{c}\ \vec{b}\ \vec{a}\right]$$

(iii) A cyclic permutation of the three vectors does not change the sign or value of the scalar triple product.

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{b}\ \vec{c}\ \vec{a}\right] = \left[\vec{c}\ \vec{a}\ \vec{b}\right]$$

7. Vector triple product

If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, $\vec{a} \times (\vec{b} \times \vec{c})$ is called a vector triple product.

(i) $\cdot\, \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\, \vec{b} - (\vec{a} \cdot \vec{b})\, \vec{c}$

(ii) $\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c}$ are different and hence placing of the brackets is important.

8. (a) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$

$$= \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}$$

(b) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \left[\vec{a}\ \vec{b}\ \vec{d}\right]\vec{c} - \left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{d}$

$$= \left[\vec{a}\ \vec{c}\ \vec{d}\right]\vec{b} - \left[\vec{b}\ \vec{c}\ \vec{d}\right]\vec{a}$$

9. (a) Vector equation of a line passing through the point $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = a + t\vec{b}$, where t is a scalar.

(b) Vector equation of a line passing through the points $\vec{a}$, $\vec{b}$ is

$\vec{r} = (1 - t)\,\vec{a} + t\vec{b}$, where t is a scalar.

(c) **(i)** Angle between the lines $\vec{r} = \vec{a}_1 + t_1\vec{b}_1$, $\vec{r} = \vec{a}_2 + t_2\vec{b}_2$ is given by

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

(ii) If the two lines are skew, then the shortest distance between the lines

$$= \pm \frac{\left(\vec{b}_1 - \vec{b}_2\right) \cdot \left(\vec{a}_1 - \vec{a}_2\right)}{|\vec{b}_1 \times \vec{b}_2|}$$

(d) **(i)** Vector equation of a plane through $\vec{a}$ containing vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + t\vec{b} + s\vec{c}$, where t and s are scalars.

(ii) Vector equation of a plane through the points $\vec{a}$, $\vec{b}$ and containing $\vec{c}$ is

$\vec{r} = (1 - t)\vec{a} + t\vec{b} + s\vec{c}$

(iii) Vector equation of a plane passing through the points $\vec{a}$, $\vec{b}$, $\vec{c}$ is

$\vec{r} = (1 - t - s)\vec{a} + t\vec{b} + s\vec{c}$

(e) Vector equation of a plane passing through $\vec{a}$ and normal to the vector $\vec{n}$ is $\left(\vec{r} - \vec{a}\right) \cdot \vec{n} = 0$

(f) Angle between the planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is given by $\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

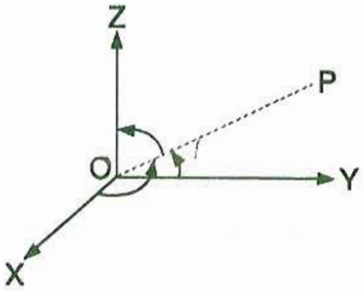
VIII. THREE DIMENSIONAL COORDINATE GEOMETRY

(1) Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(2) Section formula

Point which divides the segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $m_1 : m_2$ is

$$\left[\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right]$$



If a directed line OP makes the angles α, β, γ respectively with the positive directions of X, Y, Z axes (which are mutually perpendicular to each other), then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the direction cosines of the line OP. They are denoted by ℓ, m, n.

If P(x, y, z) is the point and if OP = r, then $\frac{x}{r} = \cos\alpha$, $\frac{y}{r} = \cos\beta$ and $\frac{z}{r} = \cos\gamma$.

Also $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2 + y^2 + z^2}{r^2} = \frac{r^2}{r^2} = 1$. i.e., $\ell^2 + m^2 + n^2 = 1$

If a, b, c are three real numbers such that $\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c}$, then a, b, c are called direction ratios of OP.

$$\text{Then } \ell = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points in space, then the direction ratios of PQ are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$. The angle θ between two lines with direction cosines ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 is given by $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$ or

$$\sin\theta = \sqrt{\Sigma(m_1n_2 - m_2n_1)^2}$$

Note: Two lines are perpendicular if $\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$ and parallel if

$\ell_1 = \ell_2$, $m_1 = m_2$, $n_1 = n_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Standard form of the equation of a plane

(1) If p is the length of the normal from the origin on the plane and ℓ, m, n are the direction cosines of this normal, then the equation of the plane is $\ell x + my + nz = p$.

Note: If $ax + by + cz + d = 0$ is the equation of a plane, then $\frac{-a}{\sqrt{\Sigma a^2}}, \frac{-b}{\sqrt{\Sigma a^2}}, \frac{-c}{\Sigma a^2}$, are the direction cosines of the normal to the plane and $p = \frac{d}{\sqrt{\Sigma a^2}}$.

(2) The equation of the plane parallel to $ax + by + cz + d = 0$ and passing through (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

(3) The equation of the plane parallel to the z-axis is $ax + by + d = 0$ etc.

(4) a, b, c are direction ratios of the normal to the plane $ax + by + cz + d = 0$.

(5) If $aa_1 + bb_1 + cc_1 = 0$, the straight line with direction ratios a_1, b_1, c_1 is parallel to the plane $ax + by + cz + d = 0$.

(6) The equation of the plane which cuts off the intercepts a, b, c on the coordinate axes is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

(7) The equation of the plane through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

(8) The angle between the two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Note: The planes are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(9) Length of the perpendicular from (x_1, y_1, z_1) on the plane $ax + by + cz + d = 0$ is

$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right|$$

Equation of a line in space

(1) If a line in space passes through the point (x_1, y_1, z_1) and parallel to a line with direction ratios a, b, c, then the equation of the line is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$$

(This is called the symmetric form of the equation of a line.)

(2) The equation of a line through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

(3) Two lines $\frac{x - x_1}{\ell_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{\ell_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

(4) **Skew lines:** Two straight lines are said to be skew lines if they are neither parallel nor intersecting.

The shortest distance between them is perpendicular to both of them.

The shortest distance between the lines

$$\frac{x - x_1}{\ell_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \text{ and}$$

$$\frac{x - x_2}{\ell_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

is $\dfrac{\Sigma(x_2 - x_1)(m_1n_2 - m_2n_1)}{\sqrt{\Sigma\left(m_1n_2 - m_2n_1\right)^2}}$

COMPENDIUM – CHEMISTRY

I. PHYSICAL CHEMISTRY

1. ATOMIC STRUCTURE

(i) (a) Mass of moving electron = $\dfrac{\text{Rest mass of electron}}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}$

where v is the velocity of the electron and c is the velocity of light.

(b) Density of the nucleus = $\dfrac{\text{Mass number}}{6.023 \times 10^{23} \times \dfrac{4}{3}\pi r^3}$

(ii) Moseley equation for characteristic X-rays $\sqrt{\nu} = a(Z - b)$ a and b are constants, Z is the atomic number and ν is the frequency of X-rays.

(iii) (a) Charge on the electron = 1.602×10^{-19} coulomb (4.8×10^{-10} esu)

(b) Mass of electron = 9.1096×10^{-31} kg, relative mass of $_{1}e^{0} = 0.000549$

(c) Mass of proton = 1.6726×10^{-27} kg, relative mass of $_{1}H^{1} = 1.00727$

(d) Mass of neutron = 1.675×10^{-27} kg, relative mass of $_{0}n^{1} = 1.00866$

(e) Radius of nucleus, $r = (1.3 \times 10^{-13})\, A^{1/3}$ cm, where A = mass number of the element.

(iv) Visible range of electromagnetic radiation = 3800 – 7600 Å.

$$E = h\nu = h\frac{c}{\lambda} = hc\bar{\nu}$$

c = Velocity of Light = 3×10^{8} m/s

h = Planck's constant = 6.626×10^{-34} Js

(v) Wave number of spectral lines in hydrogen atom

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = R\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$

Rydberg constant, R = 109678 cm^{-1} or 1.0968×10^{7} m^{-1}.

(v a) Wave number of spectral line hydrogen like species = $\bar{\nu} = \dfrac{1}{\lambda} = R_H Z^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

Z is atomic number of the species, R_H = 109678 cm^{-1} or 1.0968×10^{7} m^{-1}

(vi)

Spectral lines/Hydrogen	Spectral region	Values of n_1	Values of n_2
Lyman	UV region	1	2, 3, 4, 5 etc.
Balmer	Visible	2	3, 4, 5, 6 etc.
Paschen	IR	3	4, 5, 6, 7 etc.
Brackett	IR	4	5, 6, 7, 8 etc.
Pfund	IR	5	6, 7, 8 etc.
Humphrey	IR	6	7, 8 etc.

(vii) **Bohr's model of atom**

(a) Angular momentum, $mvr = \frac{nh}{2\pi}$, where h = Planck's constant = 6.626×10^{-27} erg sec or 6.626×10^{-34} Js

(b) Energy of the electron, $E_n = -\frac{kZe^2}{r} + \frac{kZe^2}{2r}$ (Potential energy of ${}_{-1}e^0$ + Kinetic energy of ${}_{-1}e^0$)

$k = 9.0 \times 10^9\ Nm^2/C^2$ in SI unit,

k = 1.0 in CGS units.

$$E_n = -\frac{2\pi^2k^2me^4Z^2}{n^2h^2}$$

Electronic energy of H°

when n = 1 and Z = 1, $E = -2.179 \times 10^{-18}$ J per atom

$= -13.6$ eV per atom $= -1312$ kJ/mole

In general, $E_n = \frac{E_1}{n^2} \times Z^2$

or $E_n = -13.6 \times \frac{Z^2}{n^2}$ eV/atom

The energy of the electron of hydrogen atom in the nth orbit is

$$E_n = \frac{-21.76 \times 10^{-19}\ J}{n^2}$$

(c) Radius of the Bohr orbit, $r = \frac{n^2h^2}{4\pi^2mRZe^2}$

For Z = 1 and n = 1, $r_1 = 0.529 \times 10^{-10}$ m

r_1 of $H^0 = 0.529$ Å

Radii of orbits of hydrogen like species $r_n = r_1 \times \frac{n^2}{Z} = 0.529 \times \frac{n^2}{Z}$ Å

(d) The velocity of the electron, $v = \frac{nh}{2\pi mr}$

Also, $v = \frac{Z}{n} \times \frac{2\pi e^2}{h}$ (CGS units)

Velocity of electron in its I orbit is

$Z = 1$ and $n = 1$, $v = 2.188 \times 10^8$ cm s^{-1}

Velocity of electron in hydrogen like species = $v_n = 2.188 \times 10^8 \frac{Z}{n}$ cm s^{-1}

Number of spectral lines with spectrum of an atom when an electron comes from nth level to the ground state = $\frac{n(n-1)}{2}$

(e) Rydberg constant, $R_H = \frac{2\pi^2 me^4k^2}{h^3c} = 109678$ cm^{-1} = 1.0968×10^7 m^{-1}

For other hydrogen like species, m is replaced with reduced mass, μ and $\mu = \frac{Mm}{M + m}$

where M is mass of nucleus and m is mass of electron.

(viii) Photoelectric effect: $h\nu = h\nu_0 + \frac{1}{2} mv^2$

ν_0 = threshold frequency

Also $h\nu = w_0 + \frac{1}{2}mv^2$, w_0 = work function.

(ix) Compton effect: Increase in wavelength of X-rays after scattering from the surface of a lighter element,

de Broglie principle: $\lambda = \frac{h}{p} = \frac{h}{mv}$

(x) Heisenberg's uncertainty principle: $\Delta x \cdot \Delta p = \frac{h}{4\pi}$ or $\Delta x \cdot m\Delta v = \frac{h}{4\pi}$

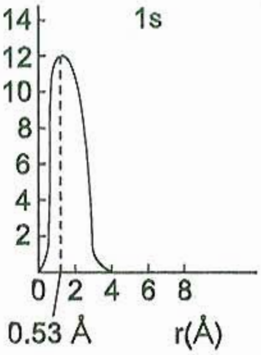
(xi) (a) Schrodinger Wave Equation: $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - v)\psi = 0$

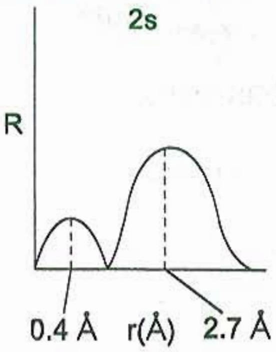
where ψ is the amplitude of the electron wave at a point with coordinates x, y, z, E = Total energy and v = potential energy of the electron. ψ is also known as wave function. ψ^2 gives the probability of finding electron at (x, y, z). The acceptable solutions of the above equation for the energy E are called **eigen values** and the corresponding wave functions are known as **eigen functions**.

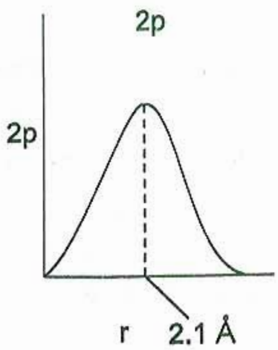
(b) Radial probability distribution curves

Radial probability (R) = $4\pi r^2\, dr\, \psi^2$

The plots of R vs distance from nucleus are as follows.







In the radial probability versus distance from the nucleus, number of peaks i.e., region of maximum probability = $n - \ell$ e.g., 2s has 2 peaks, 3s will have 3 peaks, 2p will have one peak, 3p has 2 peaks and so on.

(c) Spherical (radial) nodes and nodal plane

A spherical surface within an orbital on which the probability of finding an electron is zero is called a spherical or radial node. The number of spherical or radial nodes in an orbital is given by $(n - \ell - 1)$.

e.g., 1s orbital ($n = 1$, $\ell = 0$) has no spherical node

2s orbital ($n = 2$, $\ell = 0$) has one spherical node

2p orbital ($n = 2$, $\ell = 1$) has no spherical node

3p orbital ($n = 3$, $\ell = 1$) has one spherical node

(d) Angular node or planar node

They correspond to 'ℓ' value. It depends upon the shape of orbitals. An 's' orbital has no angular node. A 'p' orbital has one angular node; a 'd' orbital has two angular nodes.

Total no. of radial nodes = $n - \ell - 1$

Total no. of angular nodes = ℓ

Total no. of nodes = $(n - \ell - 1) + \ell = n - 1$

(xii) (a) Orbital angular momentum of electron

$$= \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi} = \sqrt{\ell(\ell+1)}\,\hbar, \quad \text{where } \hbar = \frac{h}{2\pi}$$

(b) Spin magnetic moment

$\mu_{\text{effective}} = \sqrt{4(s+1)}$ or

$\mu_{\text{effective}} = \sqrt{n(n+2)}$ (spin only)

n = number of unpaired electrons.

(c) The maximum number of emission lines = $\dfrac{n(n-1)}{2}$ where n = number of the orbit.

2. BONDING AND MOLECULAR STRUCTURE

(a) (i) Dipole Moment of Polar Molecules

$\mu = d\ell$,

where μ = dipole moment expressed in debye (one debye = 1×10^{-18} esu.cm),

= dipole moment expressed in coulomb-metre = 1×10^{-29} coulomb-metre (C.m.)

d = the charge at one end of dipole expressed in esu and

ℓ = the distance of separation of the charges expressed in cm.

1 D = 3.336×10^{-30} C.m.

(ii) % ionic character of a bond

$$= \frac{\text{Observed dipole moment}}{\text{Theoretical value assuming complete separation of charges}} \times 100$$

If bond moments μ_1, μ_2 are known, then the dipole moment of a molecule is

$$\mu = \sqrt{\mu_1^2 + \mu_2^2 + 2\mu_1\mu_2 \cos\theta}$$

Orthosubstituted compound will have the highest μ = ($\cos 60° = +\frac{1}{2}$)

(b) Percentage of ionic character of a covalent bond

It depends upon the following factors.

(I) Electronegativity difference: Greater the electronegativity difference between the two bonded atoms, greater is the percentage of ionic character. For example

Electronegativity difference	0.2	0.6	0.8	1.2	1.9	2.2	2.8	3.2		
Percentage ionic character	1	10	15	30	50	70	80	92		
Nature of bond	Covalent character increases ←					Ionic character increases →				

(II) Several empirical equations have been proposed to calculate the percentage ionic character from the electronegativity difference of the bonding atoms.

(i) Pauling equation: % ionic character = $[1 - e^{-\frac{1}{4}}(x_A - x_B)] \times 100$

(ii) Hannay and Smith equation: % ionic character = $[16(x_A - x_B) + 3.5(x_A - x_B)^2]$

Here x_A and x_B are the electronegativities of the two atoms forming a bond.

(c) Dipole moment and percentage of ionic character

$$\%\ \text{ionic character} = \frac{\text{Actual dipole moment of the bond}}{\text{Calculated dipole moment of the bond}} \times 100$$

(d) Predicting the type of hybridisation of the central atom in a molecule/ion other than complex ions:

Step (i): Add the number of valence electrons of all the atoms present in a given molecule/ion.

Step (ii): If the given ion is a cation subtract the number of electrons equal to the charge on the cation; if the ion is an anion, add number of electrons equal to the charge on the anion.

Step (iii): If the result obtained in step (ii) ≤ 8 divide by 2 and find the sum of the quotient and remainder. If the result obtained in step (ii) is between 9 and 56, divide by 8, find first quotient (Q_1). Divide the remainder (R_1) if any by 2 and find the second quotient (Q_2). Add all the quotients ($Q_1 + Q_2$) and final remainder (R_2)

i.e., $Q_1 + Q_2 + R_2 = X$

Value of X	2	3	4	5	6	7
Type of hybridisation	sp	sp^2	sp^3	sp^3d	sp^3d^2	sp^3d^3

(e) Predicting geometry of species using VSEPR theory

With the help of VSEPR theory, we can predict the geometry of various species in a systematic way. The scheme makes use of the following steps.

(i) Identify the central atom and count the number of valence electrons on the central atom.

(ii) Add to this, the number of other atoms (which form single bonds only). Here, oxygen atoms are not added as they form two bonds.

(iii) If the species is an anion, add negative charges and if it is a cation, subtract positive charges.

(iv) Divide N by 2 and we get the sum of bonding and non-bonding electron pairs. $\frac{N}{2}$ = Number of other atoms + number of lone pairs.

(v) This gives us a number, which we refer as N.

(vi) Compare the result $\left(\frac{N}{2} \text{ value}\right)$ with the value given in table, corresponding to the given number of lone pairs.

N/2 Value	No. of lone pairs	Shape of the species	Example
2	0	Linear	$HgCl_2$, $BeCl_2$
3	0	Triangular planar	BF_3, $AlCl_3$, BH_3, NO_3^-, SO_3
	1	Angular or bent	$SnCl_2$, SO_2, NO_2^-
4	0	Tetrahedral	CCl_4, BeF_4^{2-}, BF_4^-, PCl_4^+, ClO_4^-
	1	Trigonal pyramidal	NH_3, PCl_3, PF_3, ClO_3^-
	2	Angular or bent	H_2O, H_2S, OF_2, ClO_2^-
	3	Linear	ClO^-

N/2 Value	No. of lone pairs	Shape of the species	Example
5	0	Trigonal bipyramidal	PCl_5, PF_5
	1	Seesaw or irregular tetrahedron	SF_4, IF_4^+
	2	T-shaped	ClF_3, BrF_3
	3	Linear	I_3^-, Br_3^-, XeF_2
6	0	Octahedral or square bipyramidal	SF_6, PCl_6^-
	1	Square pyramidal	BrF_5, IF_5
	2	Square planar	ICl_4^-, XeF_4^-, IF_4
7	0	Pentagonal bipyramidal	IF_7

(f) M.O. Theory

(i) Bond order (B.O) $= \frac{1}{2}(N_b - N_a)$

N_b = number of electrons in the bonding molecular orbital.

N_a = number of electrons in the antibonding molecular orbital.

(ii) B.O > zero ⇒ the molecule/ion exists.

(iii) Higher the bond order, shorter is the bond length.

(iv) Higher the bond order, higher is the **bond dissociation energy**.

(v) Higher the bond order, greater is the stability.

(vi) Magnetic properties of some species and their bond order

Species	Bond order	Magnetic properties
H_2	1	Diamagnetic
H_2^+	0.5	Paramagnetic
H_2	0.5	Paramagnetic
He_2	0	does not exist
He_2	0.5	Paramagnetic
Li_2	1	Diamagnetic
Be_2	0	does not exist

Species	Bond order	Magnetic properties
B_2	1	Paramagnetic
C_2	2	Diamagnetic
N_2	3	Diamagnetic
N_2^+	2.5	Paramagnetic
O_2	2	Paramagnetic
O_2^+	2.5	Paramagnetic
O_2^-	1.5	Paramagnetic
O_2^{2-}	1.0	Diamagnetic
F_2	1.0	Diamagnetic
CN	2.5	Paramagnetic
CN^-	3.0	Diamagnetic
NO	2.5	Paramagnetic
NO^+	3.0	Diamagnetic
CO	3.0	Diamagnetic

3. NUCLEAR CHEMISTRY

(1) Radius of the nucleus = $R = R_oA^{1/3}$

R_o is the constant whose value is 1.3 fm $= 1.3 \times 10^{-13}$ cm

(2) Binding energy = $\Delta m \times 931.5$ MeV

(3) Packing fraction = $\dfrac{\text{Actual isotopic mass} - \text{Mass number}}{\text{Mass number}} \times 10^4$

Radioactivity

(i) Decay constant $\lambda = \dfrac{2.303}{t} \log \dfrac{N_o}{N_i}$

Half-life period $t_{1/2} = \dfrac{0.693}{\lambda}$

The number of half-lives in the decay time 't' is given by,

$$n = \frac{t}{t_{1/2}} = \frac{\text{Total time}}{\text{Half-life period}}$$

The amount N of the radioactive substance left after 'n' half-lives

$$N_n = \frac{N_o \text{ (initial amount)}}{2^n}$$

$$N_n = N_o \left(\frac{1}{2}\right)^n$$

(ii) Average life period (T)

$$= \frac{\text{Sum of lives of the nuclides}}{\text{Total number of nuclides}} = \frac{1}{\lambda}$$

(iii) Average life (T) = $1.44 \times T_{1/2}$

1 Ci $= 3.7 \times 10^{10}$ dps (mrd – milli rutherford, μrd – micro rutherford, Bq – Becquerel.)

$= 37 \times 10^3$ rd

1 mCi $= 3.7 \times 10^7$ dps = 37 rd

1 μCi $= 3.7 \times 10^4$ dps = 37 mrd

1 Bq = 1 disintegration per second = 1 μrd

10^6 Bq = 1 rd

3.7×10^{10} Bq = 1 Ci

(iv) **Disintegration Series**

Name of the series	Starting nuclide	Final nuclide	Particle lost	
			α	β
4n (Thorium) series	$_{90}Th^{232}$	$_{82}Pb^{208}$	6	4
4n + 1 (Neptunium)	$(_{94}Pu^{241})$ $_{92}Np^{237}$	$_{83}Bi^{209}$	8	5
4n + 2 (Uranium)	$_{92}U^{238}$	$_{82}Pb^{206}$	8	6
4n + 3 (Actinium)	$_{92}U^{235}$ $_{89}Ac^{227}$	$_{82}Pb^{207}$	7	4

(v) During disintegration, number of α-particles emitted $X = \frac{A_P - A_D}{4}$

A_P = Mass number of parent element

A_D = Mass number of daughter element

number of β-particles emitted $= D_Z - (P_Z - 2X)$

P_Z and D_Z are atomic numbers of parent and daughter element.

(vi) **Isosters**: Molecules having same number of atoms and also same number of electrons e.g., N_2 and CO.

4. BEHAVIOUR OF GASES

(i) Ideal gas equation, $PV = nRT = \frac{w}{M} RT$

$P = \frac{dRT}{M}$, d = density of the gas.

(As one mole of an ideal gas occupies at STP 22.4 L)

(a) $R = \dfrac{(1\ \text{atm.})\ (22.4\ \text{litres})}{1\ \text{mole} \times 273\ \text{K}}$

$= 0.082$ litre atm/deg/mole

(b) $R = \dfrac{(1\ \text{atm.})\ (22400\ \text{mL})}{1\ \text{mole} \times 273\ \text{K}}$

$= 82$ mL atm/deg/mole

(c) $R = \dfrac{76\ \text{cm} \times 13.595 \times 980.7 \times 22400}{1\ \text{mole} \times 273\ \text{K}}$

$= 8.314 \times 10^7$ erg/deg/mole

$= 1.987$ cal/deg/mole

($\because 4.187 \times 10^7$ ergs $= 1$ calorie)

(d) $R = \dfrac{1.013 \times 10^5\ \text{Nm}^{-2} \times 22400 \times 10^{-6}\ \text{m}^3}{1\ \text{mole} \times 273\ \text{K}}$

$= 8.314$ joule/deg/mole

Note: 1 atm. pressure = 76 cm of Hg (= 760 mm of Hg)

$= 76\ \text{cm} \times 13.595\ \text{g/ml} \times 980.7\ \text{cm/s}^2$

$= \mathbf{1.013 \times 10^6}$ **dynes/cm^2**.

Pressure = height × density × g

1 atm. pressure $= 0.76\ \text{m} \times 13.595 \times 10^3\ \text{kg/m}^3 \times 9.8\ \text{m/s}^2$

$= \mathbf{1.013 \times 10^5}$ **N/m^2** or **101.3 kPa or** $\mathbf{101.3 \times 10^3}$ **Pa**.

(ii) Dalton's law of partial pressures for a mixture of non-reacting gases

$$P_{\text{Total}} = p_A + p_B + p_C + \ldots\ldots$$

where P_{Total} = Total pressure and p_A, p_B, p_C, ... are the partial pressures of the component gases, $p_A = X_A P_t$, where $X_A = \dfrac{n_A}{n_{\text{Total}}}$ = Mole fraction of A

Mole fraction of A = Volume fraction of A.

(iii) Graham's law of diffusion (effusion)

Rate of diffusion (R) $= \dfrac{\text{Volume of the gas diffused (V)}}{\text{time taken (t)}}$

$$R = \frac{V}{t}$$

Rate of diffusion $\propto \dfrac{1}{\sqrt{d}}$, where d = density of the gas.

Hence;

$$\frac{R_1}{R_2} = \frac{V_1}{V_2} = \frac{t_2}{t_1} = \sqrt{\frac{d_2}{d_1}} = \sqrt{\frac{M_2}{M_1}}$$

where R_1 and R_2 are the rates of diffusion (or effusion), V_1 and V_2 are the volumes which diffuse in the same period of time, t_1 and t_2 are the times of diffusion for the same volume, d_1 and d_2 are the densities (or vapour densities) and M_1 and M_2 are the molecular weights of two gases 1 and 2 respectively. Here, density generally stands for vapour density or relative density.

When diffusion of gases are compared at different pressures, the rate of diffusion of the gas is directly proportional to the pressure. i.e., $R = \frac{KP}{\sqrt{M}}$ where K is a constant at constant temperature.

(iv) Avogadro's number (N_o or N_A): The number of molecules present in one mole (1 g molecular weight) of a substance or the number of atoms present in 1 g atomic weight of an element or the number of ions present in 1 g ion of an ion = 6.023×10^{23}.

Molecular weight of a gas = 2 × Vapour density.

(v) $PV = \frac{1}{3}nmu^2$ (from kinetic theory), where u_{rms} = Root mean square velocity = c

$$u = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{d}}$$

The unit of u depends upon the units of P, V, M, etc., as given below.

V	P	R	T	M	d	u
cm^3	$dynes/cm^2$	ergs	K	g	g/ml	$cm\ s^{-1}$
m^3	Nm^{-2}	joule	K	kg	kg/m^3	$m\ s^{-1}$

$u_{av} = \bar{c}$ = Average velocity = $\sqrt{\frac{8RT}{\pi M}}$

u_{mp} = Most probable velocity = $\sqrt{\frac{2RT}{M}}$

$u_{mp} = c_{mp}$

$c > \bar{c} > c_{mp}$

Ratio of $c : \bar{c} : c_{mp} = 1 : 0.9213 : 0.816$

(vi) Average kinetic energy per molecule of an ideal gas $= e_k = \frac{3}{2}kT$

$$= \frac{3}{2} \cdot \frac{R}{N} T$$

$$= 2.07 \times 10^{-23} \text{ T joules}$$

(k = Boltzmann's constant;
N = Avogadro's number)

(vii) Average kinetic energy of all the molecules in one mole of a gas $= E_k$

$= \frac{3}{2}RT = 12.47$ T joules, where T is the temperature in K.

(viii) Most probable velocity : Mean velocity : RMS velocity

= 1 : 1.128 : 1.225

(ix) The ratio of root mean square velocity for the same gas at two different temperatures will be given by the equation

$$\frac{\mu_1}{\mu_2} = \sqrt{\frac{T_1}{T_2}}$$

(x) The ratio of RMS velocity for two different gases at same temperature will be

$$\frac{\mu_1}{\mu_2} = \sqrt{\frac{M_2}{M_1}}$$

(xi) $P = \frac{1}{3} n\, mc^2$

$$PV = \frac{2}{3}\left(\frac{1}{2} mc^2\right) = \frac{2}{3} \times \text{Kinetic energy}$$

(xii) Density of a gas, $d = \frac{PM}{RT}$, where M is the molar mass.

(xiii) van der Waals' equation:

$\left(P + \frac{a}{V^2}\right)(V - b) = RT$ for one mole

$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$ for n moles. a and b are van der Waals' constants.

Units of a and b : a = atm $L^2\ mol^{-2}$; b = L mol^{-1}

non-ideal gases have large values of a and b.

(xiv) Z(compressibility factor) $= \frac{PV}{nRT}$

Z = 1 for ideal gas

(xv) **(a)** Boyle temperature, $T_B = \frac{a}{Rb}$

(b) Inversion temperature $T_i = \frac{2a}{Rb}$

(c) Critical volume $V_c = 3b$

(d) Critical temperature $T_c = \frac{8a}{27Rb}$

(e) Critical pressure $P_c = \frac{a}{27b^2}$

(f) van der Waals' constant,

$$b = 4 \times N_o \times \frac{4}{3} \pi r^3$$

where r is the radius of the molecule and N_o is Avogadro number.

(g) $T_b : T_i : T_c = 1 : 2 : \frac{8}{27}$ or 27 : 54 : 8

5. STOICHIOMETRY AND VOLUMETRIC ANALYSIS

(1) Mole concept

(i) One mol of atoms = gram atomic mass (or 1 g atom) = 6.023×10^{23} atoms

(ii) One mol of molecules = gram molecular mass (1 g molecule)

$= 6.023 \times 10^{23}$ molecules

$= 22.4$ L at STP.

(iii) One mol of ionic compound = gram formula = 6.022×10^{23} formula units

(iv) Number of moles = $\frac{W}{M} = \frac{\text{Volume of gas in L at STP}}{22.4} = \frac{\text{Number of molecules}}{N_A \text{ (Avogadro no.)}}$

(2) Calculation of molecular weight

(i) Molecular mass = 2 × vapour density

(ii) Molecular mass = mass of 22.4 L of vapour at STP.

(iii) From rates of diffusion $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$

(3) Calculation of equivalent weight

(i) Equivalent weight of an element = $\frac{\text{Atomic weight of the element}}{\text{Valency of the element}}$

(i a) Eq. wt. of metal = $\frac{\text{weight of metal}}{\text{weight of } H_2 \text{ displaced}} \times 1.008$

$$= \frac{\text{weight of metal} \times 11{,}200}{\text{volume of } H_2 \text{ displaced at NTP (mL)}}$$

(ii) Eq. wt. of metal $= \dfrac{\text{weight of metal}}{\text{weight of } O_2 \text{ combined}} \times 8$

$$= \frac{\text{weight of metal}}{\text{volume of } O_2 \text{ displaced/combined at STP}} \times 5600$$

(iii) $\dfrac{\text{weight of metal added to salt solution}}{\text{weight of metal displaced}} = \dfrac{\text{Eq. wt. of the metal added}}{\text{Eq. wt. of the metal displaced}}$

(iv) For a reaction $AB + CD \longrightarrow AD\downarrow + BC$;

$$\frac{\text{weight of salt AB added to salt CD in solution}}{\text{weight of precipitate AD formed}}$$

$$= \frac{\text{Eq. wt. of radical (A) + Eq. wt. of radical (B)}}{\text{Eq. wt. of radical (A) + Eq. wt. of radical (D)}}$$

(v) Eq. wt. = weight of element liberated or deposited by 1 Faraday (96500 C)

(vi) On passing the same quantity of electricity through different electrolytes.

$$\frac{\text{weight of x deposited}}{\text{weight of y deposited}} = \frac{\text{Eq. wt. of x}}{\text{Eq. wt. of y}}$$

(vii) Eq. wt. of an acid $= \dfrac{\text{Molecular weight}}{\text{basicity}}$

= weight of acid neutralised by 1000 cc of 1 N of base

Eq. wt. of a base $= \dfrac{\text{Molecular weight}}{\text{acidity}}$

= weight of base neutralised by 1000 cc of 1 N of acid

(viii) Eq. wt. of an organic acid (RCOOH)

$$\frac{\text{Eq. wt. of silver salt (RCOOAg)}}{\text{Eq. wt. of silver (108)}} = \frac{\text{weight of silver salt}}{\text{weight of silver}}$$

(ix) For a compound (I) being converted into another compound (II) of the same metal

$$\frac{\text{weight of the compound (I)}}{\text{weight of the compound (II)}} = \frac{\text{Eq.wt. of metal + Eq.wt. of anion of compound (I)}}{\text{Eq.wt. of metal + Eq.wt. of anion of compound (II)}}$$

(x) Eq. wt. of salt $= \dfrac{\text{Molecular wt}}{\text{Total positive valency of metal atom}}$

(xi) Eq. wt. of oxidising or reducing agent $= \dfrac{\text{Molecular weight}}{\text{no. of electrons gained/lost by the species}}$

(xii) Normality (N) $= \dfrac{\text{number of equivalents}}{\text{volume of the solution in litres}}$

(xiii) No. of equivalents = N × vol. of solution in litres = N × vol. of solution in $cm^3 \times 10^{-3}$

(xiv) Molarity (M) = $\dfrac{\text{no. of moles}}{\text{vol. of the solution in litres}}$

(xv) No. of mol = M × volume of solution in litres = M × volume in $\text{cm}^3 \times 10^{-3}$

(xvi) When a solution is diluted

$$\underset{\text{(before dilution)}}{N_1 \times V_1} = \underset{\text{(after dilution)}}{N_2 \times V_2}$$

(xvii) When the solutions of two substances (acid + base; oxidising agent + reducing agent) react completely,

$$\underset{\text{(Soln 1)}}{N_1 \times V_1} = \underset{\text{(Soln 2)}}{N_2 \times V_2}$$

(xviii) For a reaction

$$n_1 A + n_2 B \longrightarrow \text{Products}$$

$$\underset{\text{(of A)}}{\frac{M_1V_1}{n_1}} = \underset{\text{(of B)}}{\frac{M_2V_2}{n_2}}$$

(xix) **Relationship between normality and molarity**

Normality × Eq. wt = molarity × molar mass

For acids; normality = molarity × basicity

For bases; normality = molarity × acidity

For oxidising/reducing agent; normality = molarity × $\dfrac{\text{Mol. wt. of OA/RA}}{\text{Eq. wt. of OA/RA}}$

(xx) When V_1 cc of a solution having molarity M_1 is diluted to a final volume V_2 cc of the solution with molarity M_2, volume of water added = $V_2 - V_1 = \left(\dfrac{M_1 - M_2}{M_2}\right)V_1$

In terms of normality = $V_2 - V_1 = \left(\dfrac{N_1 - N_2}{N_2}\right)V_1$

6. SOLUTIONS

1. Strength of solutions

(a) % by weight of solute = $\dfrac{w_2 \times 100}{W}$

(w_2 = Weight of solute and W = Weight of solution in g)

(b) % by volume of solute = $\frac{w_2 \times 100}{V}$

(V = Volume of the solution in mL containing w_2 g of solute)

(c) Molarity = $\frac{w_2 \times 1000}{M_2 \times V} = \frac{\text{moles of solute}}{\text{Volume of solution (in L)}}$

(w_2 = Weight in g of solute whose molecular weight is M_2, V = Volume of the solution in mL)

(d) Molality = $\frac{w_2 \times 1000}{M_2 \times w_1} = \frac{\text{moles of solute}}{\text{wt. of Solvent (in kg)}}$

(w_1 = Weight in g of the solvent in which w_2 g of solute is dissolved)

(e) Normality = $\frac{w_2 \times 1000}{E_2 \times V} = \frac{\text{no. of geq}}{\text{Vol. of sol}^{n}\text{ (in L)}}$

(E_2 = Equivalent weight of the solute)

2. Colligative Properties: Raoult's law (for ideal solutions of non-volatile solutes).

(i) $p = p^o X_1$

where p = Vapour pressure of the solution, $p°$ = Vapour pressure of the solvent and X_1 = Mole fraction of the solvent.

(ii) $\frac{p^\circ - p}{p^\circ} = X_2$; X_2 = Mole fraction of solute for a dilute solution of a non-volatile non-electrolyte.

(iii) $\Delta p = \frac{p^\circ M_1}{1000}$ m.

Δp = Lowering of vapour pressure, M_1 = Molecular weight of the solvent and m = Molality of the solution.

Raoult's law (for an ideal mixture of two liquids)

$$p_A = (p^\circ)_A X_A \,;\; p_B = (p^\circ)_B X_B$$

where p_A = Partial pressure of A in the vapour

p_B = Partial pressure of B in the vapour

$(p^\circ)_A$ = Vapour pressure of A

$(p^\circ)_B$ = Vapour pressure of B

X_A = Mole fraction of A

X_B = Mole fraction of B

(iv) Relationship between mole fraction of the components of an ideal solution in the vapour phase:

$p_A = x_A\, p_A^\circ;\ p_B = x_B\, p_B^\circ$

y_A = mole fraction of the component A in vapour phase

$$y_A = \frac{p_A}{p_A + p_B} = \frac{x_A\, p_A^\circ}{x_A\, p_A^\circ + x_B\, p_B^\circ}$$

$$= \frac{x_A\, p_A^\circ}{x_A\, p_A^\circ + (1 - x_A)\, p_B^\circ}$$

$$\boxed{y_A = \frac{x_A\, p_A^\circ}{x_A(p_A^\circ - p_B^\circ) + p_B^\circ}} \text{ (or)}$$

$$\frac{1}{y_A} = \frac{x_A(p_A^\circ - p_B^\circ) + p_B^\circ}{x_A\, p_A^\circ} = \frac{p_A^\circ - p_B^\circ}{p_A^\circ} + \frac{p_B^\circ}{p_A^\circ} \cdot \frac{1}{x_A}$$

(v) Relationship between mole fraction of a component in the vapour phase and total pressure of an ideal solution:

$$y_A = \frac{p_A}{P_{\text{Total}}} = \frac{x_A\, p_A^\circ}{x_A p_A^\circ + x_B\, p_B^\circ} = \frac{x_A\, p_A^\circ}{(p_A^\circ - p_B^\circ)x_A + p_B^\circ}$$

(or) $x_A\, p_A^\circ = y_A\,(p_A^\circ - p_B^\circ)\, x_A + y_A\, p_B^\circ$

(or) $x_A = \dfrac{y_A\, p_B^\circ}{p_A^\circ + (p_B^\circ - p_A^\circ)\, y_A}$

But $y_A = \dfrac{p_A}{P_{\text{Total}}} = \dfrac{x_A\, p_A^\circ}{P_{\text{Total}}}$

Hence $y_A = \dfrac{p_A^\circ}{P_{\text{Total}}} \times \dfrac{y_A\, p_B^\circ}{p_A^\circ + (p_B^\circ - p_A^\circ)\, x_A}$

(or) $\dfrac{1}{P_{\text{Total}}} = \dfrac{p_A^\circ + (p_B^\circ - p_A^\circ)\, y_A}{p_A^\circ\, p_B^\circ}$

$$\boxed{\frac{1}{P_{\text{Total}}} = \frac{1}{p_B^\circ} + \left(\frac{p_B^\circ - p_A^\circ}{p_A^\circ\, p_B^\circ}\right) y_A}$$

(vi) Calculation of boiling point of a liquid at a given pressure, when the boiling point at some other pressure is given (use Clausius Clapeyron equation)

$$\log \frac{P_2}{P_1} = \frac{\Delta H_v}{2.303\, R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

where T_1 is the boiling point of the liquid at pressure P_1

T_2 is the boiling point of the liquid at pressure P_2

(vii) $$\frac{p^{\circ}-p}{p^{\circ}} = X_2 = \frac{\frac{w_2}{M_2}}{\frac{w_1}{M_1}+\frac{w_2}{M_2}} = \frac{w_2}{M_2} \times \frac{M_1}{w_1}$$ $\left(\text{In a dilute solution } \frac{w_2}{M_2} \text{ in the denominator can be neglected}\right)$

w_1 and w_2 are the weights in g of solvent and solute and M_1 and M_2 are molar masses.

3. (i) Depression of freezing point, $\Delta T_f = K_f m$

where ΔT_f is the decrease in freezing point, K_f is the freezing point depression constant for the solvent, m is the molality of the solution.

$$m = \frac{w_2}{M_2 M_1} \times 1000$$

(ii) Elevation of boiling point, $\Delta T_b = K_b m$

where K_b is the ebullioscopic constant of the solvent.

(iii) $K_f = \frac{RT^2}{1000\ \ell_f}$, ℓ_f = Latent heat of fusion per g.

(iv) $K_b = \frac{RT^2}{1000\ \ell_v}$, ℓ_v = Latent heat of evaporation per g at its normal boiling point.

Molecular weight of solute M_2

$$M_2 = \frac{1000\ K_b\ w_2}{w_1\ \Delta T_b}$$

$$M_2 = \frac{1000\ K_f\ w_2}{w_1\ \Delta T_f}$$

(v) Osmotic pressure, $\pi = cRT = \frac{n}{V} RT$

(where $\frac{n}{V}$ = concentration, c = concentration in molarity)

or $P = cRT$ or $\frac{P}{c} = RT$.

4. Relationship between different colligative properties

(i) Osmotic pressure (p) with relative lowering of vapour

$$p = \frac{p^0 - p}{p^0} \times \frac{dRT}{M_2} \quad (M_2 = \text{Mol. wt. of the solute})$$

(ii) Osmotic pressure (P) with elevation in boiling point (ΔT_b)

$$P = \Delta T_b \times \frac{dRT}{1000\,K_b}$$

(iii) Osmotic pressure (P) with depression in boiling point ΔT_b

$$P = \Delta T_f \times \frac{dRT}{1000\,K_f}$$

(iv) Elevation in boiling point with relative lowering of vapour pressure

$$\Delta T_b = \frac{1000\,K_b}{M_1}\left(\frac{p^\circ - p}{p^\circ}\right) \text{ (}M_1 = \text{Mol. wt. of solvent)}$$

(v) Depression in freezing point with relative lowering of vapour pressure

$$\Delta T_f = \frac{1000\,K_f}{M_1}\left(\frac{p^\circ - p}{p^\circ}\right)$$

5. (i) van't Hoff factor,

$$i = \frac{\text{Observed colligative property}}{\text{Theoretical colligative property}}$$

$$= \frac{\text{Theoretical molecular weight}}{\text{Observed molecular weight}}$$

(ii) For dissociation, degree of dissociation, $\alpha = \frac{i - 1}{n - 1}$

(iii) For association, $\alpha = \dfrac{i - 1}{\frac{1}{n} - 1}$

6. Modified expression for colligative properties using van't Hoff factor (i).

$$\frac{p^\circ - p}{p^\circ} = ix_2$$

$$\Delta T_b = i\,K_b\,m$$

$$\Delta T_f = i\,K_f\,m$$

$$\pi = i\,cRT$$

7. CHEMICAL EQUILIBRIUM

1. (i) Equilibrium constant

For the reaction

$$mA + nB \rightleftharpoons pC + qD,\ K_c = \frac{(c_C)^p(c_D)^q}{(c_A)^m(c_B)^n}$$

where c_A, c_B, c_C and c_D are moles/ℓ of A, B, C and D respectively.

$$mA(g) + nB(g) \rightleftharpoons pC(g) + qD(g)$$

$$K_p = \frac{(p_C)^p(p_D)^q}{(p_A)^m(p_B)^n}$$, where p = partial pressure, $K_p = K_c(RT)^{\Delta n}$

where $\Delta n = (p + q) - (m + n)$

= Number of moles of gaseous products – Number of moles of gaseous reactants.

(ii) $A + B \rightleftharpoons 2C$; equilibrium constant = K

$nA + nB \rightleftharpoons 2nC$; equilibrium constant = $(K)^n$

(iii) In any system at dynamic equilibrium, free energy change, $\Delta G° = 0$.

(iv) The free energy change, $\Delta G°$ and equilibrium constant, K are related as $\Delta G° = -RT\, \ell nK$.

(v) **van't Hoff equation:** This equation gives the variation of equilibrium constant with temperature. The equation is $\frac{d\ell n\, K_p}{dT} = \frac{\Delta H°}{RT}$

The integrated form of this equation is

$$\log\frac{K_2}{K_1} = \frac{\Delta H°}{2.303\,R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$$, where $\Delta H°$ = Enthalpy change for the reaction

(assumed to be constant over a temperature range $T_1 \rightarrow T_2$).

(vi) Calculation of degree of dissociation (α) from vapour density measurements:

$$\alpha = \frac{D - d}{d}$$

D = Theoretical vapour density; d = observed vapour density.

$$\alpha = \frac{M_t - M_o}{M_o}$$

where M_t = Theoretical (calculated) molecular weight

M_o = Observed (experimental) molecular weight

2. (i) **Ionic equilibrium: Ostwald's dilution law**

$$K_a = \frac{\alpha^2 C}{1 - \alpha}\ ;\ \alpha \propto \frac{1}{\sqrt{C}}$$

where K_a = Dissociation constant of a weak acid

α = Degree of dissociation at concentration C moles/L.

$$\left[\text{For a weak base , } K_b = \frac{\alpha^2 C}{1-\alpha} \text{ ; } \alpha \propto \frac{1}{\sqrt{C}}\right]$$

(ii) Relative Strengths of acids

$$\alpha = \sqrt{\frac{K_a}{C}} \text{ , } \left[H^+\right] = C\alpha = C \times \sqrt{\frac{K_a}{C}} = \sqrt{K_a C}$$

For two acids of equimolar concentration

$$\frac{\text{strength of acid } HA_1}{\text{strength of acid } HA_2} = \sqrt{\frac{K_{a(1)}}{K_{a(2)}}}, \text{ similarly } \frac{\text{strength of base } (BOH)_1}{\text{strength of base } (BOH)_2} = \sqrt{\frac{K_{b(1)}}{K_{b(2)}}}$$

(iii) Ionic product of water

$$C_{H^+} \times C_{OH^-} = K_w = 10^{-14} \text{ (at 25°C)}$$

For neutral solutions or pure water $C_{H^+} = 10^{-7}$ g ions per litre = C_{OH^-}

For acid solutions $C_{H^+} > 10^{-7}$ and for alkaline solutions $C_{H^+} < 10^{-7}$.

(iv) $pH = -\log C_{H^+}$ or $C_{H^+} = 10^{-pH}$

(v) pH of buffer solutions

Henderson-Hasselbach equation

$pH = pK_a + \log \frac{[salt]}{[acid]}$ for an acidic buffer

$(pK_a = -\log K_a)$

$pOH = pK_b + \log \frac{[salt]}{[base]}$ for a basic buffer

(vi) Buffer capacity $= \frac{\text{number of moles of the acid or base added to 1L of the buffer}}{\text{change in pH}}$

$$= \frac{n}{\Delta pH}$$

Common ion effect

By addition of X mole/L of a common ion, to a weak acid (or weak base) α becomes equal to $\frac{K_a}{X}\left(\text{or } \frac{K_b}{X}\right)$, the degree of ionisation of weak acid (or weak base) is decreased.

3. Solubility Product

$$A_xB_y \rightleftharpoons xA^{m+} + yB^{n-}$$

$$K_a = (C_Am+)^x\,(C_Bn-)^y$$

If s is the solubility in moles/L

$$K_a = (xs)^x\,(ys)^y$$

The ionic product (IP) (Product of the concentration for ions, each concentration being raised to proper power) in a saturated solution of the sparingly soluble salt = solubility product (SP)

IP > SP : Precipitation occurs

IP = SP : Solution is saturated

IP < SP : Solution is unsaturated

Common ion effect: By the addition of x moles/L of a common ion, the solubility of the sparingly soluble salt becomes equal to $\frac{K_S}{x}$ (decreased).

4. Hydrolysis of salts

(a) Salt of a weak acid and strong base

$$K_h = \frac{\alpha^2 c}{1-\alpha}$$

where K_h = hydrolysis constant,

α = degree of hydrolysis,

c = concentration of salt in moles/ℓ,

$$\alpha = \sqrt{\frac{K_h}{c}},$$

degree of hydrolysis $h = \sqrt{\frac{K_w}{K_a c}}$,

$$K_h = \frac{K_w}{K_a} \text{ and}$$

$$pH = 0.5\,(pK_w + pK_a + \log c)$$

(b) Salt of a weak base and strong acid

$$K_h = \frac{\alpha^2 c}{1-\alpha}$$

$$\alpha = \sqrt{\frac{K_h}{c}}$$

$$h = \sqrt{\frac{K_w}{K_b c}}$$

$$K_h = \frac{K_w}{K_b}$$

$$pH = 0.5\,(pK_w - pK_b - \log c)$$

(c) Salt of a weak acid and weak base

$$K_h = \frac{\alpha^2}{1-\alpha}\ ;\ \alpha = K_h$$

$$h = \sqrt{\frac{K_w}{K_a K_b}}$$

$$K_h = \frac{K_w}{K_a \cdot K_b}$$

$$pH = 0.5\,(pK_w + pK_a - pK_b)$$

8. ENERGETICS

1. Mathematical formula of first law:

$$\Delta E = q + w$$

2. Expression for work of expansion:

$$w = -P\Delta V$$

(a) Work done in isothermal, reversible expansion of an ideal gas,

$$w = -2.303\ nRT \log \frac{V_2}{V_1} \text{ or}$$

$$w = -2.303\ nRT \log \frac{P_1}{P_2}$$

(– ve sign is used as it is expansion i.e., work done by the system)

(b) $\Delta E = \dfrac{Z \times \Delta T \times M}{m}$ where Z = heat capacity of the calorimeter

ΔT = rise in temperature, M = molecular weight; m = weight of the substance

(c) $\Delta H = Z \times \Delta T \times \dfrac{M}{m}$

3. Application of first law to different types of processes

(i) For an isochoric process; $\Delta V = 0$; $\Delta E = q - P\Delta V$

$$\therefore \quad \Delta E = q_v$$

(ii) For an isobaric process : $\Delta E = q_p - P\Delta V$;

$$q_p = \Delta E + P\Delta V = \Delta H$$

(iii) For an adiabatic process:

Total work done (w) = $\int_{T_1}^{T_2} n\ C_v dT = nC_v\ (T_2 - T_1)$

$$w = \frac{nR(T_2 - T_1)}{\gamma - 1}$$

where $\gamma = \frac{C_p}{C_v}$

(iv) For adiabatic reversible expansion of gas

$$PV^{\gamma} = \text{constant}$$

$$TV^{\gamma - 1} = \text{constant}$$

$$TP^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

(v) Entropy change of an ideal gas in an isobaric process

$$\Delta S = 2.303\ C_p \log \frac{T_2}{T_1}$$

(vi) Entropy change of an ideal gas in an isochoric process

$$\Delta S = 2.303\ C_v \log \frac{T_2}{T_1}$$

(vii) Third law of Thermodynamics

$$\Delta S = \int_0^T C_p d\ \ell nT$$

$$\Delta S = 2.303\ C_p \log T$$

(viii) $\Delta S^\circ = \Sigma$ entropies of products $(S_P^\circ) - \Sigma$ entropies of reactants (S_R°)

(a) **(i)** Gibb's Free Energy (G) = $G = H - TS$

(ii) $\Delta G = \Delta H - T\Delta S$ (Gibb's Helmholtz equation)

(iii) $(-\Delta G)_{T,p} = T\Delta S_{Total}$

(iv) $\Delta G^\circ = -2.303\ RT \log K$ (K = equilibrium constant)

(v) $\Delta G^\circ = -nFE^\circ_{cell}$ (E°_{cell} = Standard e.m.f. of the galvanic cell).

(vi) ΔG° for a reaction:

$\Delta G^\circ = \sum$ Free energy of products $- \sum$ Free energy of reactants

$= \sum$ Standard free energy of formation of products $- \sum$ Standard free energy of formation of reactants

(viii) $-\Delta G = W_{useful}$

4. Relationship between ΔH and ΔE for a gaseous reaction

$$\Delta H = \Delta E + \Delta nRT$$

Δn = no. of mol. of gaseous products – no. of mol. of gaseous reactants.

5. $\Delta H = \sum H_P^\circ - \sum H_R^\circ$

$= \sum$ enthalpy of formation of products $- \sum$ enthalpy of formation of reactants

$= \sum$ enthalpy of combustion of reactants $- \sum$ enthalpy of combustion of products

$= \sum$ bond energy of reactants $- \sum$ bond energy of the products.

6. Variation of enthalpy change or internal energy change with temperature

$$\Delta C_v = \left(\frac{\partial(\Delta E)}{\partial T}\right)_V = \frac{\Delta E_2 - \Delta E_1}{T_2 - T_1} = \Delta C_v$$

$$\Delta C_p = \left(\frac{\partial(\Delta H)}{\partial T}\right)_P = \frac{\Delta H_2 - \Delta H_1}{T_2 - T_1} = \Delta C_p \text{ or } (\Delta H_2 - \Delta H_1) = \Delta C_p(T_2 - T_1)$$

ΔC_p = molar heat capacity of the products – molar heat capacity of the reactants

7. Trouton's rule: $\dfrac{\Delta H_{Vapourisation}}{T_b} = 88\ \text{Jmol}^{-1}\ \text{K}^{-1}$

where T_b is the normal boiling point of the liquid.

8. (i) Entropy change $(\Delta S) = \dfrac{q_{rev}}{T}$

(ii) Entropy of fusion $\Delta S_f = \dfrac{\Delta H_f}{T_f}$

(iii) Entropy of vaporisation $(\Delta S_v) = \dfrac{\Delta H_{vap}}{T}$

9. Efficiency of heat engine $(\eta) = \frac{w}{q_2} = \frac{T_2 - T_1}{T_2} = \frac{q_2 - q_1}{q_2}$

where q_2 = heat absorbed by the system at temperature T_2 of the source

q_1 = heat rejected by the system at temperature T_1 of the sink

w = net work done

10. **Criteria of Spontaneity:**

$(dS)_{E,V} \geq 0$ or $(dG)_{T,P} \leq 0$ or $(dE)_{S,V} \leq 0$ or $(dH)_{S,P} \leq 0$

ΔG = negative ; Spontaneous

$\Delta G = 0$; System is at equilibrium, ΔG = positive ; non-spontaneous

9. CHEMICAL KINETICS

1. **Order of a reaction:** The sum of the indices of the concentration terms in the rate equation. It is an experimental value. It can be zero, fractional or whole number.
2. **Molecularity of a reaction:** The number of molecules involved in the rate determining step of the reaction. It is a theoretical value always a whole number.
3. **Expressions for rate constant for reaction of different orders:**

Reaction	Order	Rate Law	Expression for rate constant	Unit of k
1. A → Products	0	rate = $k[A]^0$	$k = \frac{1}{t}\left[[A]_0 - [A]\right]$	mol L^{-1} $time^{-1}$
2. A → Products	1	rate = $k[A]$	$k = \frac{2.303}{t} \log \frac{[A]_0}{[A]}$	$time^{-1}$
3. 2A → Products	2	rate = $k[A]^2$	$k = \frac{1}{t}\left[\frac{1}{[A]} - \frac{1}{[A]_0}\right]$	L mol^{-1} $time^{-1}$
4. A + B → Products	2	rate = $k[A][B]$	$k = \frac{2.303}{t\left[[A]_0 - [B]_0\right]} \log \frac{[B_0][A]}{[A]_0[B]}$	L mol^{-1} $time^{-1}$
5. 3A → Products	3	rate = $k[A]^3$	$k = \frac{1}{2t}\left[\frac{1}{[A]^2} - \frac{1}{[A]_0^2}\right]$	L^2 mol^{-2} $time^{-1}$

$[A]_0$ = initial concentration of reactant A

$[B]_0$ = initial concentration of reactant B

$[A]$ = concentration of reactant at time t

$[B]$ = concentration of reactant at time 't'

4. Units of k(rate constant) $= \frac{1}{\text{time}} \cdot \frac{1}{(\text{concentration})^{n-1}}$

where n = order of reaction

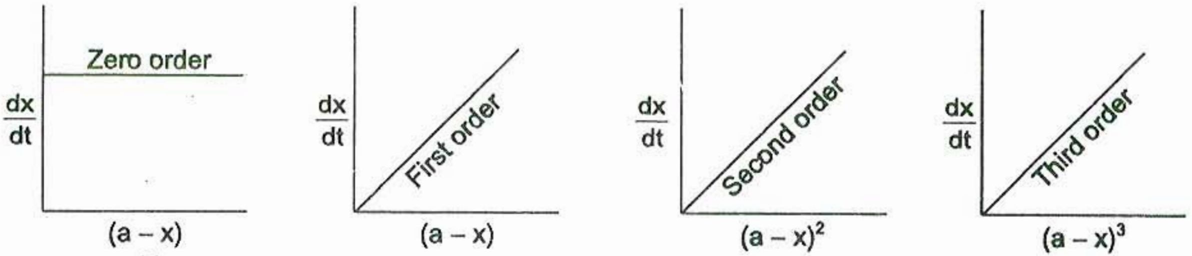
For a zero order reaction the units of k is **mol L^{-1} time^{-1}**

For a first order reaction, unit of k is **time^{-1}**

For a second order reaction, unit of k is **L mol^{-1} time^{-1}**

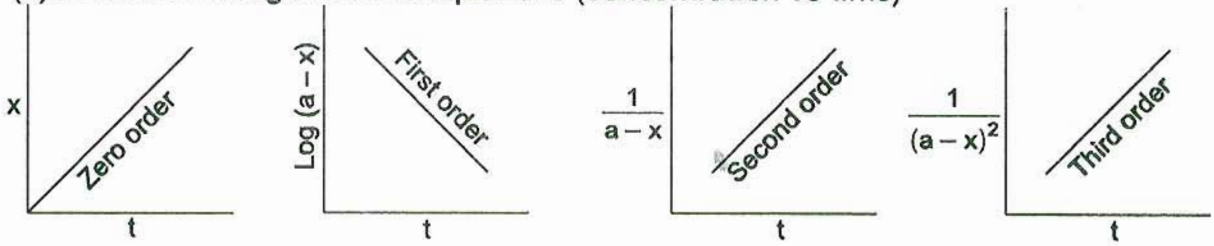
5. Some typical linear plots for reactions of different orders

(a) Plots of rate vs concentration (Rate = k (concentration)^n)



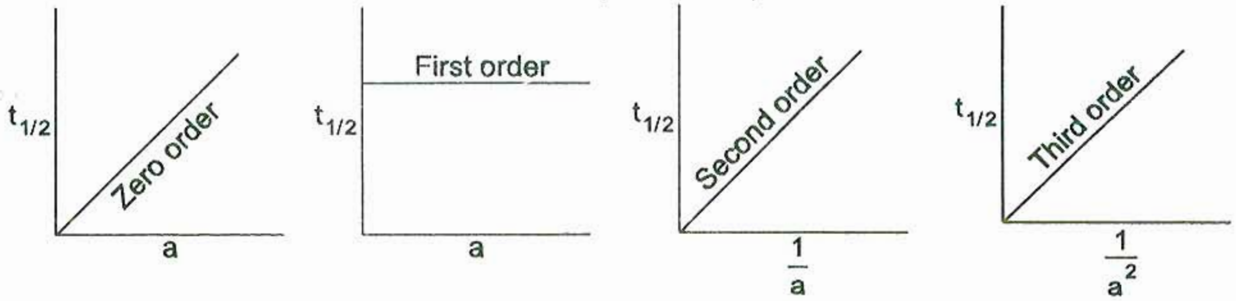
(i) (ii) (iii) (iv)

(b) Plots from integrated rate equations (concentration vs time)



(1) (2) (3) (4)

(c) Plots of half-lives vs concentration $\left(t_{\frac{1}{2}} \propto a^{1-n}\right)$



In those plots a = initial concentration of the reactant

x = amount of product formed at time 't'

a – x = amount of reactant remain at time t

6. Exponential form of expression for the rate constant (k) for first order reaction

$= [A] = [A_0]\, e^{-kt}$ or

$C_t = C_0\ e^{-kt}$

7. General expression for time taken for n^{th} fraction of a first order to complete

$$t_{1/n} = \frac{2.303}{k} \log \frac{n}{n-1}$$

8. Expression for $t_{1/2}$ for first order reaction

$$t_{1/2} = \frac{0.693}{k}$$

9. Expression for $t_{1/2}$ for nth order reaction

$$t_{\frac{1}{2}} \propto [A_0]^{1-n} \quad \text{or} \quad t_{\frac{1}{2}} \propto \frac{1}{[A_0]^{n-1}}$$

10. General expression for the amount of the substance left after 'n' half-lives: (for a first order reaction)

(a) after one half-life, amount left = $\frac{[A]_0}{2}$

(b) after two half-lives, amount left = $\frac{1}{2} \times \frac{[A_0]}{2} = \frac{[A_0]}{2^2}$

(c) after three half-lives, amount left = $\frac{1}{2}\frac{[A_0]}{2^2} = \frac{[A_0]}{2^3}$

In general, amount left after 'n' half-lives = $\frac{[A_0]}{2^n}$

$\therefore$ no. of half-lives = $\frac{\text{Total time}}{t_{\frac{1}{2}}}$

11. Calculation of order of reaction from two different half-lives:

Let a_1 be the initial concentration and the half-life be $\left(t_{1/2}\right)_1$ and a_2 be another different initial concentration of the same substance and $\left(t_{1/2}\right)_2$ be its half-life. Then

$$\boxed{n = 1 + \frac{\log\left(t_{1/2}\right)_1 - \log\left(t_{1/2}\right)_2}{\log a_2 - \log a_1}}$$

12. Expression for the degree of dissociation $\left(\frac{x}{a}\right)$ of a reactant dissociating according to first order kinetics,

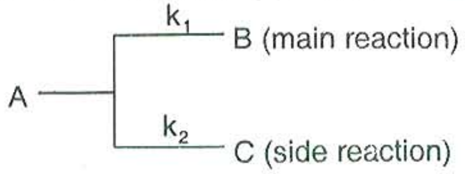
$$\frac{x}{a} = 1 - e^{-kt} \text{ or}$$

degree of dissociation at time t = $1 - e^{-kt}$

13. Modified expressions for rate constants of some common first order reactions:

	Reaction	Expression for rate constant
(i)	$N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$	$k = \frac{2.303}{t} \log \frac{V_\infty}{V_\infty - V_t}$ V_∞ = volume of O_2 collected after infinite time V_t = volume of O_2 collected at time 't'
(ii)	$NH_4NO_2 \longrightarrow 2H_2O + N_2$	$k = \frac{2.303}{t} \log \frac{V_\infty}{V_\infty - V_t}$ (V_t and V_∞ are the volume of nitrogen collected at time 't' and infinite time respectively)
(iii)	$H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$	$k = \frac{2.303}{t} \log \frac{V_0}{V_t}$ where V_0 and V_t are the volume of $KMnO_4$ used for the same volume of reaction mixture at t = 0; and after time t = t
(iv)	$CH_3COOC_2H_5 + H_2O \xrightarrow{H^+}$ $CH_3COOH + CH_3CH_2OH$	$k = \frac{2.303}{t} \log \frac{V_\infty - V_0}{V_\infty - V_t}$ (V_0, V_t and V_∞ are the volumes of sodium hydroxide solution used for titration of same volume of reaction mixtures after times, 0, t and ∞ respectively)
(v)	$C_{12}H_{22}O_{11} + H_2O \xrightarrow{H^+}$ $C_6H_{12}O_6$ + $C_6H_{12}O_6$ glucose fructose (inversion of cane sugar)	$k = \frac{2.303}{t} \log \frac{r_0 - r_\infty}{r_t - r_\infty}$
(vi)	$C_6H_5N_2Cl \longrightarrow C_6H_5Cl + N_2$	$k = \frac{2.303}{t} \log \frac{V_\infty}{V_\infty - V_t}$ V_t and V_∞ are the volumes of N_2 gas collected at time 't' and ∞ respectively

14. Rate law for reactions involving parallel or side reactions, both following first order kinetics



$$\text{rate} = -\frac{d[A]}{dt} = k_1\,[A] + k_2\,[A]$$

$$\text{rate} = (k_1 + k_2)\,[A]$$

15. Rate law for reversible reaction

$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B$$

$$\text{rate} = -\frac{d[A]}{dt} - \frac{d[B]}{dt}$$

$$= k_1[A] - k_2[B]$$

16. Rate constant (k) according to collision theory

$$k = PZe^{-E/RT}$$

P = Steric factor; Z = Collision frequency

i.e., Total number of collisions per second per unit volume; $e^{-E/RT}$ = fraction of molecules with energy greater than threshold energy.

17. (a) Effect of temperature on reaction rate : (Arrhenius equation)

$$k = Ae^{-E_a/RT}$$

A = frequency factor; E_a = energy of activation; R = gas constant; T = temperature.

(b) Logarithmic form of Arrhenius equation

$$\ell n\ k = \ell n\ A - \frac{E_a}{RT} \text{ or}$$

$$\log k = \log A - \frac{E_a}{2.303\ RT}$$

(c) Integrated form of Arrhenius equation

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303\ R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$$

(d) Another form of Arrhenius equation

$$\frac{d\ell n k}{dt} = \frac{E}{RT^2}$$

18. Quantum yield or Quantum efficiency (ϕ) of a photochemical reaction:

$$\phi = \frac{\text{number of molecules reacting in a given time}}{\text{number of quanta of light absorbed at the same time}}$$

10. ELECTROCHEMISTRY

1. Conductance (C) = $\frac{1}{\text{Resistance}} = \frac{1}{R}$

Units = ohm^{-1} or Ω^{-1} or mhos.

2. $R \propto \frac{\ell}{a}$ where 'ℓ' is the length of the conductor and 'a' is the area of cross-section

3. Specific resistance or resistivity (ρ), $R = \rho \times \frac{\ell}{a}$ or $\rho = R \times \frac{a}{\ell}$

4. Cell constant = $\frac{\ell}{a} = \frac{\text{distance between the electrodes}}{\text{surface area of the electrodes}}$

5. Specific conductivity or conductivity (κ)= $\frac{1}{\rho}$

$$\kappa = \frac{1}{R} \times \frac{\ell}{a} = \frac{\text{Cell constant}}{\text{Resistance}}$$

Units of κ = ohm^{-1} cm^{-1}

6. Relationship between equivalent conductance (Λ) and specific conductance:

$\Lambda = \kappa \times V$ where 'V' is the volume containing one gram equivalent of the electrolyte.

(a) If 'C' gm equivalents present in 1000 cm^3

$$\Lambda = \frac{\kappa \times 1000}{C} = \frac{\kappa \times 1000}{\text{Normality}}$$

(b) If 'C' gm equivalents present in 1 dm^3 or 1 L

$$\Lambda = \frac{\kappa}{C}$$

(c) If 'c' gm equivalents are present in 10^{-3} m^3, then $\Lambda = \frac{\kappa}{1000\ C}$

7. For a uni-univalent electrolyte molar conductivity and equivalent conductance are the same.

8. Debye-Huckel-Onsager equation. (For a uni-univalent electrolyte)

$$\Lambda_m = \Lambda_m^{\circ} - b\sqrt{c}$$

Λ_m = molar or equivalent conductance

Λ_m° = molar or equivalent conductance at infinite dilution

b = constant, depending upon the nature of the solvent and temperature

c = concentration of the electrolyte in mol L^{-1} or gm eq.L^{-1}

9. Kohlrausch's Law: (mathematical form)

$$\Lambda_m^\circ = x\lambda_A^\circ + y\lambda_B^\circ$$

Λ_m° = molar conductivity at infinite dilution

x = no. of cations produced

y = no. of anions produced

λ_A° = ionic conductance of the cation

λ_B° = ionic conductance of the anion

10. Application of Kohlrausch's law

(i) Calculation of Λ_m° of weak electrolytes: e.g., CH_3COOH

$$\Lambda_m^\circ = \lambda^\circ_{CH_3COO^-} + \lambda^\circ_{H^+}$$

$$\Lambda^\circ_{m\,(CH_3COOH)} = \Lambda^\circ_{(CH_3COONa)} + \Lambda^\circ_{(HCl)} - \Lambda^\circ_{(NaCl)}$$

(ii) α (degree of dissociation of the weak electrolyte) = $\dfrac{\Lambda^\circ_C}{\Lambda^\circ_\infty}$

$$\alpha = \frac{\text{molar conductivity at a given concentration}}{\text{molar conductivity at inifnite dilution}}$$

(III) Solubility of a sparingly soluble salt (s)

$$S(\text{mol L}^{-1}) = \frac{\kappa \times 1000}{\Lambda_m^\circ}$$

11. Representation of a galvanic cell and construction of a galvanic cell

(i) Metal | electrolyte (1) || electrolyte (2) | Metal

(Anode)	(Cathode)
(LHS electrode)	(RHS electrode)
Low reduction potential value	High reduction potential value

(ii) E.M.F of the cell (E_{cell}) = Reduction potential of RHS electrode – Reduction potential of LHS electrode

(iii) EMF of standard hydrogen electrode, $H^+ + 1e \rightleftharpoons \frac{1}{2} H_2$; $E^\circ = 0$ V

12. Nernst equation for electrode potential

$$M^{n+} + ne \longrightarrow M$$

$$E_{el} = E^\circ_{el} - \frac{0.059}{n} \log \frac{1}{[M^{n+}]} \text{ or}$$

$$E_{el} = E^\circ_{el} + \frac{0.059}{n} \log [M^{n+}]$$

13. Nernst equation for the cell reaction

$$xA + yB \rightleftharpoons \ell C + mD$$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n} \log \frac{[C]^{\ell}\,[D]^{m}}{[A]^{x}\,[B]^{y}} \text{ or}$$

$$E_{cell} = E^{\circ}_{cell} + \frac{0.059}{n} \log \frac{[A]^{x}\,[B]^{y}}{[C]^{\ell}\,[D]^{m}}$$

Note: Molar concentration of pure solids, pure liquids and gases at one atmospheric pressure are equal to one.

14. Relationship between E°_{cell} and equilibrium constant

$$E^{\circ}_{cell} = \frac{0.059}{n} \log K, \text{ where K is the equilibrium constant}$$

15. Relationship between free energy change and e.m.f. of cell

$$-\Delta G = nFE_{cell} \text{ or}$$

$$-\Delta G^{\circ} = nFE^{\circ}_{cell}$$

16. Predicting the relative oxidising and reducing powers from the values of standard reduction potential values

Greater the reduction potential, greater is the ability of the species to get reduced and hence is a stronger oxidising agent.

Lesser the reduction potential, greater is the ability of the species to get oxidised and hence is a stronger reducing agent.

17. EMF of a concentration cell

$$E = \frac{0.059}{n} \log \frac{c_2}{c_1}$$

where c_2 is the concentration of the electrolyte in RHS electrode and c_1 is the concentration of the electrolyte in LHS electrode.

18. (a) Calculation of pH of an electrolyte by e.m.f measurement using a hydrogen electrode

$$E_{H^+/H_2} = -0.059 \text{ pH}$$

(b) Calculation of pH of an electrolyte by using a calomel electrode

$$pH = \frac{E_{cell} - 0.2415}{0.0591}$$

(c) Calculation of pH of an electrolyte by using a quinhydrone electrode

$$E_{cell} = 0.6996 - 0.059 \text{ pH}$$

19. Faraday's Laws of electrolysis

(1) I law of Faraday $W = eIt$

where W = amount of the substance liberated at the electrode,

e = electrochemical equivalent of the substance,

I = current strength in amperes and

t = time in seconds.

(2) II law of Faraday

The amount of substances liberated at the electrodes are proportional to their chemical equivalent, when the same quantity of current is passed through different electrolytes. 1 gram equivalent weight of the element will be liberated by passage of 96500 coulombs of electricity.

Combining the two laws, we have

$$w = \frac{ItE}{96500}$$

where w = weight in g of any material deposited at an electrode,

I = current strength in ampere,

t = time in seconds and

E = gram equivalent of the substance deposited.

20. Z (electrochemical equivalent) $= \frac{\text{Eq.wt of substance}}{96500}$

21. Predicting products of electrolysis:

(a) During the electrolysis of an aqueous solution of an electrolyte, if the reduction potential of the cation is higher than H_2O (– 0.83 V), the cation (M^{n+}) is reduced/liberated at cathode. (e.g., Cu^{+2}, Ag^+ ions), otherwise H_2 gas is liberated due to the reduction of H_2O.

(b) If the anion has a lower reduction potential than H_2O (– 1.23), the anion is liberated (e.g. Br^-), otherwise, water is oxidised to O_2 gas.

(c) The products of electrolysis may also be predicted based on the minimum voltage required to discharge an ion (discharge potential). The discharge potential of certain cations and anions is of the order

$Li^+ < K^+ < Na^+ < Ca^{+2} < Mg^{+2} < Al^{+3} < Zn^{+2} < Fe^{+2} < Ni^{+2} < H^+ < Cu^{+2} < Hg^{+2} < Ag^2 < Au^{+3} < SO_4^{-2} < NO_3^- < OH^- < Br^- < I^-$

11. SOLID STATE

1. Crystal Systems and their Maximum Symmetry Elements

System	Maximum Symmetry Elements	Examples
1. Cubic or Regular	Nine planes Thirteen axes	NaCl, KCl, CaF_2, ZnS, Cu_2O, Diamond, Alums, Pb, Ag, Au, Hg
2. Ortho-rhombic (or Rhombic)	Three planes Three axes	KNO_3, K_2SO_4, $BaSO_4$, $PbCO_3$, Mg_2SiO_4, Rhombic sulphur
3. Tetragonal	Five planes Five axes	SnO_2, TiO_2, $ZrSiO_4$, KH_2PO_4, $PbWO_4$, Sn
4. Monoclinic	One plane One axis	$Na_2SO_4 \cdot 10H_2O$, $Na_2B_4O_7 \cdot 10H_2O$, $CaSO_4 \cdot 2H_2O$, Monoclinic sulphur
5. Triclinic	No planes No axes	$CuSO_4 \cdot 5H_2O$, $K_2Cr_2O_7$, H_3BO_3
6. Hexagonal	Seven planes Seven axes	ZnO, CdS_2, HgS, Graphite, Ice, PbI_2, Beryl, Mg, Zn, Cd
7. Rhombohedral (or Trigonal)	Seven planes Seven axes	$NaNO_3$, ICl, Calcite, Magnesite, Quartz, As, Sb, Bi

2. The Seven Crystal Systems

Crystal System	Bravais Lattices	Parameters of Unit Cell	
		Intercepts	Crystal Angles
1. Cubic	$\left.\begin{array}{l}\text{Primitive}\\ \text{Face-centred,}\\ \text{Body-centred}\end{array}\right\} = 3$	$a = b = c$	$\alpha = \beta = \gamma = 90°$
2. Ortho-rhombic	$\left.\begin{array}{l}\text{Primitive}\\ \text{Face-centred}\\ \text{Body-centred}\\ \text{End-centred}\end{array}\right\} = 4$	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$
3. Tetragonal	$\left.\begin{array}{l}\text{Primitive,}\\ \text{Body-centred}\end{array}\right\} = 2$	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$
4. Monoclinic	$\left.\begin{array}{l}\text{Primitive,}\\ \text{End-centred}\end{array}\right\} = 2$	$a \neq b \neq c$	$\alpha = \gamma = 90°$ $\beta \neq 90°$
5. Triclinic	Primitive = 1	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$
6. Hexagonal	Primitive = 1	$a = b \neq c$	$\alpha = \beta = 90°$ $\gamma = 120°$
7. Rhombohedral	Primitive = 1	$a = b = c$	$\alpha = \gamma = 90°$ $\beta \neq 120°$
	Total = 14		

3. Relationship between radius of void (r) and the radius of the sphere (R):

r (tetrahedral) = 0.225 R

r (octahedral) = 0.414 R

4. Relation between nearest neighbour distance (d) and edge length (a) of cubic unit cell

Simple cubic	b.c.c	f.c.c
$d = a$	$d = \frac{\sqrt{3}a}{2}$	$d = \frac{a}{\sqrt{2}}$

5. Relation between atomic radius (r) and edge length (a) in case of pure elements having cubic unit cell

Simple cubic	b.c.c	f.c.c
$r = \frac{a}{2}$	$r = \frac{\sqrt{3}\,a}{4}$	$r = \frac{a}{2\sqrt{2}}$

(For pure elements r = d/2)

6. Relationship between radius ratio, coordination number and structural arrangement

Radius ratio $\left(\frac{r^+}{r^-}\right)$	**Co-ordination no. of cations**	**Arrangement**
upto 0.15	2	Linear
0.15 to 0.23	3	Triangular
0.23 to 0.41	4	Tetrahedral
0.41 to 0.73	6	Octahedral
greater than 0.73	8	Cubic

7. Expression for density (ρ) of a cubic crystal in terms of its edge.

$$\rho = \frac{nM}{a^3N_o}$$

where ρ = Density

M = Molar mass

a = Side length

N_o = Avogadro number

n = Number of species = 1 for simple cube

= 2 for BCC

= 4 for FCC

Superconductivity: A superconductor is a material that loses abruptly its resistance to the electric current when cooled to a specific characteristic temperature. Superconductors are non-stoichiometric compounds consisting of rare earthen silicates with transition metal oxides.

II. INORGANIC CHEMISTRY

1. PERIODIC CLASSIFICATION AND ELECTRONIC CONFIGURATION

(i) Long form of the periodic table: IUPAC has recommended a revised form of periodic table with the groups numbered from 1 to 18.

(ii) Quantum numbers are used to label the various energy levels of the electrons.

(iii) Pauli's exclusion principle : No two electrons in an atom can have all the four quantum numbers identical.

(iv) Hund's rule of maximum multiplicity – pairing of electrons does not take place in any p, d or f-orbital until each such orbital contains atleast one electron each.

(v) Aufbau principle – building up.

(vi) $n + l$ rule : The sum of quantum numbers $n + l$ determines the energy of the orbital.

(vii) Elements are classified into four blocks as s, p, d and f based on the type of subshell into which the differentiating electron enters.

(viii) Except zero group elements, all other s and p-block elements are called representative elements.

(ix) The general electronic configuration of

(a) d-block elements is $ns^{1-2}(n-1)d^{1-10}$. (Transition elements)

(b) f-block elements is $ns^{2}(n-1)d^{0-1}(n-2)f^{1-14}$. (Inner-transition elements)

(a) Atomic radius

(i) Order of radius: van der Waals' radius > metallic radius > covalent radius

(ii) Cation radius is always less and anion radius is always greater than its parent neutral atom.

(iii) In the case of isoelectronic species, when the number of protons increases, radii decreases.

(b) Ionisation potential

1. Ionisation potential decreases

   (i) when atomic size increases

   (ii) screening effect increases

   (iii) in the group from top to bottom

2. Ionisation potential increases

   (i) when nuclear charge increases

   (ii) in the period from left to right

   (iii) when the element has half-filled or completely filled sub-shells

3. Order of ionisation potential is $I_1 < I_2 < I_3 < \ldots\ldots < I_n$ always

(c) Electron affinity

(1) Second electron affinity is always negative.

(2) Electron affinity of a neutral atom is equal to ionisation potential of its anion.

(3) For inert gases and for atoms having filled orbitals, electron affinity is zero.

(4) Electron affinity of chlorine is greater than fluorine (small atomic size).

(d) Electronegativity

(1) The commonly accepted electronegativity values are obtained as follows

$$E_N = \frac{I(\text{kJ mol}^{-1}) + E_a(\text{kJ mol}^{-1})}{544}$$

$$E_N = \frac{I(\text{kcal mol}^{-1}) + E_a(\text{kcal mol}^{-1})}{125}$$

(2) If the electronegativity difference between two atoms is greater than 1.9, the bond is ionic bond.

(e) Oxidation state

(1) Oxidation state of s-block elements is equal to group number.

(2) p-block elements show multivalency, the oxidation state change by two numbers. (8-Group number)

(3) The common oxidation state of d-block elements is +2. All transition elements show variable oxidation states.

(4) The common oxidation state of f-block elements is +3.

(5) No element exceeds its group number in the oxidation state.

(6) Ruthenium and osmium exhibit maximum oxidation state + 8.

(7) The most electronegative element fluorine show – 1 oxidation state only.

(f) Diagonal relationship

(1) The first element of a group has similar properties with the second element of the next group. This is called diagonal relationship.

(2) It is due to similar sizes of atoms or ions and electronegativities of the participating elements.

(3) The diagonal relationship disappears after IV group.

2. TYPE OF COMPOUNDS

(a) Fajan's rules explain the nature of compound, (whether it is ionic or covalent) as follows

(1) A compound is more ionic (or) less covalent if it contains

(i) large cation and small anion

(ii) cations having inert gas configuration

(2) A compound is less ionic (or) more covalent if it contains

(i) small cation and large anion

(ii) cations having pseudo inert gas configuration (a cation with 18-electron valence shell configuration).

(b) According to VSEPR theory

(i) The order of repulsion between electron pairs on central atom follows the order.

Lone pair – lone pair > lone pair – bond pair > bond pair – bond pair

(ii) The number of lone pairs of electrons and bond pairs of electrons (only σ-electrons) on central atom determines the shape of molecule or ion.

(c) Bond order = $\frac{N_b - N_a}{2}$

N_b = Total number of electrons in all the bonding molecular orbitals

N_a = Total number of electrons in all the anti-bonding molecular orbitals.

(d) Hydrides

(1) ionic hydrides (salt-like)

(i) Group 1 metals form ionic hydrides $M^+ H^-$ and the heavier Group 2 metals form hydrides like MH_2.

(ii) They are powerful reducing agents

(2) Covalent hydrides (molecular)

Hydrides of the p-block element are covalent.

(3) Metallic (or interstitial Hydrides)

(i) Many of the elements in the d-block and f-block form metallic hydrides.

(ii) In many cases, the compounds are non-stoichiometric, where the chemical composition is variable. Such compounds are called interstitial hydrides.

(e) Oxides

(i) Normal oxides: The oxidation number of M can be deduced from the empirical formula M_xO_y.

(ii) Peroxides: It contain O_2^{2-} peroxide ion and (O – O) linkage.

(iii) Superoxide: It contains more oxygen than would be expected.

(iv) Suboxides: It contains less oxygen than would be expected from the oxidation number of M and M – M bonds in addition to M – O bonds.

(v) Basic oxides are metallic oxides and are usually ionic.

(vi) Nonmetallic oxides are acidic and covalent.

(vii) Many metals yield oxides which are amphoteric.

(viii) A few covalent oxides are neutral.

(f) Carbides

(i) Carbides are of three main types

(1) ionic or salt-like

(2) interstitial or metallic

(3) covalent

(ii) Interstitial carbides are formed mostly by transition elements and some of the rare earths and actinides.

(g) **(i)** About 95% of the earth's crust is composed of silicate materials.

(ii) Granite, mica, clay, slates, bricks, cement, ceramics, glass .... are silicates.

(h) Organic or inorganic compounds containing noble gas atoms trapped in their crystal structures are called clathrate compounds.

3. EXTRACTIVE METALLURGY

(i) A mineral is a naturally occurring inorganic compound with a definite crystalline structure. An ore is a rock or a mineral from which a metal or an element can be produced economically.

(ii) Gangue: The worthless silicate portion of an ore is called gangue. Flux is a chemical compound used for removing non-fusible impurities from the ore.

(iii) Froth floatation is a physical method of separating a mineral from the gangue depending on differences in their wettabilities by a liquid solution.

(iv) Roasting is the process of heating a mineral in the presence of air.

(v) Calcination is the process of heating the ore in the absence of air.

(vi) Smelting is the process by which the metal is extracted in the fused state.

(vii) Carbon reduction process is the one by which the oxide is reduced to the metal by coal or coke.

(viii) Self-reduction is the process by which a mixture of the oxide and sulphide is heated together to get the metal without the addition of a reducing agent.

(ix) Electrolytic reduction: Highly electropositive metals are extracted by the electrolysis of their oxides, hydroxides or oxides.

4. GRADATION IN PROPERTIES OF ELEMENTS AND THEIR COMPOUNDS (GROUP WISE)

Table 1

Comparison of properties of compounds of Alkali and Alkaline earth metals

	Property	Alkali metals	Alkaline earth metals
1.	Stability of carbonates	$Li_2CO_3 < Na_2CO_3 < K_2\ CO_3 < Rb_2CO_3 < Cs_2\ CO_3$	$BeCO_3 < MgCO_3 < CaCO_3 < Sr\ CO_3 < BaCO_3$ (Sulphates also follow the same order)
2.	Solubility of carbonates and bicarbonates	$Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3 < Cs_2CO_3$ $LiHCO_3 < NaHCO_3 < KHCO_3 < RbHCO_3 < CsHCO_3$	$BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$
3.	Solubility and basic strength of oxides	$Li_2O < Na_2O < K_2O < Rb_2O < Cs_2O$	$BeO < MgO < CaO < SrO < BaO$
4.	Solubility, stability and basic strength of hydroxides	$LiOH < NaOH < KOH < RbOH < CsOH$	$Be(OH)_2 < Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$
5.	Solubility of sulphates	Li_2SO_4 is insoluble	$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$
6.	Melting point of halides	$MF > MCl > MBr > MI$	$MF_2 > MCI_2 > MBr_2 > MI_2$

Table 2

Boron Family

	Property	Gradation
1.	Stability of + 3 oxidation state	B > Al > Ga > In > Tl
2.	Stability of + 1 oxidation state	Ga < In < Tl
3.	Reducing nature	Al > Ga > In > Tl
4.	Basic nature of the oxides and hydroxides	B < Al < Ga < In < Tl
5.	Relative strength of Lewis acid	$BF_3 < BCl_3 < BBr_3 < BI_3$

Table 3

Carbon Family

	Property	Gradation
1.	Reactivity	C < Si < Ge < Sn < Pb
2.	Metallic character	C < Si < Ge < Sn < Pb
3.	Acidic character of the oxides	$CO_2 > SiO_2 > GeO_2 > SnO_2 > PbO_2$ Weaker acidic (amphoteric)

	Property	**Gradation**
4.	Thermal stability and volatility of hydrides	$CH_4 > SiH_4 > GeH_4 > SnH_4 > PbH_4$
5.	Reducing nature of hydrides	$CH_4 < SiH_4 < GeH_4 < SnH_4 < PbH_4$
6.	Thermal stability of tetrahalides	$CCl_4 > SiCl_4 > GeCl_4 > SnCl_4 > PbCl_4$
7.	Thermal stability and volatility of tetrahalide with a common central atom	$MF_4 > MCl_4 > MBr_4 > MI_4$
8.	Oxidising character of M^{+4} species	$GeCl_4 < SnCl_4 < PbCl_4$
9.	Ease of hydrolysis of tetrahalides	$SiCl_4 < GeCl_4 < SnCl_4 < PbCl_4$
10.	Reducing character of dihalides	$GeCl_2 > SnCl_2 > PbCl_2$

Table 4
Nitrogen Family

	Property	**Gradation**
1.	Acidic strength of trioxides	$N_2O_3 > P_2O_3 > As_2O_3$
2.	Acidic strength of pentoxides	$N_2O_5 > P_2O_5 > As_2O_5 > Sb_2O_5 > Bi_2O_5$
3.	Acidic strength of oxides of nitrogen	$N_2O < NO < N_2O_3 < N_2O_4 < N_2O_5$
4.	The stability of pentoxides	$P_2O_5 < As_2O_5 > Sb_2O_5 > N_2O_5 > Bi_2O_5$
5.	Basic nature, bond angle, thermal stability and dipole moment of hydrides	$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
6.	Reducing power, poisonous nature, covalent nature of hydrides	$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$
7.	Stability of trihalides of nitrogen	$NF_3 > NCl_3 > NBr_3$
8.	Lewis base strength	$NF_3 < NCl_3 > NBr_3 < NI_3$
9.	Ease of hydrolysis of trichlorides	$NCl_3 > PCl_3 > AsCl_3 > SbCl_3 > BiCl_3$ (NF_3 & PF_3 are not hydrolysed)
10.	Lewis acid strength of trihalides of P, As and Sb	$PCl_3 > AsCl_3 > SbCl_3$
11.	Lewis acid strength among phosphorus trihalides	$PF_3 > PCl_3 > PBr_3 > PI_3$
12.	Bond angle, among the halides of phosphorus	$PF_3 < PCl_3 < PBr_3 < PI_3$

Table 5
Oxygen Family

	Property	**Gradation**
1.	Melting and boiling point of hydrides	$H_2O > H_2Te > H_2Se > H_2S$
2.	Volatility of hydrides	$H_2O < H_2Te < H_2Se < H_2S$
3.	Thermal stability of hydrides	$H_2O > H_2S > H_2Se > H_2Te$

	Property	Gradation
4.	Reducing nature of hydrides	$H_2S < H_2Se < H_2Te$
5.	Covalent character of hydrides	$H_2O < H_2S < H_2Se < H_2Te$
6.	Bond angle & Dipole moment of hydrides	$H_2O > H_2S > H_2Se > H_2Te$ (104°) (92°) (91°) (90°)
7.	Ease of hydrolysis of hexahalides	$SF_6 > SeF_6 > TeF_6$
8.	The acidic character of oxides (elements in the same oxidation state)	$SO_2 > SeO_2 > TeO_2 > PoO_2$ $SO_3 > SeO_3 > TeO_3$
9.	Acidic character of oxide of a particular element (e.g. S)	$SO < SO_2 < SO_3$
10.	Stability of dioxides	$SO_2 > TeO_2 > SeO_2 > PoO_2$

Table 6
Halogen Family

	Property	Gradation
1.	Bond energy of halogens	$Cl_2 > Br_2 > F_2 > I_2$
2.	Bond length in X_2 molecule	$F_2 < Cl_2 < Br_2 < I_2$
3.	Solubility of halogen in water	$F_2 > Cl_2 > Br_2 > I_2$
4.	Oxidising power	$F_2 > Cl_2 > Br_2 > I_2$
5.	Enthalpy of hydration of X^- ion	$F^- > Cl^- > Br^- > I^-$
6.	Reactivity of halogens	$F > Cl > Br > I$
7.	Ionic character of M – X bond in halides	$M-F > M-Cl > M-Br > M-I$
8.	Reducing character of X^- ion	$I^- > Br^- > Cl^- > F^-$
9.	Boiling point or volatility of hydrides	$HCl < HBr < HI < HF$
10.	Thermal stability of hydrides	$HF > HCl > HBr > HI$
11.	Acidic strength of halogen acids	$HI > HBr > HCl > HF$
12.	Conjugate base strength of halogen acids	$I^- < Br^- < Cl^- < F^-$
13.	Reducing property of hydrogen halides	$HF < HCl < HBr < HI$
14.	Dipole moment of hydrogen halides	$HF > HCl > HBr > HI$
15.	Oxidising power of oxides of chlorine	$Cl_2O > ClO_2 > Cl_2O_6 > Cl_2O_7$ (decreasing order)
16.	Acidic character of oxyacids of chlorine	$HClO < HClO_2 < HClO_3 < HClO_4$
17.	Strength of conjugate bases of oxyacids of chlorine	$ClO^- > ClO_2^- > ClO_3^- > ClO_4^-$
18.	Oxidising power of oxyacids of chlorine	$HClO > HClO_2 > HClO_3 > HClO_4$
19.	Thermal stability of oxyacids of chlorine	$HClO < HClO_2 < HClO_3 < HClO_4$
20.	Stability of anions of oxyacids of chlorine	$ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$

5. TRANSITION ELEMENTS (d-BLOCK ELEMENTS)

(1) The elements with exceptional configuration are

$Cr^{24}[Ar]\ 3d^5 4s^1$; $Cu^{29}[Ar]\ 3d^{10} 4s^1$

$Mo^{42}[Kr]\ 4d^5 5s^1$; $Pd^{46}[Kr]\ 4d^{10} 5s^0$

$Ag^{47}[Kr]\ 4d^{10} 5s^1$; $Pt^{78}[Xe]\ 4f^{14}\ 5d^{10}\ 6s^0$

$Au^{79}[Xe]\ 4f^{14} 5d^{10} 6s^1$

(2) Transition elements act as good catalysts due to the presence of incomplete 'd' orbitals and variable oxidation states.

(3) Ferromagnetic substances are those in which there are large number of electrons with unpaired spins and whose magnetic moments are aligned in the same direction.

(4) Bohr Magneton is the unit of magnetic moment.

(5) Many compounds of transition metals are coloured due to the presence of unpaired d-electrons.

6. COORDINATION COMPOUNDS

(i) Coordination compound is a compound consisting either of complex ions and other ions of opposite charge or of a neutral complex species.

(ii) Coordination number is the number of the nearest atoms or groups in the coordination sphere.

(iii) Ligand is a Lewis base donor of electrons that bonds to a central metal atom in a coordination compound.

(iv) Octahedral geometry is the geometry of a molecule in which six groups or atoms occupy the vertices of a regular octahedron with the central atom at the centre of the octahedron.

(v) Paramagnetic substance is one that is attracted to the magnetic field, this results on account of unpaired electrons present in the atom/molecule/ion.

(vi) Polydentate ligand is one that can bond with two or more atoms or groups to a central metal atom.

(vii) Square planar geometry is one in which a central atom is surrounded by four other atoms or groups arranged in a square and in a plane.

(viii) Stability constant is also known as formation constant – is the equilibrium constant for the formation of a complex ion.

(ix) Effective atomic number EAN

$$= (Z - \text{Oxidation number}) + (2 \times \text{Coordination number})$$

(x) (1) Type of hybridisation of M^{+n} ion in complexes and the number of unpaired electrons present

No.	Complex	M^{n+}	Type of hybridisation	No. of unpaired electrons	Inner orbital or outer orbital complex
1.	$[V(H_2O)_6]^{+3}$	$V^{+3}(d^2)$	d^2sp^3	2	Inner orbital (low spin or spin paired)
2.	$[Cr(NH_3)_6]^{+3}$	$Cr^{+3}(d^3)$	$d^2\ sp^3$	3	Inner orbital (low spin or spin paired)
3.	$[Mn(CN)_6]^{-3}$	$Mn^{+3}(d^4)$	d^2sp^3	2	Inner orbital (low spin or spin paired)
4.	$[Fe(CN)_6]^{-3}$	$Fe^{+3}(d^5)$	d^2sp^3	1	Inner orbital (low spin or spin paired)
5.	$[Fe(CN)_6]^{-4}$	$Fe^{+2}(d^6)$	d^2sp^3	0	Inner orbital (low spin or spin paired)
6.	$[FeF_6]^{-3}$	$Fe^{+3}(d^5)$	sp^3d^2	5	Outer orbital (high spin or spin free)
7.	$[Co(NH_3)_6]^{+3}$	$Co^{+3}(d^6)$	d^2sp^3	0	Outer orbital (high spin or spin free)
8.	$[CoF_6]^{-3}$	Co^{+3}	sp^3d^2	4	Outer orbital (high spin or spin free)
9.	$[Ni(CN)_4]^{-2}$	$Ni^{+2}(d^8)$	dsp^2	0	Outer orbital (high spin or spin free)
10.	$Ni(CO)_4$	Ni	sp^3	0	Outer orbital (high spin or spin free)
11.	$[Cu(NH_3)_2]^+$	$Cu^+(d^{10})$	sp	0	Outer orbital (high spin or spin free)
12.	$[Cu(NH_3)_4]^{+2}$	$Cu^{+2}(d^8)$	dsp^2	1	Outer orbital (high spin or spin free)
13.	$[Cu(CN)_4]^{-3}$	Cu^+	sp^3	0	Outer orbital (high spin or spin free)
14.	$[Zn(NH_3)_4]^{+2}$	$Zn^{+2}(d^{10})$	sp^3	0	Outer orbital (high spin or spin free)
15.	$[Cr(CN)_6]^{-3}$	$Cr^{+3}(d^3)$	d^2sp^3	3	Outer orbital (high spin or spin free)
16.	$[Cr(H_2O)_6]^{+3}$	Cr^{+3}	d^2sp^3	3	Outer orbital (high spin or spin free)
17.	$[Co(NO_2)_6]^{-3}$	$Co^{+3}(d^6)$	d^2sp^3	1	Outer orbital (high spin or spin free)

No.	Complex	M^{n+}	Type of hybridisation	No. of unpaired electrons	Inner orbital or outer orbital complex
18.	$[Co(NH_3)_6]^{+2}$	Co^{+2}	d^2sp^3	1	Outer orbital (high spin or spin free)
19.	$[Ni(NH_3)_6]^{+2}$	Ni^{+2}	sp^3d^2	2	Outer orbital (high spin or spin free)
20.	$[Fe(H_2O)_6]^{+2}$	Fe^{+2}	sp^3d^2	4	Outer orbital (high spin or spin free)
21.	$[Fe(NH_3)_6]^{+2}$	Fe^{+2}	sp^3d^2	4	Outer orbital (high spin or spin free)
22.	$[Cu(NH_3)_6]^{+2}$	Cu^{+2}	sp^3d^2	1	Outer orbital (high spin or spin free)
23.	$[MnF_6]^{-3}$	$Mn^{+3}(d^4)$	sp^3d^2	4	Outer orbital (high spin or spin free)
24.	$[Ni(H_2O)_6]^{+2}$	Ni^{+2}	sp^3d^2	2	Outer orbital (high spin or spin free)
25.	$[NiCl_4]^{-2}$	Ni^{+2}	sp^3	2	Outer orbital (high spin or spin free)
26.	$[Ni(NH_3)_4]^{+2}$	Ni^{+2}	sp^3	2	Outer orbital (high spin or spin free)
27.	$[FeCl_4]^{-}$	Fe^{+3}	sp^3	5	Outer orbital (high spin or spin free)
28.	$[MnCl_4]^{-2}$	Mn^{+2}	sp^3	5	Outer orbital (high spin or spin free)
29.	$[Pt(NH_3)_2Cl_2]$	Pt^{+2}	dsp^2	0	Outer orbital (high spin or spin free)
30	$[CuCl_4]^{-2}$	Cu^{+2}	sp^3	1	Outer orbital (high spin or spin free)

(2) **Inner and outer orbital Complexes**

Inner orbital Complexes: (n – 1)d-orbitals are used up in hybridisation (d^2sp^3) Generally diamagnetic or reduced paramagnetism also known as low spin or spin paired complexes

Outer orbital Complexes: nd-orbitals are used up in hybridisation (sp^3d^2) paramagnetic, known as high spin or spin free complex

(3) Factors affecting stability of complex

(i) Greater the charge on the central metal ion, greater is the stability.

(ii) Greater the ability of the ligand to donate electron pair (basic strength) greater is the stability.

(iii) Formation of chelate rings increases the stability.

(4) Spectrochemical series

$I^- < Br^- < Cl^- < F^- < OH^- < OX^{-2} < H_2O < Py < NH_3 < en < NO_2^- < CN^- < CO$

Weak field ligands form outer orbital octahedral complexes	Strong field ligands form inner orbital complexes

III. ORGANIC CHEMISTRY

1. CONCEPTS

(i) Hybridisation

(a) The phenomenon of intermixing of orbitals of the same atom having the slightly different energies to form new orbitals which have identical shapes and equivalent energies is known as hybridisation.

(b) The more the 's' character in the atomic/hybrid orbital, the lower the energy.

∴ the order of decreasing energy is $p > sp^3 > sp^2 > sp > s$

The order of decreasing electronegativity of hybrid orbitals is $sp > sp^2 > sp^3$

Type of hybridisation	Bond Angle	Geometry	Type of bond formed
sp^3	109.5°	Tetrahedral	σ
sp^2	120°	Trigonal planar	σ and one π
sp	180°	Linear (diagonal)	σ and two π

Isomerism

(ii) Conformational isomers are those isomers which arise due to rotation around a single bond.

(iii) Configuration is the spatial arrangement of atoms in a molecule.

(iv) Any object which is non-superimposable on its mirror image or lacks any elements of symmetry is chiral and is also known as dissymetric object.

(v) Stereo-isomers which are non-superimposable mirror images of each other are called enantiomers.

(vi) A meso compound is optically inactive, even though it has asymmetric centres (due to internal compensation of rotation of plane polarised light). It has plane of symmetry.

(vii) An equimolar mixture of enantiomers is called racemic mixture, which is optically inactive. (It can be separated into optically active forms.)

(viii) Anamer is the stereoisomer at the α-carbon atom of glucose.

(ix) Epimer is the stereoisomer at any other stereocentre (other than α).

(x) Tautomerism is the type of isomerism arising by the migration of labile hydrogen giving rise to functional group, isomerism.

(xi) Reaction Intermediates and Reagents:

Homolytic fission $\longrightarrow$ Free radicals (in presence of hν, high temperature or in presence of peroxide).

Heterolytic fission $\longrightarrow$ ions (Carbocation, Carbanion) (polar solvent).

(xii) Nucleophiles – electron rich

Two types 1. Anions (X^-); 2. Neutral molecules

with lone pair of electrons (Lewis bases)

Electrophiles : electron deficient

Two types 1. Cations 2. Neutral molecules with vacant orbitals (Lewis acids).

(xiii) Inductive effect is due to σ electron displacement along a chain and is permanent effect.

(xiv) + I (Inductive effect) increases basicity, – I effect increases acidity of compounds.

(xv) Resonance is a phenomenon in which two or more structures can be written for the same compound but none of them actually exists.

2. ALKANES

(i) Pyrolytic cracking is a process in which alkane decomposes to a mixture of smaller hydrocarbons, when it is heated strongly, in the absence of oxygen in presence of catalyst.

(ii) Combustion, is a process in which hydrocarbons form carbon dioxide and $H_2O(l)$ when they are completely burnt in air /O_2.

(iii) Alkanes undergo substitution reaction in vapour phase by free radical mechanism.

3. ALKENES

(i) In dehydration and dehydrohalogenation the preferential order for removal of hydrogen is 3° > 2° > 1° (Saytzeff's Rule).

(ii) The lower the ΔH_h (heat of hydrogenation) the more stable the alkene is.

(iii) The alkene with more alkyl groups on the double bond is more stable.

(iv) Stereospecificity of additions;

(a) Addition of Br_2 is trans (anti).

Addition of **Br_2** to

(i) **Cis** isomer (alkene) results **Racemic** mixture,

(ii) **Trans** isomer results **Meso** compound.

(b) Addition of H_2 in the presence of catalyst is cis (syn). Addition of H_2 to

(i) **Trans** isomer (alkene) results **Racemic** mixture,

(ii) **Cis** isomer results **Meso** Compound.

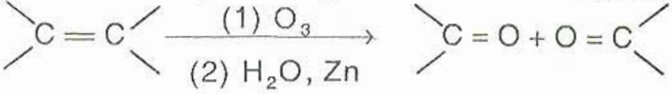
(c) Formation of glycols from alkenes with

(a) Peroxy acids is anti addition

(b) Baeyer's reagent is syn addition.

(v) Alkenes undergo anti-Markownikoff addition only with HBr in the presence of peroxides.

(vi) Alkenes undergo ozonolysis and form carbonyl compounds as follows



(vii) Oxidation of alkenes with hot $KMnO_4$ solution proceeds in general as follows

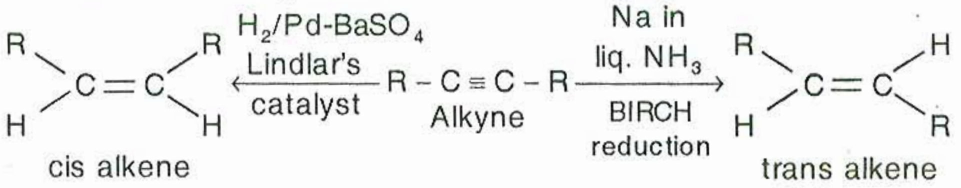
$H_2C =$ gives CO_2

$RCH =$ gives $RCOOH$

$R_2C =$ gives $R_2C = O$

4. ALKYNES

(i) Hydrogenation of Alkynes



(ii) Alkynes add water molecule in presence of mercuric sulphate and dil. H_2SO_4 and form carbonyl compounds.

(iii) Terminal alkynes have acidic H-atoms, so they form metal alkynides with Na, ammoniacal cuprous chloride solution and ammoniacal silver nitrate solution, or liberate H_2 with alkali metals, with Grignard reagent form RH.

(iv) Alkynes are acidic because of H-atoms which are attached to sp 'C' atom which **(a)** has more electronegativity **(b)** has more 's' character than sp^2 and sp^3 'C' atoms.

5. ARENES

(i) Types of Electrophilic substitutions

Reaction	**Reagent**	**Electrophile**	**Catalyst**
(a) Halogenation	$X_2 (X = Cl, Br)$	X^+	FeX_3
(b) Nitration	HNO_3 (conc.)	NO_2^+	H_2SO_4 (conc.)
(c) Sulfonation	H_2SO_4 (or) $H_2S_2O_7$	SO_3	–
(d) Friedel-Crafts Alkylation	RX	R^+	$AlCl_3$ (anhydrous)
	$ArCH_2X$		$AlCl_3$ (anhydrous)
	ROH $RCH = CH_2$		HF, H_2SO_4 BF_3
(e) Friedel-Crafts Acylation	RCOCl	$R\overset{+}{C}O$	$AlCl_3$

(ii) All o- and p-directing groups are ring activating groups (except – X).

They are – OH, – NH_2, – X, – R, – OR, etc.

(iii) All m-directing groups are ring deactivating groups. They are

– CHO, – COOH, – NO_2, – CN, – $\overset{+}{N}R_3$, – SO_3H, etc.

(iv) Aromatic compounds are the ones which are

(a) Cyclic and planar,

(b) Having continuous overlapping of 'p' orbitals, (conjugated system)

(c) Obey Huckel's rule $(4n + 2)\ \pi$ electron.

(v) Anti-aromaticity: Planar cyclic conjugated species, less stable than corresponding, acyclic unsaturated species are called anti-aromatic (4n Rule).

6. HALOGEN COMPOUNDS

(i) The order of reactivity is

(a) RI > RBr > RCl > RF

(b) Allyl halide > Alkyl halide > Vinyl halide

(c) Aralkyl halide > Aryl halide

(ii) S_N1 reaction: Mainly 3° alkyl halides undergo this reaction and form racemic mixture. S_N1 is favoured by polar solvent and low concentration of nucleophile.

(iii) S_N2 reaction: Mainly 1° alkyl halides undergo this substitution. Walden inversion takes place. S_N2 reaction is preferred by non-polar solvents and high concentration of nucleophile.

7. ALCOHOLS

(i) Alkenes are converted to alcohols in different ways as follows

Reagent	**Types of addition**
dil. H_2SO_4	– Markownikoff (also with rearrangement)
B_2H_6 and H_2O_2, OH^-	– Anti-Markownikoff
Oxymercuration demercuration	– Markownikoff (no rearrangement)

(ii) H of OH is very weakly acidic.

The order of decreasing acidity is phenols > H_2O > 1° alcohol > 2° alcohol > 3° alcohol > Alkyne

(iii) Oxidation by chromic acid ($Na_2Cr_2O_7$, con. H_2SO_4)

1° alcohol $\longrightarrow$	aldehyde $\longrightarrow$	carboxylic acid
	(with same number of 'C' atom)	(with same number of 'C' atom)
2° alcohol $\longrightarrow$	ketone $\longrightarrow$	carboxylic acid
	(with same number of 'C' atom)	(with less number of 'C' atom)

3° alcohol $\longrightarrow$ ketone $\longrightarrow$ carboxylic acid

(with less number of 'C' atom) (with less number of 'C' atom)

(iv) Intramolecular dehydration of alcohols $\longrightarrow$ alkenes

(v) Intermolecular dehydration of alcohols $\longrightarrow$ Ethers

(vi) Alcohols react with acids or acid derivatives in presence of acid or alkali and form esters (esterification)

8. PHENOLS

(i) Phenol $\xrightarrow{CHCl_3/OH^{\ominus}/H^+}$ Phenolic aldehyde (Reimer-Tiemann reaction)

(ii) Phenol $\xrightarrow{CCl_4\ +\ OH^-/H^+}$ Phenolic acid (Reimer-Tiemann reaction)

(iii) Phenol $\xrightarrow[\Delta]{CO_2}$ Phenolic carboxylic acid (Kolbe reaction)

(iv) Acidity of phenols

(a) increase by electron withdrawing substituents like

$-NO_2, -CN, -CHO, -COOH, -X, -\overset{+}{N}R_3$ ....

(b) decrease by electron releasing substitutents like

$-R, -OH, -NH_2, -NR_2, -OR$

9. ETHERS

$R_2O + HI \longrightarrow ROH + RI$

(conc. and hot)

mixed ethers form RI with smaller alkyl group

10. CARBONYL COMPOUNDS

(i) Formation of alcohols using RMgX.

(a) Formaldehyde + RMgX $\xrightarrow{\text{Hydrolysis}}$ 1° alcohol

(b) Aldehyde + RMgX $\xrightarrow{\text{Hydrolysis}}$ 2° alcohol
(other than HCHO)

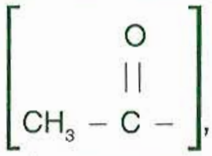
(c) Ketone + RMgX $\xrightarrow{\text{Hydrolysis}}$ 3° alcohol

(ii) Haloform (Iodoform) reaction.

Reagent – $NaOH + I_2$

Product – Iodoform (CHI_3)
(Yellow precipitate)

Methyl ketones



$\left[CH_3-\overset{\overset{O}{||}}{C}-\right]$, and compounds which are oxidised to CH_3-CO- group only answer this test.

(iii) Cannizzaro reaction (Disproportionation) aldehyde with no α H-atom HCHO, C_6H_6CHO

Aldehyde $\xrightarrow[\text{alkali}]{\text{hot conc.}}$ Alcohol + Salt of Acid (auto oxidation and reduction)

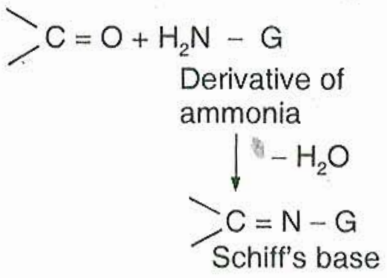
Crossed-Cannizzaro reaction gives alcohol with aryl group or bigger alkyl group.

(iv) Aldol condensation

carbonyl compound, + dil. alkali
(with α H-atom)
↓
β-hydroxy
carbonyl compound

iso-butyraldehyde is an exception as it undergoes both Cannizzaro reaction and aldol condensation.

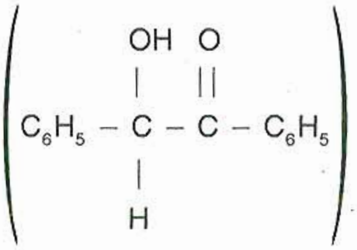
(v) Formation of Schiff's bases (condensation reaction)



(G = any group like H, C_6H_5NH, OH, $NH_2NHCONH_2$)

(vi) Benzoin condensation

Benzaldehyde $\xrightarrow[\text{NaCN}]{\text{ethanolic}}$ Benzoin



$\left(C_6H_5-\underset{\underset{H}{|}}{\overset{\overset{OH}{|}}{C}}-\overset{\overset{O}{||}}{C}-C_6H_5\right)$

(vii) Perkin's reaction

Aromatic aldehyde $C_6H_5HO+(CH_3CO)_2O\xrightarrow[\Delta]{\text{NaOEt}}C_6H_5CH=CHCOOH$

Cinnamic acid
Unsaturated acid

11. CARBOXYLIC ACIDS

(i) The rate of esterification decreases when alcohol, acid or both have branched substitutents.

(ii) Saponification: Alkaline hydrolysis of a fat of high molecular weight acid gives a carboxylate salt (soap).

(iii) Ortho effect: All ortho substituted benzoic acids (irrespective of type of substituent) are stronger than benzoic acid.

12. NITROGEN COMPOUNDS

(i) Order of basicity in aqueous medium

$(R = -CH_3 \text{ or } -C_2H_5)$

$2° > 1° > 3° > NH_3$

(ii) Hofmann degradation

Amides $\xrightarrow{Br_2/KOH}$ 1° amine (one carbon atom less)

(iii) Gabriel Synthesis

phthalimide + R – X $\xrightarrow{\text{Hydrolysis}}$ 1° amine + salt of phthalic acid

(iv) Aromatic amines are less basic than aliphatic amines.

(v) The basicity of amines is

(a) decreased by electron withdrawing groups.

(b) increased by electron releasing groups.

(vi) Carbylamine reaction

1° amine $\xrightarrow{CHCl_3 + KOH}$ Isocyanide
(foul smelling)

(vii) Diazotisation reaction

Aromatic 1° amine (where NH_2 is attached to the aromatic nucleus)

$\xrightarrow[0°C]{NaNO_2 + HCl}$ Diazonium salt.

(viii) Reduction of nitrobenzene in different media gives different products.

Medium – **Product**

Acidic – Aniline

Basic – Azoxy, azo and finally hydrazobenzene

Neutral – Phenyl hydroxylamine

13. CARBOHYDRATES, AMINO ACIDS AND POLYMERS

(i) Carbohydrates are polyhydroxy aldehydes or ketones.

(ii) Monosaccharides are simple sugars, containing three to nine carbon atoms.

(iii) Mutarotation is the change in value of specific rotation observed in all reducing sugars.

(iv) Oligosaccharides are sugars containing two to ten simple sugar units.

(v) Polymer is a chemical species of high molecular weight made up from repeating units of low molecular weight.

(vi) Polypeptides are polymers formed by linking of many amino acids by peptide bonds.

(vii) Proteins are biological polymers formed by amino acids.

(viii) Zwitter ion is a doubly ionized form in which there is a positive and a negative charge in the same species – observed in amino acids.

(ix) Bakelite, Teflon, Nylon, Terylene are examples of a variety of polymers.

(x) Urethanes are the esters of carbamic acid, H_2NCOOH.

Characteristic reactions

Homologous series		Type of reaction
(1) Alkanes	–	Substitution (Mostly free radical)
(2) Alkenes and Alkynes	–	Electrophilic addition
(3) Arenes	–	Electrophilic substituion
(4) Alkyl halides	–	Nucleophilic substitution
(5) Aldehydes and ketones	–	Nucleophilic addition

Identification Tests

Homologous series	Reagent	Observation
(1) Unsaturated compound (Alkenes, Alkynes)	Baeyer's reagent (alkaline $KMnO_4$)	Decolourising the reagent
(2) Alcohols	Ceric ammonium nitrate solution	Red colouration
(3) Phenols	Neutral $FeCl_3$ solution	Violet/deep blue colouration
(4) Aldehydes and Ketones	2, 4-dinitrophenyl hydrazine	Orange precipitate
(5) Acids	$NaHCO_3$ solution	Brisk effervescence (CO_2 is evolved)
(6) 1° amine	$CHCl_3$ + KOH	Foul smell (isocyanide)
(7) 2° amine	$NaNO_2$ + HCl (HNO_2)	Yellow oily liquid (Nitrosoamine)

Tests to differentiate

(1) 1°, 2° and 3° Alcohols	–	(1) Lucas Test (2) Victor Meyer's Test
(2) 1°, 2° and 3° Amines	–	Hinsberg Test
(3) 1°, 2° and 3° Nitro compounds	–	Test with HNO_2 and KOH
(4) Aryl halides and Aralkyl halides	–	Test with $AgNO_3$ solution
(5) Aldehydes and Ketones	–	Tollen's Test/Fehling's Test
(6) Aromatic aldehydes and Aliphatic aldehydes	–	Fehling's Test

COMPENDIUM – PHYSICS

1. UNITS AND DIMENSIONS

Mechanics

Sl. No.	Quantity	Symbol	Unit	Dimension
1.	Velocity	$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$	ms^{-1}	$[LT^{-1}]$
2.	Acceleration	$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$	ms^{-2}	$[LT^{-2}]$
3.	Momentum	$\vec{p} = m\vec{v}$	kg ms^{-1} or Ns	$[MLT^{-1}]$
4.	Force	$\vec{F} = m\vec{a}$	newton, N	$[MLT^{-2}]$
5.	Work	$W = \vec{F} \cdot \vec{S}$	joule, J	$[ML^2T^{-2}]$
6.	Torque	$\tau = \vec{r} \times \vec{F}$	Nm	$[ML^2T^{-2}]$
7.	Energy	$E = FS$	J	$[ML^2T^{-2}]$
8.	Power	$P = \frac{\Delta E}{\Delta t}$	watt, W	$[ML^2T^{-3}]$
9.	Moment of Inertia	$I = mR^2$	kg m^2	$[ML^2]$
10.	Density	$\rho = \frac{m}{V}$	kg m^{-3}	$[ML^{-3}]$
11.	Pressure	$p = \frac{F_n}{A}$	pascal (Pa) or Nm^{-2}	$[ML^{-1}T^{-2}]$
12.	Young's Modulus	Y	Nm^{-2}	$[ML^{-1}T^{-2}]$
13.	Frequency	ν, f	hertz(Hz)	$[T^{-1}]$

Electricity and Magnetism

Sl. No.	Quantity	Symbol	Unit	Dimension
1.	Current	i	ampere, A	[A]
2.	Charge	q	coulomb, C	[AT]
3.	Electric field intensity	$E = \frac{F}{q}$	Vm^{-1} or NC^{-1}	$[MLT^{-3}A^{-1}]$
4.	Electric Potential	$V = \frac{W}{q}$	volt, V	$[ML^2T^{-3}A^{-1}]$
5.	Electrical Resistance	$R = \frac{V}{i}$	ohm, Ω	$[ML^2T^{-3}A^{-2}]$

6.	Inductance	L	henry, H	$[ML^2T^{-2}A^{-2}]$
7.	Permittivity	ε	Fm^{-1}	$[M^{-1}L^{-3}T^4A^2]$
8.	Permeability	μ	Hm^{-1}	$[MLT^{-2}A^{-2}]$
9.	Magnetic Flux	ϕ	weber, Wb	$[ML^2T^{-2}A^{-1}]$
10.	Magnetic Induction	B	tesla or Wb m^{-2}	$[MT^{-2}A^{-1}]$
11.	Electric Dipole Moment	$p_e = ql$	Cm	[LTA]
12.	Magnetic Dipole Moment	M	Am^2	$[L^2A]$
13.	Capacitance	C	farad, F	$[M^{-1}L^{-2}T^4A^2]$

Physical constants

1.	Angstrom	Å	10^{-10} m
2.	Avogadro number	N_o	6.023×10^{23} mol^{-1}
3.	Bohr-radius	a_o	5.292×10^{-11} m
4.	Boltzmann's constant	k	1.381×10^{-23} J/K or 8.617×10^{-5} eV/K
5.	Electron charge	e	1.602×10^{-19} C
6.	Electron rest mass	m_e	9.109×10^{-31} kg
7.	Electron volt	eV	1.602×10^{-19} J
8.	Permeability of free space	μ_0	1.257×10^{-6} T m/A
9.	Permittivity of free space	ε_0	8.854×10^{-12} $C^2/N\text{-}m^2$
10.	Planck's constant	h	6.626×10^{-34} Js
11.	Rydberg constant	R	1.097×10^7 m^{-1}
12.	Speed of light in free space	c	2.998×10^8 m/s
13.	Stefan–Boltzmann constant	σ	5.7×10^{-8} $Wm^{-2}K^{-4}$
14.	Gravitational constant	G	6.67×10^{-11} N kg^{-2} m^2
15.	Gravitational acceleration	g	9.8 ms^{-2}

Multiples of Basic Unit

Prefix	Multiples of basic unit	Symbol
Femto	10^{-15}	f
Pico	10^{-12}	p
Nano	10^{-9}	n
Micro	10^{-6}	μ
Milli	10^{-3}	m
Centi	10^{-2}	c
Deci	10^{-1}	d
Deca	10	da
Hecto	10^{2}	h
Kilo	10^{3}	k
Mega	10^{6}	M
Giga	10^{9}	G
Tera	10^{12}	T

1.1 SIGNIFICANT FIGURES

- All the nonzero digits are significant.
- All the zeros between two nonzero digits are significant, no matter where the decimal point is, if at all.
- If the number is less than 1, then the zeros on the right of decimal point but to the left of the first nonzero digit are not significant.
- The terminal or trailing zeros in a number without a decimal point are not significant.
- The trailing zeros in a number with a decimal point are significant.
- For a number greater than 1, without any decimal, the trailing zeros are not significant.
- For a number with a decimal, the trailing zeros are significant.
- By a change in the position of decimal point alone, the number is significant, figures does not change.
- Power of ten (such as 10^{-19}) are not counted in significant figures s.f. (Thus 2.6×10^{-19} has only two s.f's.)
- In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.
- In addition or subtraction, the final results should retain as many decimal places as are there in the number with the least decimal places.

1.2 ERROR ANALYSIS

Mean

$$a_{mean} = \frac{a_1 + a_2 + \cdots\cdots + a_n}{n}$$

Absolute errors

$$\Delta a_1 = a_{mean} - a_1$$

$$\vdots \qquad \vdots$$

$$\Delta a_n = a_{mean} - a_n$$

$$(\Delta a)_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots\cdots + |\Delta a_n|}{n}$$

Then $a = a_{mean} \pm (\Delta a)_{mean}$

Percentage error or relative error,

$$\delta a = \frac{(\Delta a)_{mean}}{a_{mean}} \times 100\%$$

Standard deviation

$$\sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2\right]^{\frac{1}{2}}, \text{ where } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Then error, $\alpha = \frac{\sigma}{\sqrt{n}}$

Mean deviation (or) Mean error (or) Average error

$$\bar{\delta} = \frac{|\delta_1| + |\delta_2| + \ldots\ldots\ldots + |\delta_n|}{n}, \text{ where } \delta_1 = x_1 - \bar{x} \text{ (etc.)}$$

- If $Z = A + B$, then $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$
- If $Z = A - B$, then $Z \pm \Delta Z = (A - B) \pm \Delta A \pm \Delta B$
- For $Z = AB$ or $\frac{A}{B}$, we have fractional error $\pm \frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$
- When indices (powers) are involved, for example: if $Z = \frac{A^p B^q}{C^r}$,

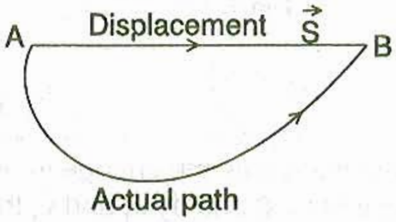
  then $\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

2. MECHANICS

2.1 SOME DEFINITIONS

Distance between two points is a scalar.

Displacement is a vector $\vec{S}$ drawn from the initial position (A) to the final position (B).



Unit: m(metre)

Velocity is the rate of change of displacement.

It is a vector $\vec{v}$.

$$\vec{v} = \frac{d\vec{S}}{dt}$$

Unit: ms^{-1}

Acceleration is the rate of change of velocity.

It is a vector $\vec{a}$.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Unit: ms^{-2}

$$\textbf{Average speed} = \frac{\text{Total distance covered}}{\text{Total time}}$$

$$\textbf{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

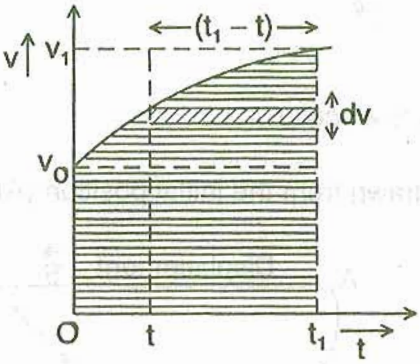
$$\textbf{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time}}$$

Uniform velocity: A particle with uniform velocity undergoes equal displacements in equal intervals of time, however small the intervals may be.

Uniform acceleration: A particle has uniform acceleration if it undergoes equal changes in velocity in equal intervals of time, however small the intervals may be.

2.2 GRAPHICAL SOLUTION OF RECTILINEAR MOTION PROBLEMS

v-t Curve

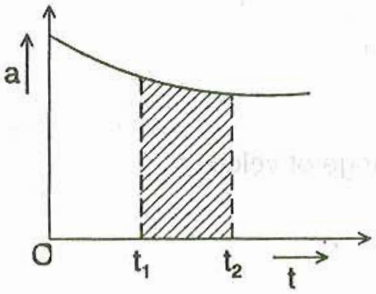


The area under the v-t curve measures the change in displacement x. Denoting, respectively by x_0 and v_0 the values of x and v at t = 0 and by x_1 and v_1 their values at $t = t_1$,

$$x_1 - x_0 = \text{area under v-t curve} = v_0 t_1 + \int_{v_0}^{v_1} (t_1 - t)\,dv$$

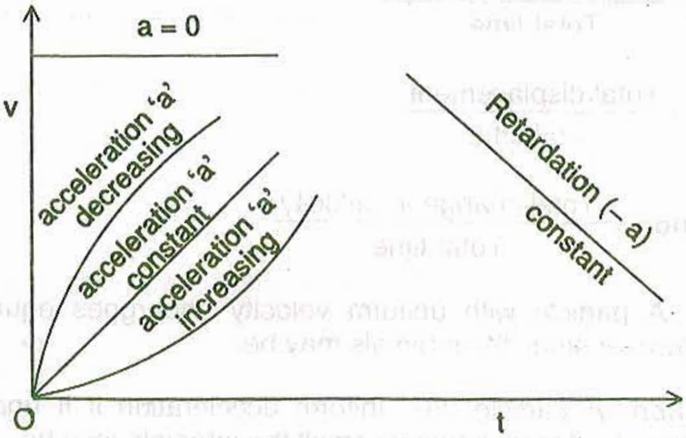
a–t Curve

Area under the a-t curve measures the change in velocity.



$$v_2 - v_1 = \text{area under (a-t) curve} = \int_{t_1}^{t_2} a\,dt$$

Characteristics of v-t graph



Characteristics of a-t graph

(1) If the graph obtained is a line parallel to x-axis, the acceleration is constant.

(2) If the graph obtained is an oblique straight line of positive slope, the acceleration is uniformly increasing and if it is of negative slope, the acceleration is uniformly decreasing.

2.3 EQUATIONS FOR RECTILINEAR MOTION

If a particle changes its velocity from **u** to **v** in time **t**, with uniform acceleration **a** and **undergoes** a displacement **S** in that time,

$$v = u + at \quad \text{... (i)}$$

$$S = \frac{1}{2}t(u + v) \quad \text{... (ii)}$$

$$S = ut + \frac{1}{2}at^2 \quad \text{... (iii)}$$

$$S = vt - \frac{1}{2}at^2 \quad \text{... (iv)}$$

$$v^2 = u^2 + 2aS \quad \text{... (v)}$$

$$S_n = u + \frac{a}{2}(2n - 1) \quad \text{... (vi)}$$

where S_n is the displacement in the nth second.

The quantities u, v, a and S are in the same straight line, but they may be positive or negative. Choosing a given direction as positive, all quantities in this direction are positive and those in the opposite direction are negative.

2.4 MOTION UNDER GRAVITY

For vertical motion under gravity, taking **upward direction positive**,

$$v = u - gt \quad \text{... (i)}$$

$$S = ut - \frac{1}{2}gt^2 \quad \text{... (ii)}$$

$$v^2 = u^2 - 2gS \quad \text{... (iii)}$$

where v and u are upward velocities and S is upward displacement.

The following results may be obtained.

(i) For a body projected upward from the ground with velocity u,

Maximum height reached = $\frac{u^2}{2g}$

Time taken to reach maximum height = $\frac{u}{g}$

Time to go up and come down = $\frac{2u}{g}$

Velocity when reaching the ground = – u

(ii) For a body dropped from a height H,

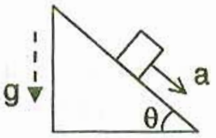
taking the downward direction as positive,

time of fall, $t = \sqrt{\dfrac{2H}{g}}$

Velocity when reaching the ground

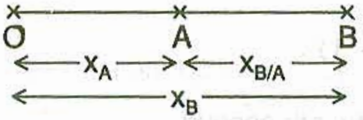
$v = \sqrt{2gH}$

(iii) For a body moving down a smooth inclined plane of angle θ, the acceleration of the body $a = g \sin\theta$



(iv) The value of $g = 9.8\ m/s^2$ unless otherwise specified.

2.5 RELATIVE MOTION OF TWO PARTICLES



When two particles A and B move along the same straight line, denoting by $x_{B/A}$, the relative position coordinate of B with respect to A, we had $x_B = x_A + x_{B/A}$.

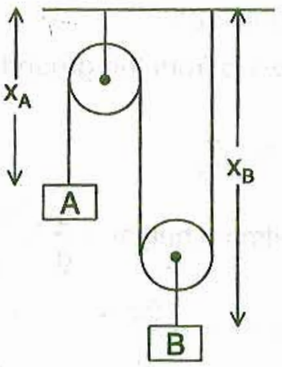
Denoting by $v_{B/A}$ and $a_{B/A}$ respectively, the relative velocity and the relative acceleration of B with respect to A,

$$v_B = v_A + v_{B/A}$$

$$a_B = a_A + a_{B/A}$$

2.6 DEPENDENT MOTIONS

The position of a particle will depend upon the position of another or several other particles. The motions are then said to be dependent.



(a)

In **Figure (a)** the position of block B depends upon the position of block A.

$x_A + 2x_B$ = length of rope

Differentiating twice with respect to time t

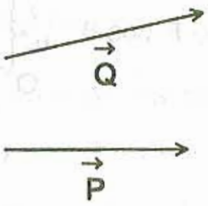
$$v_A + 2v_B = 0$$

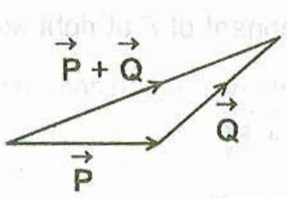
$$a_A + 2a_B = 0$$

Since only one of the two coordinates x_A and x_B may be chosen arbitrarily, the system shown in **Figure** has one degree of freedom.

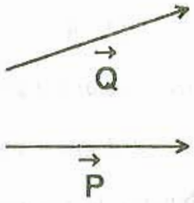
2.7 VECTORS

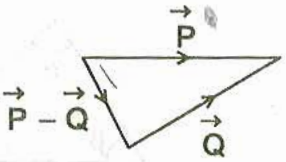
(i) Addition of Vectors





(ii) Subtraction of Vectors

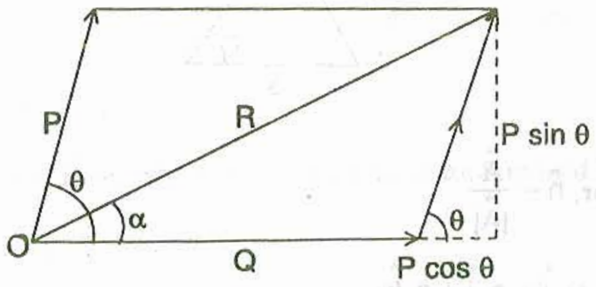




(iii) Parallelogram of Vectors

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{P\sin\theta}{Q + P\cos\theta}$$

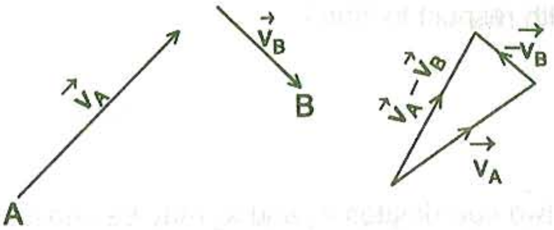


Value of R lies between (P + Q) and (P – Q)

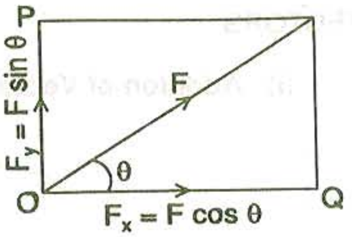
(it may be equal to (P + Q) or (P – Q) also) [depending on the value of θ]

(iv) Relative velocity

The velocity of A relative to B is $\vec{v}_A - \vec{v}_B$.



(v) Resolution of Vectors



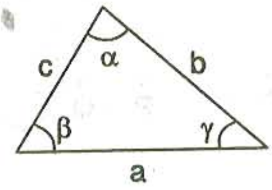
The component of F in a direction making an angle θ is F cos θ.

The other component of F at right angles to F cos θ is F sin θ.

$$F = \sqrt{F_x^2 + F_y^2}$$

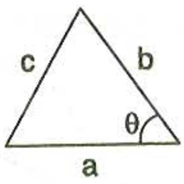
(vi) Sine Law of Triangle

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



(vii) Cosine Law of Triangle

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

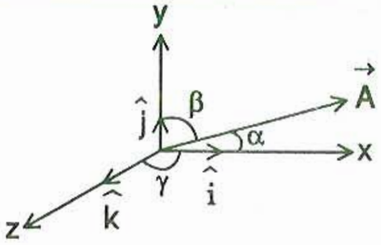


(viii) Unit vector, $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$

where $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$.

$$a = |\vec{A}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$



(ix) Direction cosines (d.c's) of a vector

We know components are

$$a_x = a\cos\alpha,$$

$$a_y = a\cos\beta,$$

$$a_z = a\cos\gamma.$$

$\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines of a vector $\vec{A}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Two vectors will be parallel, if their direction cosines are same.

(x) Products of scalars and vectors

scalar × scalar = scalar ... (1)

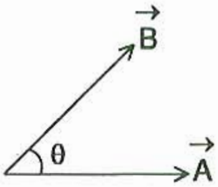
example: power × time = work

scalar × vector = vector ... (2)

example: mass × velocity = momentum

On the other hand, the product of two vectors can be **either** scalar **or** vector.

Scalar product or Dot product



$$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta$$

$$= a_xb_x + a_yb_y + a_zb_z$$

where $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$,

$$\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}.$$

Example: (1) Magnetic flux, $\Phi = \vec{B}\cdot\vec{A}$, where $\vec{B}$ is magnetic induction and $\vec{A}$ is directed area.

(2) Work done, $W = \vec{F}\cdot\vec{S}$, $W = \vec{\tau}\cdot\vec{\theta}$

(3) Power, $P = \vec{F}\cdot\vec{v}$, $P = \vec{\tau}\cdot\vec{\omega}$

Note: $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = \hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k} = 0.$$

(xi) Vector product or Cross product

$$\vec{A} \times \vec{B} = |A|\ |B|\ \sin\theta\ \hat{n}$$

where $\hat{n}$ = unit vector perpendicular to plane containing $\vec{A}$ and $\vec{B}$.

Example:(1) Angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

(2) Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

(3) Magnetic force, $F = q\vec{v} \times \vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \left[\hat{i}\,(a_y b_z - a_z b_y) - \hat{j}\,(a_x b_z - a_z b_x) + \hat{k}\,(a_x b_y - a_y b_x)\right]$$

If it is traversed anticlockwise, such as in product $\hat{i} \times \hat{j}$, the result is positive $\hat{k}$.

$$\Rightarrow \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

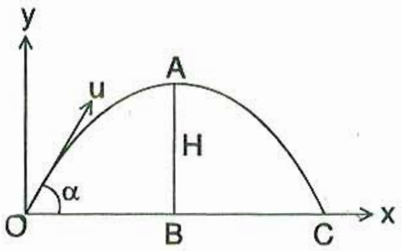
$$\hat{k} \times \hat{i} = \hat{j}$$

2.8 PROJECTILE MOTION

If a body is projected with a velocity u at an angle of elevation α, the path of motion is given by

$$y = x \tan\alpha - \frac{1}{2} g \frac{x^2}{u^2 \cos^2\alpha}$$

Here y is the vertical displacement and x is the horizontal displacement. The path is a parabola.



(i) Time taken to reach maximum height = $\frac{u \sin\alpha}{g}$

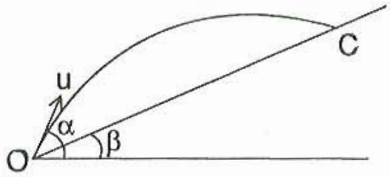
(ii) Time taken to reach point C on the horizontal plane passing through the point of projection = $\frac{2u \sin\alpha}{g}$

(iii) Greatest height reached, $H = \dfrac{u^2 \sin^2 \alpha}{2g}$.

(iv) Horizontal range, $R = \dfrac{u^2 \sin 2\alpha}{g}$.

(v) Maximum horizontal range for a given velocity of projection u is $\dfrac{u^2}{g}$.

(vi)



For an inclined plane of angle β with the horizontal and for a projectile with a velocity u and angle of elevation α to the horizontal, the range OC is

$$\frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g \cos^2 \beta}$$

(vii) Initial kinetic energy of projectile $E_k = \dfrac{1}{2} mu^2$.

Kinetic energy of projectile at highest point,

$$E_k' = \frac{1}{2} m(u \cos \alpha)^2 \Rightarrow E_k' = E_k \cos^2 \alpha.$$

2.9 CIRCULAR MOTION

(i) For uniform circular motion, the angular velocity, $\omega = \dfrac{v}{r}$, where v is the linear speed and r is the radius of the circle. The centripetal acceleration is $\dfrac{v^2}{r} = \omega^2 r$. The unit of angular velocity is radian/s.

Here,

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \text{(where } \theta \text{ is angle)}$$

$$v = \frac{\Delta s}{\Delta t} \quad \text{(where s is arc length)}$$

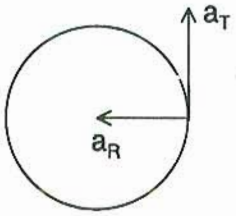
$$v = r\omega$$

$$\text{Period } T = \frac{1}{n} \quad \text{(where n is frequency)}$$

$$1 \text{ radian} = \frac{180°}{\pi} = 57.3°$$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

$\vec{v}, \vec{\omega}, \vec{r}$ are perpendicular to each other.



$$a = \frac{\Delta v}{\Delta t}$$

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

$$a = \alpha r$$

Radial acceleration, $a_R = \omega^2 r$

Tangential acceleration, $a_T = r\alpha$

Resultant acceleration, $a = \sqrt{a_R^2 + a_T^2}$

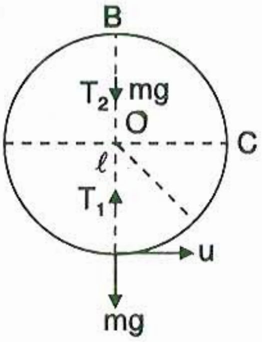
(ii) Motion in vertical circle

A body of mass m is tied to a string of length ℓ and is projected with a horizontal velocity u from A.

(a) If $u > \sqrt{5g\ell}$, it will perform complete revolution.

(b) If $u < \sqrt{5g\ell}$, the particle will either oscillate about the lowest point A or will leave the circular path. The condition of oscillation is that the velocity should vanish earlier than tension i.e., $h_1 < h_2$

where $h_1 = \frac{u^2}{2g}$, $h_2 = \frac{u^2 + g\ell}{3g}$



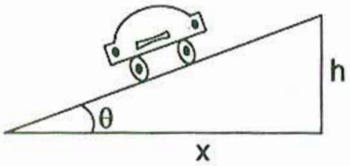
(c) If $\sqrt{2g\ell} < u < \sqrt{5g\ell}$, the particle will leave circular path somewhere between C and B.

(iii) Analogy between translatory and angular motions in terms of equations.

$v = u + at$	$\omega = \omega_0 + \alpha t$
$S = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 - u^2 = 2as$	$\omega^2 - \omega_0^2 = 2\alpha\theta$

(iv) Banking

When a vehicle moving with speed takes turn, the circular path has to be banked, as otherwise the vehicle **may skid or topple**.



To avoid skidding: $\mu_s \geq \frac{v^2}{gr}$

If the radius of the circular path is r and the speed of the vehicle is to reach the value v, the angle of banking θ is given by

$\tan\theta = \frac{v^2}{rg} = \frac{h}{x}$ [This is the condition of no skidding.]

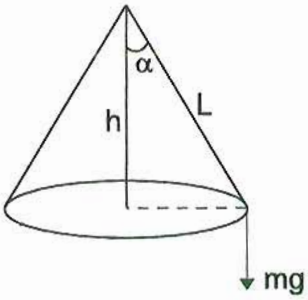
where x is the distance between two edges (or rails) and h is height of one edge relative to the other.

(v) If a vehicle is **to avoid skidding** its speed $v < \sqrt{\mu rg}$, where μ is the coefficient of friction between the wheels and the road (**Note:** skidding is also called slipping).

(vi) If a vehicle is to avoid toppling, $v < \sqrt{\frac{rag}{h}}$

where 2a is the distance between the left and right wheels and h is the height of the centre of gravity of the vehicle from the track.

(vii) Conical pendulum: $T = 2\pi\sqrt{\frac{L\cos\alpha}{g}} = 2\pi\sqrt{\frac{h}{g}}$



Momentum of a body of mass m and velocity $\vec{v}$ is $m\vec{v}$.

Newton's second law: $\vec{F} = m\dfrac{d\vec{v}}{dt} = \dfrac{dp}{dt} = m\,\vec{a}$

where $\vec{F}$ is the resultant impressed force and $\vec{a}$ is the acceleration.

Force is that which changes or tends to change the state of rest or of uniform motion of a body in a straight line.

Unit: 1 newton.

In F = ma, the quantity m is called "inertial mass" of body.

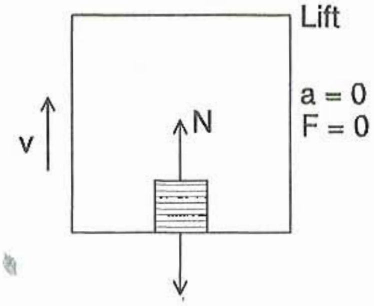
In weight, W = mg, the quantity m is called "gravitational mass" of body.

(I) Elevator or lift

If $W_0 = mg$ = actual weight of a body, N = apparent weight of a body in the lift (which is also reactionary force).

Case (i): Let the lift be at rest or moves up or down with uniform (constant) velocity, v

$$N = Mg = W_0$$

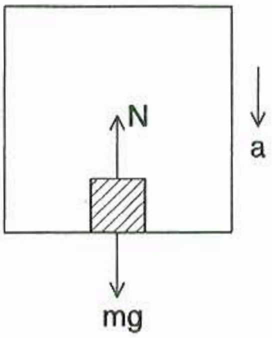


⇒ The apparent weight is the same as true weight.

Case (ii): If the lift moves downwards with acceleration 'a' (which is equivalent to saying that it has upward retardation, a)

$$N = m(g - a)$$

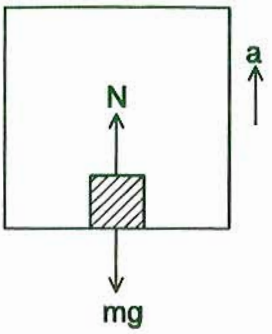
$$\Rightarrow N < W_0$$



Case (iii): If the lift moves upwards with acceleration 'a' (or downward retardation, a)

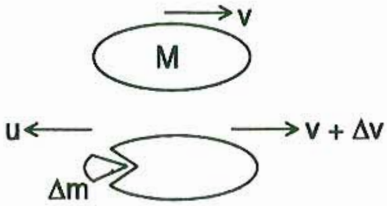
$$N = m(g + a)$$

$$\Rightarrow N > W_0$$



(II) Systems of varying mass

In some systems, such as rocket, sand piling on conveyor belt, the mass varies with time as shown.



$$M\frac{dv}{dt} = F_{ext} + u_{rel}\frac{dM}{dt}$$

where $u_{rel} = u - (-v) = v + u$.

Example

Rocket: The velocity of the rocket at any time t is given by,

$$v = v_0 + u\log_e\left(\frac{m_0}{m}\right)$$

where v_0 = initial velocity of the rocket,

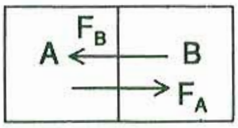
m_0 = initial mass of the rocket at time ($t = 0$),

m = mass of rocket at instant t.

The thrust on the rocket $F = u\frac{dm}{dt}$.

(III) Forces and Free body diagrams (FBD)

Forces occur in pairs. If a force F_A is exerted by a body A on body B, the body B exerts a force F_B on A.

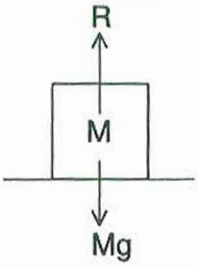


Here, $F_B = -F_A$

$\Sigma F_x = ma_x$

$\Sigma F_y = ma_y$

(IV) Normal Reaction

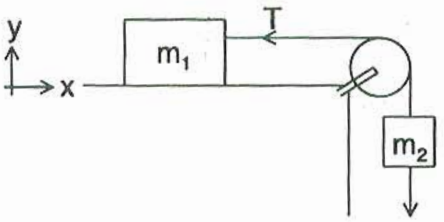


When a body of mass M is placed on the horizontal surface of a table, the weight Mg acts on the table. This is opposed by the table with the normal reaction R.

R = Mg but acts upwards.

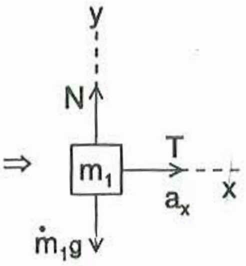
Choice of axes: X-axis along surface (whether horizontal surface or inclined plane)

Example (i)



"given system"

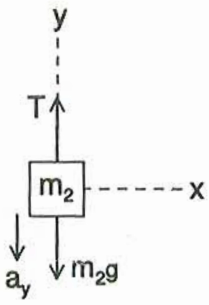
$\Rightarrow$



"FBD for m_1"

$\Rightarrow \Sigma F_x = m_1 a_x,$ $\qquad \Sigma F_y = m_2 a_y$

$\Rightarrow T = m_1 a,$ $\qquad T - m_2 g = -m_2 a$



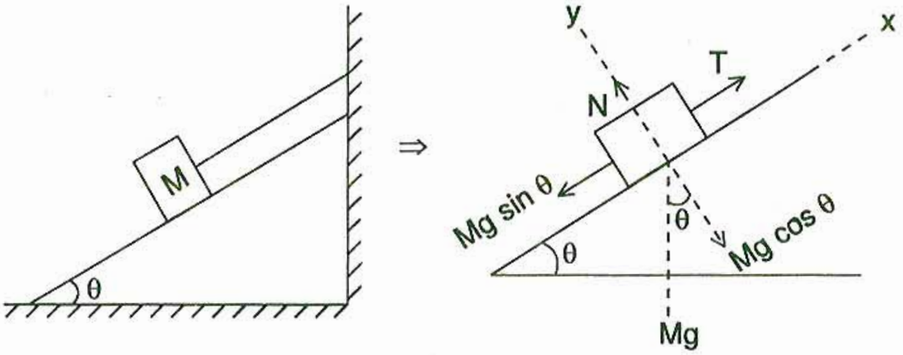
"FBD for m_2"

$a_x = a_y = a$

$$\Rightarrow a = \left(\frac{m_2}{m_1 + m_2}\right)g,$$

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g$$

Example (ii)



$\Rightarrow \Sigma F_x = 0$ (since it is at rest on inclined plane)

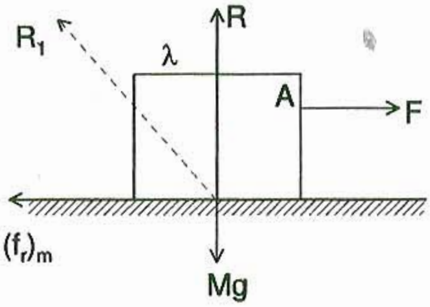
(or) $T - Mg \sin \theta = 0$

and $\Sigma F_y = 0$ (no motion normal to the plane)

(or) $N - Mg \cos \theta = 0$

(V) Maximum frictional force

When a horizontal force F is applied to the block A and if its magnitude is gradually increased, a force f_r comes into play opposing the tendency of the block to move.



This is due to friction. If the applied force F reaches a limiting value, the block starts to move. This is because f_r has a maximum value. If $(f_r)_m$ is the maximum force of friction, $\mu = \frac{(f_r)_m}{R}$ is the coefficient of static friction.

$$(f_r)_m = \mu R = \mu Mg$$

In general, $(f_r)_m = \mu \times$ normal contact force.

Let R_1 be resultant of limiting friction $(f_r)_m$ and normal reaction R,

λ be angle between R and R_1.

Then, $\mu = \tan \lambda$

When the body is at rest with friction present, static friction force $F_s = \mu_s R$.

When the body is in motion with friction present, force of kinetic force $F_k = \mu_k R$

$$\mu_k < \mu_s$$

For plane inclined at an angle θ with horizontal, if the body just slides (acceleration = 0) $\therefore \mu = \tan\theta$.

If $\theta < \tan^{-1}\mu$, the body will not slide.

If $\theta = \tan^{-1}\mu$, the body will just slide down.

If $\theta > \tan^{-1}\mu$, the body will slide with an acceleration a.

$$a = g\sin\theta - \mu g\cos\theta.$$

(VI) Pulley systems

Unless stated otherwise, pulleys and strings are massless.

Example: Atwood's machine;

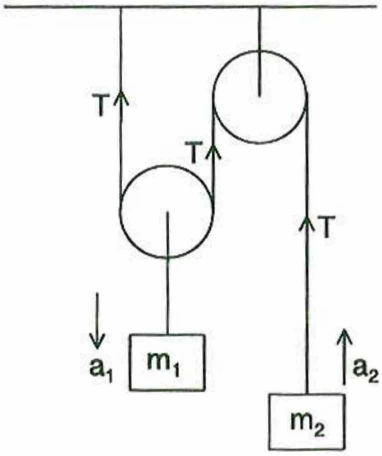
Accelerations are related as:

$$\vec{a}_2 = -2\vec{a}_1$$

$$m_2a_2 = T - m_2g$$

$$m_1a_1 = m_1g - (T + T)$$

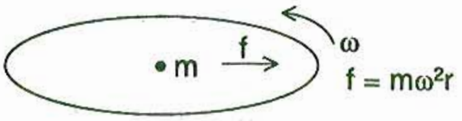
$$\Rightarrow m_1a_1 = m_1g - 2T$$



2.10 FICTITIOUS FORCES

In an accelerated frame of reference, Newton's laws are not applicable unless we introduce what are called fictitious forces.

If we are studying the motion of a body of mass m, in a frame of reference with acceleration a, the fictitious force to be introduced = – ma.



Thus for a rotating frame of reference with angular velocity ω, the fictitious force to be introduced is $m\omega^2r$ which is away from the centre. We say then that a centrifugal force $m\omega^2r$ acts on the body, when viewed from the rotating frame.

2.11 WORK, POWER, ENERGY

Definitions

Work: The work done by a force $\vec{F}$ whose point of application has a displacement $\vec{S}$ is $W = \vec{F}.\vec{S} = FS\cos\phi$, where ϕ is the angle between the vectors $\vec{F}$ and $\vec{S}$. When F = 1 N, and S = 1 m, with $\phi = 0$, W = 1 joule.

Area of F-S graph gives W.

Power is the rate of doing work. Power produced by a moving force $(P) = \vec{F}.\ \vec{v}$, where $\vec{v}$ is the velocity of the point of application of the force.

$$\left(\because P = \frac{W}{t} = \frac{FS}{t}\right)$$

1 horse power = 745 watts

Unit of power is 1 watt = 1 joule/s

$= 10^7$ergs/s

Energy is the capacity for doing work.

(i) **Kinetic energy** of a body of mass m having a velocity, v is $\frac{1}{2}mv^2$.

$$\Rightarrow E_k = \frac{1}{2}mv^2 \Rightarrow E_k = \frac{p^2}{2m} \text{ (since } p = mv)$$

1 kWh = 3.6×10^6 J, 1 cal = 4.2 J

Unit of energy is the same as **unit of work**.

(ii) **Potential energy** of a body of mass m at a height h from the ground or above a certain reference level is mgh.

(iii) Law of conservation of Energy

Energy can neither be created nor destroyed. Energy can be transformed from one form to another. Thus in a dynamo, mechanical energy is converted into electrical energy. In an electric bulb, electrical energy is converted into heat and light energy.

In mechanics, the sum of kinetic energy and potential energy remains constant.

(iv) The Work Energy Theorem

The work done by the resultant force acting on a body is equal to the change in its kinetic energy.

$$\text{Work} = FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

(v) Conservative force F is one that can be derived from a function U(x) by relation

$$F_x = -\frac{dU}{dx}$$

(This can be extended to dimensions of y, z also.)

Example of conservative force: Gravitational force and Electrostatic force.

Example of nonconservative force: Force of kinetic friction.

2.12 CONSERVATION OF MOMENTUM

Definition

Impulse: I is the product of the force and the time during which the force acts. If the force is variable, then $I = \int F\,dt \Rightarrow$ The area under F-t graph gives impulse.

Collisions or impacts

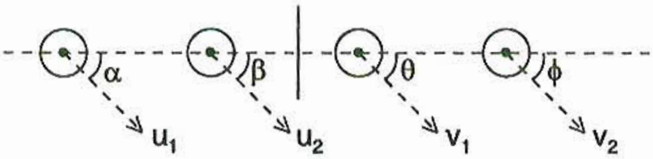
(i) Principle of conservation of momentum

The total momentum of a system of particles is a constant in the absence of external forces.

(ii) Newton's experimental law

The coefficient of restitution,

$$e = -\frac{\text{relative velocity after impact}}{\text{relative velocity before impact}}$$



$$e = -\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

but there are angles involved as shown in **Figure**,

$$|e| = \left[\frac{v_1 \cos\theta - v_2 \cos\phi}{u_1 \cos\alpha - u_2 \cos\beta}\right]$$

For perfect elastic impact, e = 1 and conservation of kinetic energy holds.

(iii) Force due to a jet of water on a wall = $u^2 Ad$, where u is the velocity of the jet, A is the area of cross-section of jet and d is the density of the fluid.

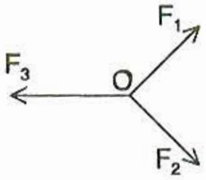
(iv) In the absence of external forces, the centre of mass of a system has a constant velocity.

Rigid body statics

Conditions for equilibrium of n concurrent forces

(i) For point O to be at rest

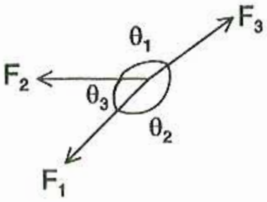
$$\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = 0$$



(ii) The algebraic sum of moments of all the forces about any arbitrary point is zero.

Lami's theorem for equilibrium of non-parallel coplanar forces

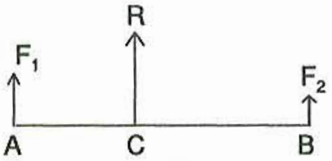
$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$



Resultant of parallel forces

(i) Two like parallel forces

The line of action of resultant R divides the line AB internally in the inverse ratio of the forces and magnitudes.

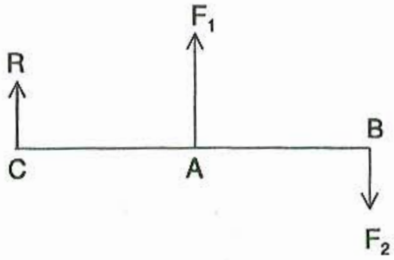


$$R = F_1 + F_2$$

$$\frac{AC}{BC} = \frac{F_2}{F_1}$$

(ii) Two unlike parallel forces

$$R = F_1 - F_2$$



$$\frac{AC}{BC} = \frac{F_2}{F_1}$$

The line of action of resultant divides the line AB externally in the inverse ratio of the forces.

2.13 RIGID BODY DYNAMICS

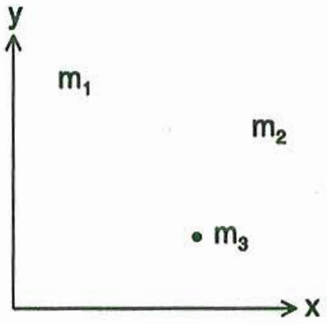
(a) Centre of Mass of a System of Particles

If there are a number of particles of masses m_1, m_2, m_3 etc. at positions having the coordinates (x_1, y_1, z_1),(x_2, y_2, z_2), (x_3, y_3, z_3) etc. respectively, the centre of mass of the particles is given by the coordinates X, Y, Z obtained as follows.

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$$

$$Y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...}$$

$$Z = \frac{m_1z_1 + m_2z_2 + m_3z_3 + ...}{m_1 + m_2 + m_3 + ...}$$



In the absence of external forces, the centre of mass of a system is constant.

If there is an external force $\vec{F}$ acting on the system $\vec{F} = M\,\vec{a}_c$,

where M is the total mass Σm_i and $\vec{a}_c$ is the acceleration of the centre of mass (CM).

(b) A system of two bodies with force of interaction only

If the particle m_2 exerts a force F_{12} on mass m_1, $\vec{F}_{12} = m_1\vec{a}_1$

Similarly, $\vec{F}_{21} = m_2\vec{a}_2$

Here, $\vec{F}_{12} = -\vec{F}_{21}$

$\vec{a}_1 - \vec{a}_2$ the relative acceleration $(\vec{a})$ is

$$\frac{\vec{F}_{12}}{m_1} - \frac{\vec{F}_{21}}{m_2} = \vec{F}_{12}\left[\frac{1}{m_1} + \frac{1}{m_2}\right]$$

$$\vec{F}_{12} = \frac{m_1m_2}{m_1 + m_2}\,\vec{a}$$

The reduced mass of the system,

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$= \frac{m_2}{1 + \frac{m_2}{m_1}}$$

If $m_2 << m_1$, $\mu \approx m_2$.

(c) Two bodies with CM moving with uniform velocity

Velocity of centre of mass, $v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$,

where v_1 and v_2 are the velocities of m_1 and m_2 in external inertial frame. Their velocities relative to the centre of mass are,

$$v_1 - v_{CM} = v_1 - \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_1 v_1 + m_2 v_1 - m_1 v_1 - m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_2(v_1 - v_2)}{m_1 + m_2} = \frac{\mu}{m_1}(v_1 - v_2),$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Similarly,

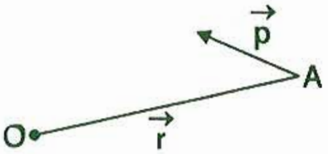
$$v_2 - v_{CM} = \frac{m_1 v_2 + m_2 v_2 - m_1 v_1 - m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_1(v_2 - v_1)}{m_1 + m_2} = \frac{\mu}{m_2}(v_2 - v_1)$$

2.14 ANGULAR MOMENTUM, $\vec{L}$

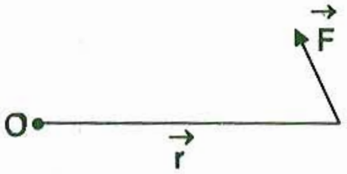
The angular momentum of a particle about a point, O = $\vec{r} \times \vec{p}$, where $\vec{r}$ is the radius vector drawn from the point O to the particle A and $\vec{p}$ is the linear momentum.

$$|\vec{L}| = |\vec{r} \times \vec{p}| = I\omega$$



Torque ($\vec{\tau}$): If a force $\vec{F}$ acts on a body so that the body tends to rotate about an axis O, the turning effect or the torque = $\vec{r} \times \vec{F}$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$



Conservation of angular momentum

In the absence of external torque, the angular momentum of a particle does not change.

If $\vec{\tau} = 0$, then $\vec{L}$ is constant.

Moment of Inertia: $I = \Sigma mr^2$

TABLE: MOMENTS OF INERTIA

Sl.No.	Nature of body	Mass	Dimension	Axis	I
1	Uniform solid sphere	M	radius r	passing through centre of mass	$\frac{2}{5}Mr^2$
2	Uniform solid cylinder or disc	M	radius r	coincides with the geometric axis (i.e., longitudinal axis passing through centre of mass)	$\frac{1}{2}Mr^2$
3	A cylindrical shell or hoop	M	radius r	coincides with the longitudinal axis passing through the centre of mass	Mr^2
4	A thin straight rod	M	length 2ℓ	passing through centre of mass and perpendicular to longitudinal axis.	$\frac{M\ell^2}{3}$
5	A thin straight rod	M	length 2ℓ	passing through one of its ends and perpendicular to longitudinal axis.	$\frac{4M\ell^2}{3}$

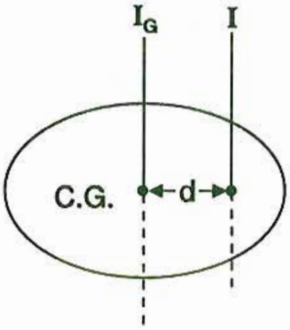
Torque is measured by the product of the magnitude of the force and the perpendicular distance between the axis and the line of action of the force.

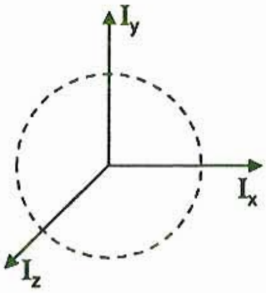
$$\vec{\tau} = \vec{r} \times \vec{F} = I\alpha$$

Radius of gyration (K)

$$K^2 = \frac{\Sigma mr^2}{\Sigma m} = \frac{I}{M}$$

where K depends on the shape of the body and the axis of rotation.





Parallel axes theorem: $I = I_G + Md^2$

Perpendicular axes theorem: $I_z = I_x + I_y$

Kinetic energy of rotation = $\frac{1}{2} I \omega^2$

Angular momentum,

$$\vec{L} = I\omega = |\vec{r} \times \vec{p}|$$

Rotational mechanics

In pure rolling, (i.e., without slipping)

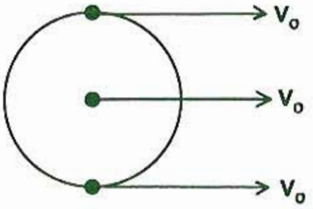
The velocity of the contact point = 0,

the velocity of centre of mass, $v_{cm} = R\omega$,

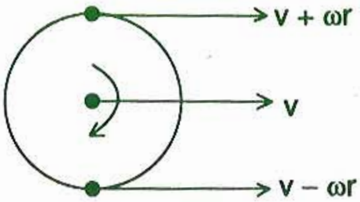
the angular acceleration about centre, $\alpha = \frac{a}{R}$

For rolling body in an inclined plane, acceleration = $\left(\frac{g \sin\theta}{1 + \frac{K^2}{R^2}}\right)$.

For sliding body in an inclined plane, acceleration = $g \sin\theta$



Pure sliding



Sliding + Rotation

Acceleration (a) of a body rolling down on an inclined plane

$$a = \frac{g \sin \theta}{1 + \left(\frac{K^2}{R^2}\right)}$$

where R = radius of body,

K = radius of gyration.

⇒ Different bodies of same radius such as ring, hollow sphere, solid sphere and cylinder will acquire different accelerations (as they have different K values) and hence take different time to reach the bottom of the plane.

Example: For ring, $a_1 = \left(\frac{g \sin \theta}{2}\right)$

For cylinder, $a_2 = \frac{2}{3}(g \sin \theta)$

As $a_2 > a_1$, cylinder reaches bottom faster.

Quantity	Sliding	Rolling
Energy conservation	$mgh = \frac{1}{2}mv^2$	$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
Velocity at bottom	$v = \sqrt{2gh}$	$v = \sqrt{\frac{2gh}{1 + \left(\frac{K^2}{R^2}\right)}}$
Acceleration of body down the plane	$a = g \sin \theta$	$a = \frac{g \sin \theta}{1 + \left(\frac{K^2}{R^2}\right)}$
Time taken in reaching bottom	$t = \sqrt{\frac{2\ell}{g \sin \theta}}$	$t = \sqrt{\frac{2\ell\left(1 + \frac{K^2}{R^2}\right)}{g \sin \theta}}$

2.15 GRAVITATION

(i) **Kepler's 3rd law for planetary motion**

$$T^2 \propto d^3 \Rightarrow T \propto d^{3/2}$$

where T = Time period of revolving planet,

d = mean distance of planet from the sun.

(ii) The gravitational force of attraction between a body of mass M and another of mass m, when separated by a distance d from each other is given by,

$$F = G\frac{Mm}{d^2},$$

where G is the universal gravitational constant,

$$G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}.$$

(iii) Weight of a body of mass m $= G\dfrac{Mm}{R^2} = mg$

Acceleration due to gravity on the surface of the earth,

$g_0 = \dfrac{GM}{R^2}$, where M is the mass and R is the radius of the earth.

Here, $M = \dfrac{4}{3}\pi R^3 \rho$, where ρ is density of earth.

(iv) The acceleration due to gravity, g at a height h above the surface of the earth is given by,

$$\frac{g_0}{g} = \left(1 + \frac{h}{R}\right)^2$$

If h << R,

$$g = g_0\left(1 - \frac{2h}{R}\right)$$

(v) Earth's rotation affects the value of g.

At the equator, $g' = g_0 - \omega^2 R$, where $\omega^2 R = 0.034\ \text{m/s}^2$

At latitude λ, $g_\lambda = g_0 - \omega^2 R \cos^2\lambda$.

(vi) **Orbital period** at distance r from the centre of the earth for a satellite,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

(vii) **Orbital velocity of a satellite** ·

$$v_0 = \sqrt{\frac{g_0 R^2}{R + h}}$$

(viii) Escape velocity

$$v_e = \sqrt{2g_0R}$$

(ix) Kinetic energy of a satellite (of mass m)

$$E_k = \frac{1}{2} \cdot \frac{GMm}{r}$$

(x) Gravitational potential energy of a satellite

$$E_p = -\frac{GMm}{r}$$

(xi) Gravitational potential (m = 1)

$$U = -\frac{GM}{r}$$

(xii) Total energy of a satellite

$$E = -\frac{GMm}{2r} = -E_k = \frac{E_p}{2}$$

2.16 ELASTICITY

(i) Hooke's Law

The force developed within a body is directly proportional to the deformation of the body within a certain limit is called the elastic limit.

(ii) Force constant of a spring is the force required to produce unit extension in it.

(iii) Young's modulus, $Y = \frac{F/A}{dL/L}$ N/m^2

where F is the stretching force, A is the area of cross-section of rod or wire, dL is the elongation, and L is the original length of rod or wire.

(iv) Bulk modulus, $B = -\frac{\Delta P}{dV/V}$ N/m^2

where ΔP is the change in pressure, dV is the change in volume and V is the original volume.

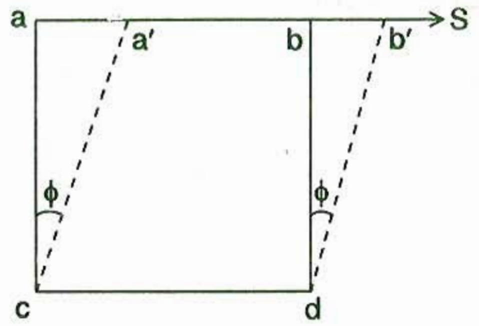
$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}}$$

(v) Shear modulus (or) modulus of rigidity

$$n = \frac{S/A}{\phi}\ \text{N/m}^2 = \frac{F/A}{\ell/L}$$

where S is the shearing force, A is the area of the surface on which the force is applied, and ϕ is the angle of shear.

$$\text{Resilience per unit volume} = \text{Elastic energy} = \frac{1}{2} \times \text{stress} \times \text{strain}$$



(vi) **Elastic potential energy** U in a spring = $\frac{1}{2}kx^2$, where k is the force constant and x is the extension or compression.

Also, $U = \frac{F^2}{2k}$ (since $F = -kx$)

⇒ graph of U – x is a parabola.

(vii) **Poisson's ratio (σ)**

When a rod or a wire is subjected to a tensile stress, its length increases in the direction of the tensile force but the length perpendicular to the tensile force decreases.

(Fractional change in transverse length) ∝ (fractional change in longitudinal length)

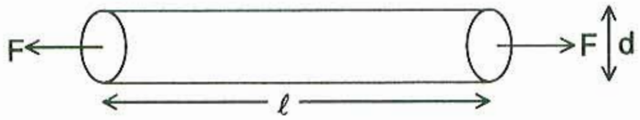
$$\Rightarrow \left(\frac{\Delta d}{d}\right) = \sigma\left(\frac{\Delta \ell}{\ell}\right)$$

The value of σ for many materials lies between 0.1 and 0.3.

Relations between Y, n, σ and B

$$Y = 2n(1+\sigma)$$

$$Y = 3B(1-2\sigma)$$



2.17 SIMPLE HARMONIC MOTION

A body is said to execute a simple harmonic motion when it experiences a restoring force which is proportional to its displacement and is directed to the equilibrium point.

(i) Displacement $y = A \sin 2\pi ft$; where A is the amplitude and f is the frequency. It is also written as,

$y = A \sin \omega t$; where ω is the angular frequency.

Particle velocity, $v = \omega\sqrt{A^2 - y^2}$

Particle acceleration = $-\omega^2 y$

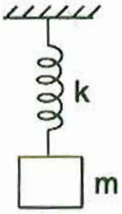
Restoring force = $-m\omega^2 y$

Differential equation of SHM

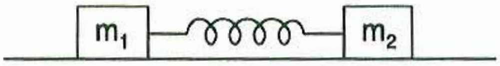
$$\frac{d^2y}{dt^2} = -\omega^2 y, \text{ or } \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \text{ (angular SHM)}$$

(ii) Motion of a body suspended by a spring

(a) $T = 2\pi\sqrt{\frac{m}{k}}$



(b) If two masses m_1 and m_2 are connected by a spring, then the time period is given by



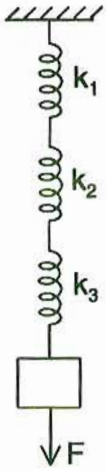
$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

where μ is known as reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2}$

(c) If a spring of force constant, k is divided into n equal parts and one such part is attached to a mass m, the time period

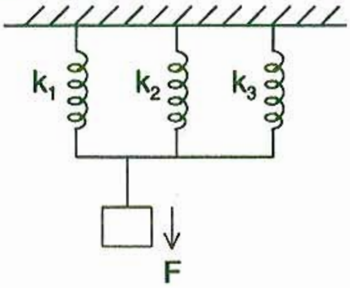
$$T = 2\pi\sqrt{\frac{m}{nk}}$$

(d) **Springs** (k's are spring constants) **in series**



$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

(e) Springs in parallel



$$k_{eff} = k_1 + k_2 + k_3$$

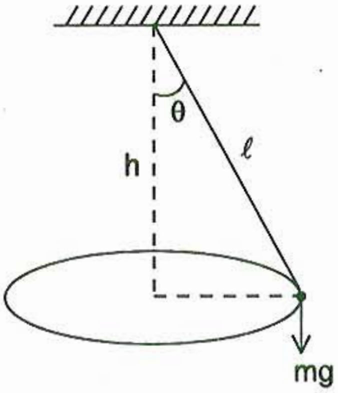
(f) Period of mass (M) spring system, when mass m_s of spring is taken into account

$$T = 2\pi\sqrt{\frac{\left(M + \frac{m_s}{3}\right)}{k}}$$

(iii) SHM of conical pendulum

(θ is constant, bob describes horizontal circle.)

Time period, $T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$



Torsional pendulum

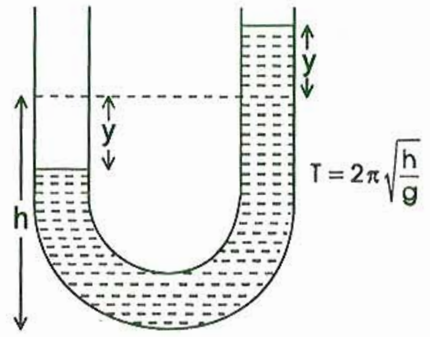
Restoring torque, $\tau = -C\theta$, where C is torsional rigidity of wire

$$T = 2\pi\sqrt{\frac{I}{C}}$$

(iv) SHM of a particle in a tunnel along a diameter (in a tunnel) of earth (of radius R)

$$T = 2\pi\sqrt{\frac{R}{g}}$$

(v) SHM of water in U-tube



2.18 FLUID MECHANICS

(i) **An ideal liquid** is one in which a shearing force cannot be exerted.

(ii) Density (d) = $\frac{\text{mass}}{\text{volume}} = \frac{M}{V}$

Unit: g/cm^3 (C.G.S.)

: kg/m^3 (S.I.)

Density of water = 1 g/cm^3 (C.G.S.) = 1000 kg/m^3 (S.I.)

Relative density = $\frac{\text{density of substance}}{\text{density of water}}$

(iii) Pressure = $\frac{\text{Force}}{\text{Area}} = \frac{\Delta F}{\Delta A}$

Pressure at a point in a liquid of density d, at rest = hdg, where 'h' is the depth of the point from the free surface of the liquid.

Unit: dyne/cm^2 (C.G.S.)

newton/metre^2 or pascal (S.I.)

One bar = 10^6 dyne/cm^2

(iv) **Principle of Archimedes**

When a body is partly or wholly immersed in a liquid at rest, it suffers an upward thrust equal to the weight of the displaced liquid. This thrust acts vertically through the centre of gravity of the displaced liquid.

(v) **Pascal's law:** It states that the pressure exerted on any part of an enclosed fluid is transmitted undiminished to every portion of fluid and walls of containing vessel.

When a block of mass M floats in a liquid, the total pressure at the bottom of liquid,

$P = P_0 + hdg + \frac{Mg}{A}$

where P_0 = atmospheric pressure,

A = area of bottom,

h = height of liquid column.

(vi) Equation of Continuity

As per conservation of mass, we have

$$\rho Av = \text{constant}$$

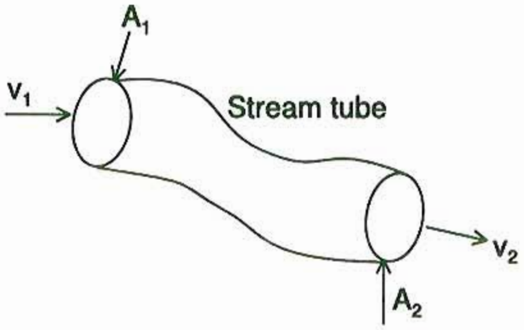
where ρ is density,

A is area of cross-section,

v is velocity over cross-section.

If flow is incompressible (ρ = constant), then Av = constant

$$\Rightarrow A_1v_1 = A_2v_2$$



(vii) Bernoulli's equation

For the steady flow of incompressible fluid, this equation relates the pressure p, the fluid speed v, and the height y at any two points on the same streamline as below.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

(viii) Velocity of efflux (From Torricelli's theorem derived from Bernoulli's equation)

$$v = \sqrt{2gd}$$

where d is the depth of interest.

Surface tension (T) is force per unit length of line on liquid surface.

Capillary rise method

$$T = \frac{hr\rho g}{2\cos\theta}, \text{ generally } \theta \approx 0°$$

Jaeger's method

$$T = \frac{gr}{2}(h\rho - h_1\rho_1)$$

Surface energy,

$$\sigma_s = \frac{[\text{Potential energy of liquid due to surface tension}]}{\text{Area of surface}}$$

Bonding energy, E_0

If each molecule in liquid has n near neighbours, and if new surface contains N molecules per unit area, then

$$\sigma_s = \frac{1}{4} nNE_0$$

Numerically: $\sigma_s = T$

Bubble: excess pressure, $P = \frac{4T}{r}$

Drop: excess pressure, $P = \frac{2T}{r}$

Viscosity

$$F = -\eta A \frac{dv}{dx}$$

where F is tangential force, A is area on which force acts, η is the coefficient of viscosity and $\frac{dv}{dx}$ is velocity gradient.

Poiseuille's formula

$$V = \frac{\pi \mathbf{Pr}^4}{\mathbf{8}\eta\ell}$$

where V is volume of liquid flowing per second, P is the difference of pressure at the ends of tube, r is the radius, and ℓ is the length of tube.

Stoke's Law

For a small sphere of radius r falling through a viscous medium, opposing viscous force, $F = 6\pi\eta rv$, where v is velocity.

Terminal velocity, $v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$

where ρ is density of material of sphere and σ is density of fluid.

Critical velocity (where streamline motion changes a turbulent motion), $v_c = \frac{R\eta}{\rho r}$

where R is called Reynold's number.

3. THERMAL PHYSICS

3.1 EXPANSIONS OF SOLIDS AND LIQUIDS

Temperature

It is the degree of hotness or coldness of a body. Heat flows from a body at a higher temperature to a body at a lower temperature.

Temperature is measured by some property of a substance which varies with temperature.

If temperature scale uses the property X of a substance, then

$$t = \frac{X_t - X_0}{X_N - X_0} N.$$

Here N is the number of divisions between the ice point and the steam point X_0, X_N and X_t are the measures of its property at the ice point, the steam point and the unknown temperature, t.

The ideal gas scale is defined as,

$$T = \frac{p}{p_{tr}} \times 273.16 \text{ K}$$

where p_{tr} is the pressure at the triple point of water.

(ii) Linear expansivity, $\alpha = \frac{\Delta L}{L_0 t} = \frac{L_2 - L_1}{L_1(t_2 - t_1)}$

(Similarly coefficient of aerial expansion, β is defined.)

The pendulum clock loses time in summer and gains time in winter.

The time lost by a pendulum clock when the temperature rises by t°C is $\frac{1}{2}\alpha t \times 86400$ seconds per day, where α is the linear expansivity of the pendulum.

(iii) Thermal stress = $\alpha Y t$

where α is the linear expansivity, Y is the Young's modulus and t is the range of temperature.

(iv) Calorimetry

Law of mixtures states that Heat lost = Heat gained

For change of temperature θ, heat required = $ms\theta$

For change of state (with latent heat, L) heat required: $Q = mL$

For example; formulae to be used at various stages are,

$$\boxed{\begin{matrix}\text{Ice}\\ -\theta^\circ\text{C}\end{matrix}} \xrightarrow{ms_i\theta} \boxed{\begin{matrix}\text{Ice}\\ 0^\circ\text{C}\end{matrix}} \xrightarrow{ML_f} \boxed{\begin{matrix}\text{Water}\\ 0^\circ\text{C}\end{matrix}} \xrightarrow{ms\theta} \boxed{\begin{matrix}\text{Water}\\ 100^\circ\text{C}\end{matrix}} \xrightarrow{mL_v} \boxed{\begin{matrix}\text{Steam}\\ 100^\circ\text{C}\end{matrix}}$$

(where L_f is latent heat for fusion of ice and L_v is for vaporisation of water and s_i is specific heat of ice and s is that of water)

(v) Including electronic contribution, specific heat of a solid is of form

$$C = \beta_1 T + \beta_2 T^3$$

(vi) Change in density of a liquid with temperature is given by,

$$d_2 = \frac{d_1}{1 + \gamma\,(t_2 - t_1)}$$

where γ is the cubical expansivity of the liquid.

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

3.2 IDEAL GASES AND THERMODYNAMICS

(i) The ideal gas equation is

$PV = \mu RT$, where μ is the number of moles of the gas and $R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

(ii) Volume Coefficient, $\alpha_V = \dfrac{V_t - V_0}{V_0 t}$ at constant pressure.

(iii) Pressure Coefficient, $\alpha_p = \dfrac{P_t - P_0}{P_0 t}$ at constant volume.

(iv) **A mole** of a substance is the amount of the substance that contains as many particles as there are atoms in exactly 12 grams of the isotope of carbon, C^{12}

(v) **A kilomole** is the mass in kg that is numerically equal to the molecular weight of the substance.

(vi) First law of Thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

ΔQ is the quantity of heat supplied or taken from the gas, ΔU is the change in internal energy and ΔW is the external work done.

(vii) **Molar Heat Capacities**

$C_v = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T}$ at constant volume

$C_p = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T}$ at constant pressure

(viii) $C_p - C_v = R$

(ix) $\frac{C_p}{C_v} = \gamma;\ C_v = \frac{R}{\gamma - 1};\ C_p = \frac{\gamma R}{\gamma - 1}$

(x) In an isothermal change, there is no change in temperature.

$$P_1V_1 = P_2V_2,\ \Delta U = 0$$

(xi) In an adiabatic change, the quantity of heat supplied or taken out is zero.

$$P_1V_1^{\gamma} = P_2V_2^{\gamma},\ \Delta Q = 0$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T_1^{\gamma}}{P_1^{\gamma-1}} = \frac{T_2^{\gamma}}{P_2^{\gamma-1}}$$

(xii) Work done in adiabatic change,

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

(xiii) Work done in isothermal change,

$$W = 2.303\ n\ RT \log_{10}\left(\frac{P_1}{P_2}\right)$$

$$= 2.303\ P_1V_1 \log_{10}\left(\frac{P_1}{P_2}\right)$$

(xiv) If PV^n = constant, it is called polytropic process (where $n \neq \gamma,\ \gamma = \frac{C_p}{C_v}$)

In this process,

molar heat capacity,

$$C = \left(\frac{R}{\gamma - 1}\right) - \left(\frac{R}{n - 1}\right).$$

(xv) Work done in a closed cycle is the area enclosed by PV graph.

(xvi) **Slope of PV diagram**

For isothermal process: $\left(\frac{dP}{dV}\right)_{\text{isoth.}} = -\frac{P}{V}$

For adiabatic process: $\left(\frac{dP}{dV}\right)_{\text{adia.}} = -\frac{\gamma P}{V}$

For isobaric process: $\left(\frac{dP}{dV}\right)_{isob.} = 0$

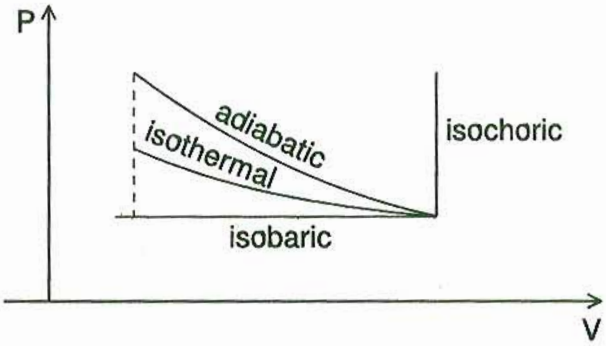
For isochoric process: $\Delta V = 0$ or V = constant $\Rightarrow W = 0$

(xvii) For Carnot cycle (or engine)

Efficiency, $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

(xviii) Change in entropy (S) = $\Delta S = \frac{\Delta Q}{T}$

(xix) Typical P-V diagrams will be as below.



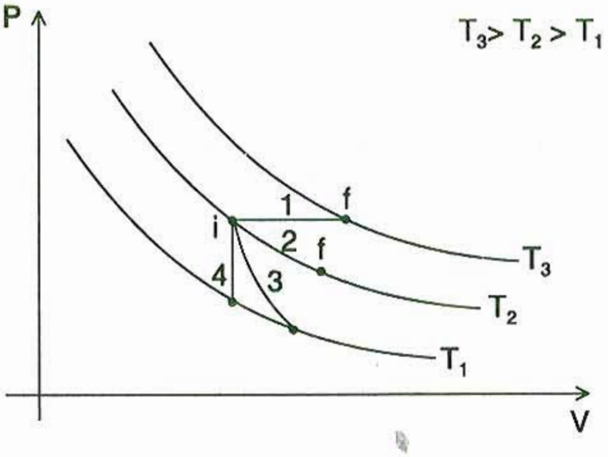
(xx)

Gas molecule	f = degrees of freedom	C_v	C_p	γ
Monoatomic, e.g., He	3	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Diatomic, e.g., O_2	5	$\frac{5}{2}R$	$\frac{7}{2}R$	1.4
Triatomic, e.g., CH_4	6	3R	4R	1.33

(xxi) The four thermodynamic processes for ideal gases and formulae for them.

Path in figure below	Constant quantity	Process type	$\left[\begin{array}{c}\text{Equations for all}\\ \Delta U = Q - W\\ \Delta U = nC_v\Delta T\end{array}\right]$
1	P	Isobaric	$Q = nC_p\Delta T$ $W = P\Delta V$
2	T	Isothermal	$Q = W = nRT\ \ln\left(\frac{V_f}{V_i}\right)$ $\Delta U = 0$

Path in figure below	Constant quantity	Process type	$\begin{bmatrix} \text{Equations for all} \\ \Delta U = Q - W \\ \Delta U = nC_v\Delta T \end{bmatrix}$
3	PV^γ; $TV^{\gamma-1}$	Adiabatic	$Q = 0$ $W = -\Delta U$
4	V	Isochoric	$Q = \Delta U$ $= nC_V\,\Delta T$ $W = 0$



If work done on the system is obtained as positive, work done by the system is negative.

⇒ We shall identify one of the bodies as our system to apply sign convention. Then, any transfer of energy out of the system is said to be positive work done by the system (or negative work done on the system).

3.3 KINETIC THEORY OF GASES

(i) Pressure of an ideal gas,

$$P = \frac{1}{3}\rho\,\overline{c^2}$$

where P is the pressure, ρ is the density of the gas and $\overline{c^2}$ is mean square velocity of the molecules.

(ii) **Root Mean Square Velocity**

$$c_{rms} = \sqrt{\frac{3P}{\rho}}$$

$$= \sqrt{\frac{3RT}{M_m}}$$

where R is the universal gas constant and M_m is the molecular weight.

(iii) Average kinetic energy of molecule, $\frac{1}{2}m_0\overline{c^2} = \frac{3\mu RT}{2N}$, where μ is the number of moles of the gas and N is the number of molecules in μ moles.

or $\frac{1}{2}m_0\overline{c^2} = \frac{3RT}{2N_A}$, where N_A is the Avogadro number $= \frac{3}{2}k_BT$,

where k_B is the Boltzmann constant.

(iv) Equipartition of Energy

The number of degrees of freedom of a point is three.

A rigid body, whose moment of inertia about each of the three mutually perpendicular axes passing through the centre of mass is not negligible, has six degrees of freedom.

If two spheres are connected by a rigid rod, the total number of degrees of freedom is five.

If two spheres are connected by a spring, the system has three degrees of freedom of translational motion, two degrees of rotational motion and one vibrational motion.

When the number of particles is large and Newtonian mechanics is applicable, the available energy which depends only on temperature distributes itself in equal shares to each degree of freedom. For one molecule, the energy is $\frac{1}{2}k_BT$ for one degree of freedom.

$\Rightarrow$ Each degree of freedom contributes $\frac{R \cdot T}{2}$ per mole, to the energy of the gas.

(v) Real or van der Waals' gas,

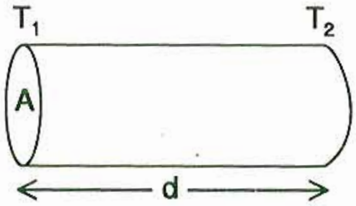
$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a, b are constants.

3.4 TRANSMISSION OF HEAT

(i) Thermal conductivity, K is given by,

$$Q = KA\,\frac{T_1 - T_2}{d}\,t$$



where K is the coefficient of thermal conductivity, T_1 and T_2 are the temperatures at two points down the length of the rod, separated by a distance d and A is the area of cross-section of the rod.

(ii) **Thermal resistance, (R)**

$R = \frac{\ell}{KA}$, where ℓ is length of rod.

(iii) Radiation of heat energy takes place in the form of electromagnetic waves which are called infrared waves.

(iv) Blackbody is one that absorbs all the radiation of all wavelengths falling on it.

(v) **Stefan Boltzmann Law**

$$E = \sigma (T^4 - T_0^4)$$

Here E is the energy radiated per second per unit area by a body at T K, surrounded by an environment at temperature T_0 K. σ is called Stefan's constant.

(vi) **Prevost's Theory:** Every body radiates energy at a rate controlled by its absolute temperature, T.

(vii) Newton's law of cooling,

$$\frac{dQ}{dt} = k(\theta - \theta_R)$$

where $\frac{dQ}{dt}$ is the rate of loss of heat of a body at temperature and θ, θ_R being the temperature of the surroundings.

(viii) Wien's displacement law and blackbody radiation,

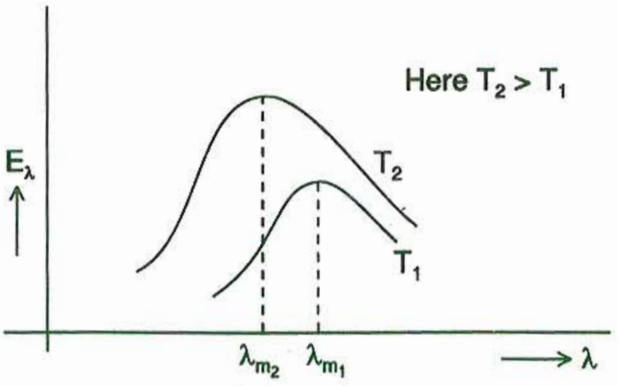
$\lambda_m T$ = a constant = 2.9×10^{-3} m-K

where λ_m is the wavelength in the radiation of a blackbody having maximum intensity.

(ix) **Kirchhoff's law in blackbody radiation**

It states that the ratio of emissive power (e_λ) to the absorptive power a_λ for the radiations at a particular wavelength and at a particular temperature is constant for all bodies and is equal to the emissive power, E_λ of a blackbody.

$$E_\lambda = \frac{e_\lambda}{a_\lambda}$$



(x) Mechanical equivalent of heat (J)

$J = \frac{W}{H}$, where W is work done, H is heat produced.

4. SOUND

4.1 WAVE MOTION

(i) The displacement due to a progressive wave travelling in the positive x-direction is

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

where y is the displacement at point x, at time t, A is the amplitude, T is the period and λ is the wavelength. The frequency is $n = \frac{1}{T}$ and the velocity of the wave is $\frac{\lambda}{T}$.

$$v_\omega = \frac{\lambda}{T} = n\lambda$$

Phase difference of waves,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

Particle velocity, $v_p = \frac{dy}{dt}$,

Wave velocity, $v_\omega = \frac{\omega}{k}$

$$v_p = v_\omega \times \frac{dy}{dx} = v_\omega \times \text{strain}$$

Group velocity, $v_g = \frac{d\omega}{dk} = v_\omega - \lambda \frac{dv_\omega}{d\lambda}$

(ii) The displacement due to a stationary wave is

$$y = \left(2A \cos \frac{2\pi x}{\lambda}\right) \sin \frac{2\pi t}{T}$$

(iii) Pitch, Intensity and Timbre are the characteristics of a musical note. Pitch depends on the frequency. Intensity is proportional to the square of the amplitude and timbre depends on the frequency and intensity of the constituent overtones.

(iv) Resonance occurs when the forcing frequency is equal to the natural frequency of a vibrating body.

(v) Velocity of propagation of sound in a gas = $\sqrt{\frac{\gamma P}{\rho}}$, where ρ is the density of the gas and γ is the ratio of specific heats.

(vi) Pressure variation in longitudinal wave,

$P = E\frac{dy}{dx}$, where $E = \frac{\text{stress}}{\text{strain}}$

If $y = A\sin(\omega t - kx)$,

P varies as: $P = P_0\cos(\omega t - kx)$

$\Rightarrow$ P differs in phase from y by $\frac{\pi}{2}$.

4.2 VIBRATING AIR COLUMNS

Distance between node and antinode = $\frac{\lambda}{4}$

(i) In a pipe of length L closed at one end, the fundamental note has a frequency $f_1 = \frac{v}{4L}$, where v is the velocity of sound in air.

The first overtone, $f_2 = \frac{3v}{4L} = 3f_1$ (III harmonic)

The second overtone, $f_3 = \frac{5v}{4L} = 5f_1$ (V harmonic),

end correction = 0.3R, where R is the radius of pipe at open end.

(ii) In a pipe of length L open at both ends, the fundamental note has a frequency $f_1 = \frac{v}{2L}$ (I harmonic).

The first overtone, $f_2 = \frac{v}{L} = 2f_1$ (II harmonic)

The second overtone, $f_3 = \frac{3v}{2L} = 3f_1$ (III harmonic)

end correction, x = 0.6R, where R is radius of open end.

4.3 PROPAGATION OF SOUND IN SOLIDS

(i) The velocity of propagation of a longitudinal wave in a rod of Young's modulus Y and density ρ is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

(ii) The velocity of propagation of a transverse wave in a stretched string,

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension in the string and m is the mass per unit length of the string.

(iii) In a sonometer wire of length L and mass per unit length m under tension T vibrating in n loops,

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

4.4 PROPAGATION OF SOUND IN GASES

Laplace formula, $v = \sqrt{\frac{\gamma P}{\rho}}$

where γ is the ratio of specific heats, P is the pressure and ρ is the density.

The velocity of sound in a gas is independent of pressure but is directly proportional to the square root of absolute temperature.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$

4.5 DOPPLER EFFECT

(i) When a source of sound moves with a velocity v_s in a certain direction, the wavelength decreases in front of the source and increases behind the source.

$$\lambda' \text{ (in front)} = \frac{v - v_s}{n}\ ;\ f' = \frac{v}{\lambda'} = \frac{v}{v - v_s}n$$

$$\lambda'' \text{ (behind)} = \frac{v + v_s}{n};\ f'' = \frac{v}{\lambda''} = \frac{v}{v + v_s}n$$

Here v is the velocity of sound in air.

(ii) When an observer moves with a velocity v_0 away from a source which is stationary, the number of waves reaching the observer per second is less than when the observer is at rest.

The apparent frequency $= \frac{v - v_0}{v}n$

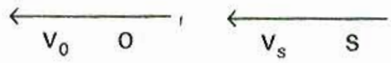
If the observer moves towards the source with a velocity v_0, the number of waves reaching him per second is more than when he is at rest.

The apparent frequency $= \frac{v + v_0}{v}n$

As a consequence of (i) and (ii) above we have the following results.

(a) When the source is moving towards the observer and the observer is moving away from the source, the apparent frequency

$$n' = \frac{v - v_0}{v - v_s} n$$



(b) When the source and the observer are moving towards each other,

$$n' = \frac{v + v_0}{v - v_s} n$$

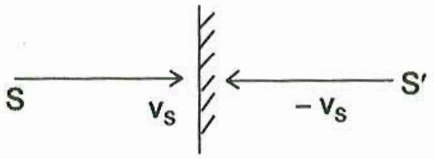
(c) When the source and observer are moving away from each other,

$$n' = \frac{v - v_0}{v + v_s} n$$

(d) When the source is moving away from the observer and the observer is moving towards the source,

$$n' = \frac{v + v_0}{v + v_s} n$$

(iii) When a source is moving towards a reflecting surface, with a velocity v_s, the echo may be considered to be equivalent to a source behind the wall moving with a velocity $(-v_s)$.



4.6 INTENSITY OF SOUND

The intensity level, B of sound is expressed in decibels,

$$B = 10 \log \frac{I}{I_0}$$

where I is the intensity, I_0 is a reference intensity.

$(I_0 = 10^{-12}\ W/m^2)$

4.7 BEATS

When two tuning forks of close but different frequencies, f_1 and f_2 are vibrating simultaneously at nearby places, a listener observes a fluctuation in the intensity of sound, called **beats**. The number of beats heard per second is $f_1 \sim f_2$.

5. ELECTRICITY AND MAGNETISM

5.1 ELECTROSTATICS

(i) Force between two electric point charges at rest: Coulomb's Law

$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ where q_1 and q_2 are the charges in **Coulomb**, r is the distance between the point charges in **metre**.

ε_0 is called the permittivity in free space. The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ metre/faraday.

F is obtained in **newton**.

Relative permittivity, $\varepsilon_r = k = \frac{\varepsilon}{\varepsilon_0}$

(ii) Field due to a point charge, $E = \frac{q}{4\pi\varepsilon_0 r^2}$ NC^{-1}

(iii) Electrical potential at a point, $V = \frac{W}{q_0}$ volt

where W is the work done in taking a test charge q_0 from infinity to that point. (W in joule, q_0 in coulomb, V in volt)

(iv) Potential due to a point charge,

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

q in coulomb, r in metre and V in volt.

(v) Capacitance of a sphere of radius R is $C = 4\pi\varepsilon_0 R$

R in metre, capacitance in **farad**.

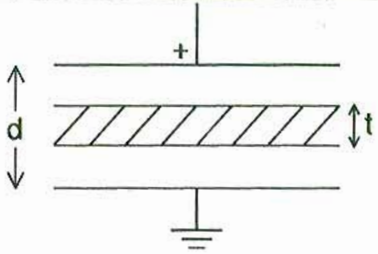
(vi) The capacitance of a parallel plate capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

A is the area of the plate in m², d is in metre and C is in farad.

(a) The capacitance of a parallel plate capacitor partly filled with dielectric of dielectric constant k.

$$C = \frac{\varepsilon_0 A}{\left[(d - t) + \frac{t}{k}\right]}$$



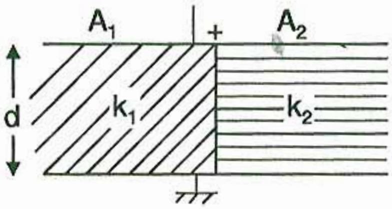
(b) If several slabs of dielectric constants k_1, k_2, k_3 ... and respective thickness t_1, t_2, t_3, ... be placed between the two plates, then the capacitance

$$C = \frac{\varepsilon_0 A}{\left[\left(d - (t_1 + t_2 + t_3 + \ldots)\right) + \frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \ldots\right]}$$

If $d = t_1 + t_2 + t_3 + \ldots$

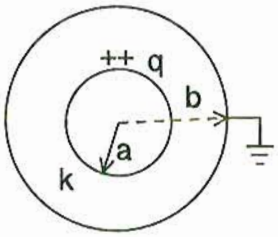
$$C = \frac{\varepsilon_0 A}{\left[\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \ldots\right]}$$

(c) Capacitance $C = \frac{\varepsilon_0 k_1 A_1}{d} + \frac{\varepsilon_0 k_2 A_2}{d}$



(d) Capacitance of a spherical capacitor

$$C = \frac{4\pi\varepsilon_0 kab}{b - a}$$



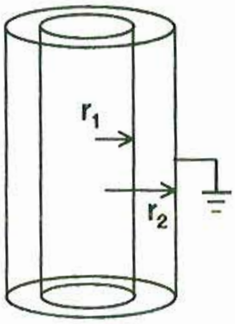
if the outer sphere is earthed.

If the inner sphere is earthed and the outer sphere is charged, then the capacity of condenser becomes

$$C = 4\pi\varepsilon_0 b + \frac{4\pi\varepsilon_0 ab}{b - a} = \frac{4\pi\varepsilon_0 b^2}{b - a}$$

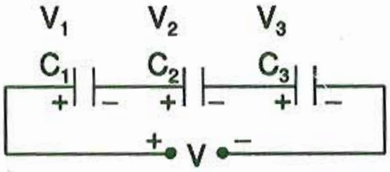
(e) Capacitance of a cylindrical condenser,

$$C = \frac{2\pi\varepsilon_0 \ell k}{\log_e \frac{r_2}{r_1}}$$



(vii) Condensers in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

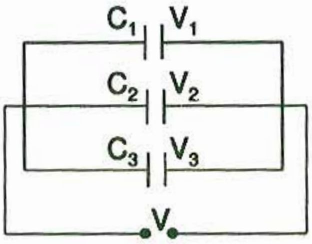


In series connection, the charge on each plate is the same. $V = V_1 + V_2 + V_3$

(viii) Condensers in parallel,

$C = C_1 + C_2 + C_3$

$V = V_1 = V_2 = V_3$



(ix) For a capacitor, $Q = CV$

(x) Energy in a charged capacitor,

$$W = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\cdot\frac{Q^2}{C}$$

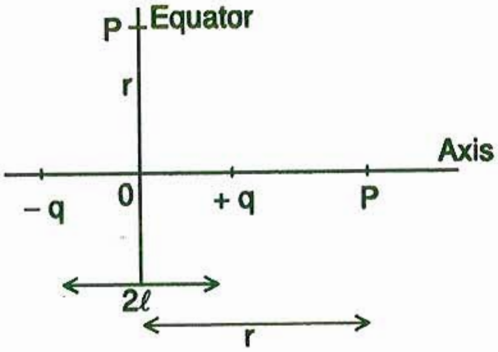
(xi) **Electric Dipole**

Two equal and opposite charges separated by a finite distance (2ℓ) constitute an electric dipole of moment, $\vec{p} = q \cdot 2\vec{\ell}$

Electric field due to dipole

(a) at a point P on axis

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - \ell^2)^2}$$



For short dipole (r >> ℓ) on axis

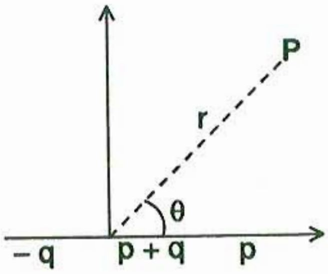
$$E = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}$$

$$\text{Potential at P} = V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$$

(b) at a point on equator (at distance r)

$$E = \frac{p}{4\pi\varepsilon_0 (r^2 + \ell^2)^{\frac{3}{2}}}$$

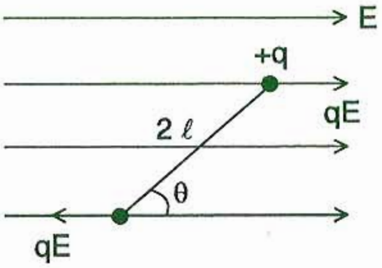
$$V = 0$$



$$\text{At any point P, } E = \frac{p}{4\pi\varepsilon_0} \frac{\sqrt{3\cos^2\theta + 1}}{r^3}$$

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

(xii) Electric dipole in uniform electric field



Torque $\vec{\tau} = \vec{p} \times \vec{E}$

Work in rotating from θ_1 to θ_2

$W = PE\ (\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 90°$, $\theta_2 = \theta$

Potential energy, $U = -\,pE\cos\theta$

$$= -\,\vec{p}\cdot\vec{E}$$

(xiii) Electric flux (and Gauss law)

$$\phi_E = \int_S \vec{E}\cdot\vec{ds}$$

$$\int_S \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\ (\Sigma q) \text{ enclosed by S}$$

(xiv) Near a thin flat sheet of charge of surface charge density σ,

$$E = \frac{\sigma}{2\,\varepsilon_0}$$

(xv) Near a conductor of any shape,

$$E = \frac{\sigma}{\varepsilon_0}$$

(xvi) Motion of a charged particle in an electric field

$$\vec{F} = q\,\vec{E}$$

$$\vec{a} = \frac{q\,\vec{E}}{m}$$

If it enters the field E in a perpendicular direction with speed u, its path is a parabola,

$$y = \frac{qE}{2mu^2}\,x^2$$

(xvii) Force on a surface charge per unit area

$$F = \frac{1}{2}\varepsilon E^2 \text{ N/m}^2$$

Energy stored in electric field per unit volume

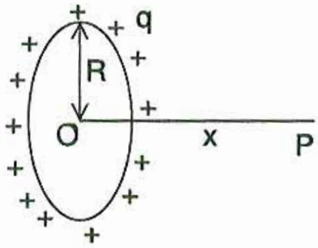
$$U_E = \frac{1}{2}\varepsilon E^2 \text{ J/m}^3$$

Note: $1\ \mu F = 10^{-6}$ F

$1\ pF = 10^{-12}$ F

(xviii) Electric field due to a ring of charge q at a distance x along the axial line

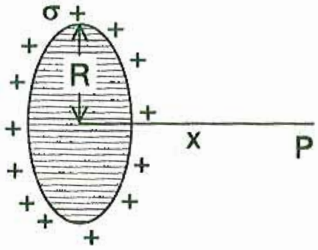
$$E = \frac{qx}{4\pi\varepsilon_0(R^2 + x^2)^{3/2}}$$



$$\text{Potential, } V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{R^2 + x^2}}$$

(xix) Electric field due to a disc of charge density σ at a distance x along the axial line

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$



$$\text{Potential } V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + x^2} - x\right]$$

(xx) Electric field due to finite line of charge of linear charge density λ

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 x}[\sin\alpha + \sin\beta]$$

(xxiii) Self-energy of a uniformly charged sphere

$$U = \frac{3}{5} \cdot \frac{Q^2}{4\pi\varepsilon_0 R}$$

Self-energy of a uniformly charged spherical cond

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

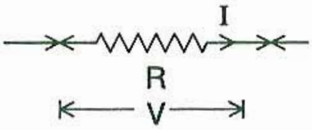
5.2 ELECTRIC CURRENTS

(i) **Current** in a conductor is the charge flowing across a poi[illegible] unit time

$I = \frac{Q}{t}$; unit is ampere

When a current of 1 ampere flows across a point for 1 s, the charge transformed is 1 coulomb.

(ii) **Ohm's Law:** $I = \frac{V}{R}$



If V is in volt, I in ampere and R is in ohm, R is the electrical resistance of the conductor.

(iii) Specific resistance (resistivity) ρ of a conductor is given by,

$\rho = \frac{RA}{L}$, where A is the area of cross-section and L, the length of the conductor. If R is in ohm, A in m^2 and L in m, ρ is in ohm-metre.

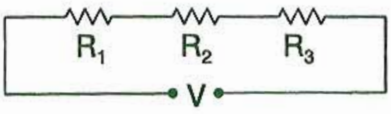
$R = \rho\frac{L}{A}$, $\sigma = \frac{1}{\rho} = \frac{J}{E}$, $J = nev_{drift}$

$$J = \frac{I}{A}$$

(iv) Temperature and resistance

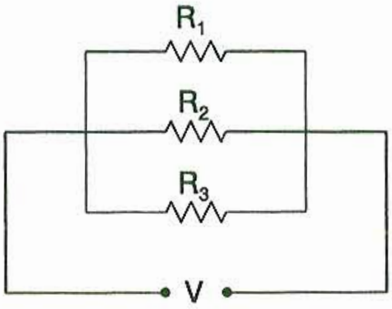
$R_t = R_0(1 + \alpha t)$; α is the temperature coefficient of resistance.

(v) Series connection: $R = R_1 + R_2 + R_3$



(vi) Parallel connection

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$



Note: When it is not that much clear about series or parallel connections in a network but if there is symmetry (mirror image like) in the network, follow equipotential method to draw equivalent network and find resultant resistance between two points.

(vii) Terminal potential difference of a battery of emf ε and internal resistance r

$= \varepsilon - ir$, where 'i' is the current in the battery.

(viii) **Joule's Law:** Heat produced in a conductor of resistance R, when a current I flows for t seconds = $I^2 Rt$ joules.

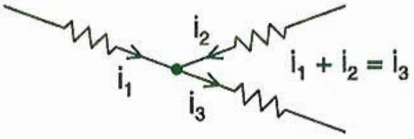
(ix) Power in a circuit = VI watts

$= I^2R$ watts

$= \frac{V^2}{R}$ watts

(x) **Kirchhoff's Rules**

(a) The sum of the currents that flow into a junction is equal to the sum of the currents that flow out of the junction.



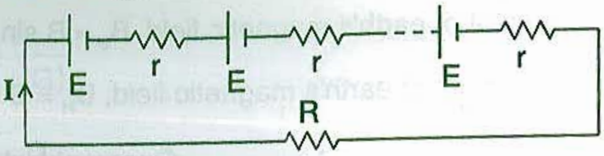
(b) The sum of the e.m.f.'s around a loop is equal to the sum of the IR potential drops around the loops.

In writing $\Sigma I_k = 0$, the current 'into' a node (or junction) is taken to be positive.

In applying Kirchhoff's voltage law ($\Sigma E_k = \Sigma I_\ell R_\ell$), if there are more than one source when the directions do not agree, the voltage of source is taken as positive, if it is in the direction of assumed current.

Note: Kirchhoff's current law (KCL) and voltage law (KVL) can be applied to electric as well as electronic circuits.

(xi) (a) Combination of n cells in series



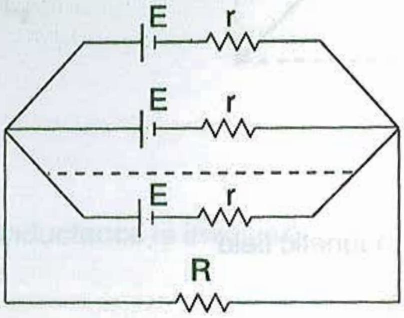
E_t = Total e.m.f. = nE

R_i = Total internal resistance = nr

R_t = Total resistance of circuit = R + nr

$$I = \frac{nE}{R + nr}$$

(b) m cells in parallel



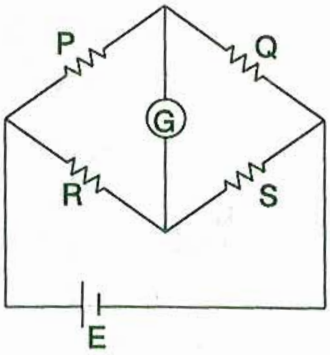
Total e.m.f., $E_t = E$ (only)

Total internal resistance, $R_i = \frac{r}{m}$

Total resistance of circuit, $R_t = R + \frac{r}{m}$

$$I = \frac{E}{R + \frac{r}{m}}$$

(xii) Wheatstone bridge (applied to metre bridge and for capacitor also)

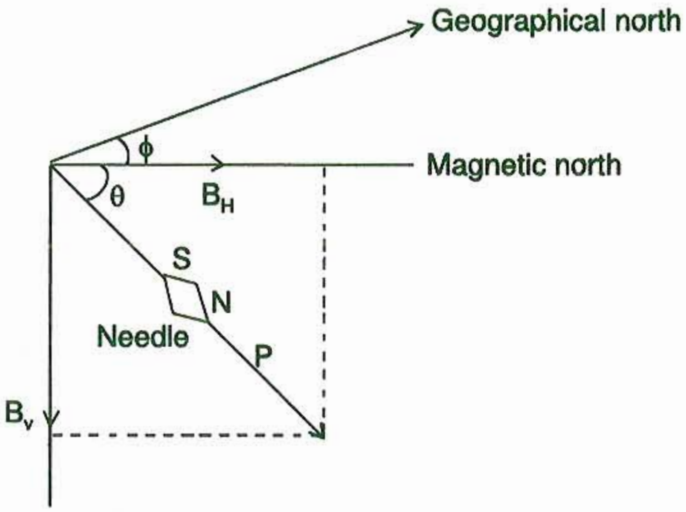


For null deflection in (G), $\frac{P}{Q} = \frac{R}{S}$.

(xiii) Terrestrial magnetism

Vertical component of earth's magnetic field, $B_v = B \sin\theta$

Horizontal component of earth's magnetic field, $B_H = B \cos\theta$



where $B = \sqrt{B_H^2 + B_v^2}$

= Resultant magnetic field

θ = angle of dip

ϕ = angle of declination

At magnetic equator, $\theta = 0°$

At magnetic pole, $\theta = 90°$

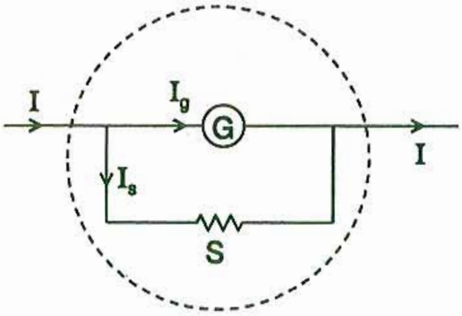
$$\tan\theta = \frac{B_v}{B_H}.$$

(xiv) Meters to measure current and voltage

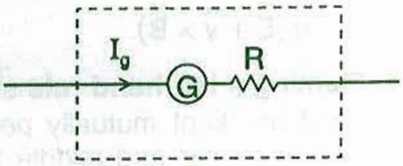
(a) Ammeter

$$I_g G = (I - I_g) S$$

$$I_g = \frac{IS}{(S+G)}$$



(b) Voltmeter

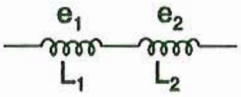


$$I_g = \frac{V}{(R+G)}$$

$$R = \frac{V}{I_g} - G$$

(xv) Inductances in groups

(a) Series

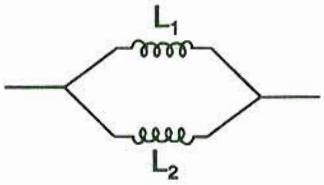


Effective e.m.f., $e = e_1 + e_2$

$$\Rightarrow -L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt}$$

$$\Rightarrow L = L_1 + L_2$$

(b) Parallel



$$L = \frac{L_1 L_2}{L_1 + L_2}$$

(xvi) When mutual (M) inductance is involved

(i) Series

(a) When flux linkage is in same sense

$$L = L_1 + L_2 + 2M$$

(b) When flux linkage is in opposite sense

$$L = L_1 + L_2 - 2M$$

(ii) Parallel: Coefficient of coupling,

$$K = \frac{\pm M}{\sqrt{L_1 L_2}}$$

$K \leq 1$ (equality holds for 'tight' coupling)

5.3 MAGNETISM AND ELECTROMAGNETIC INDUCTION

(i) If a force F is exerted by a magnetic field on a small length L of wire carrying an electric current I, the magnetic induction, $B = \frac{F}{IL \sin\theta}$

where θ is the angle between I and B. If F is in newtons, I in amperes, L in m, B is in tesla.

(ii) **Lorentz Force:** A charge q moving in a region where there is an electric field $\vec{E}$ as well as a magnetic field $\vec{B}$ experiences a force,

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

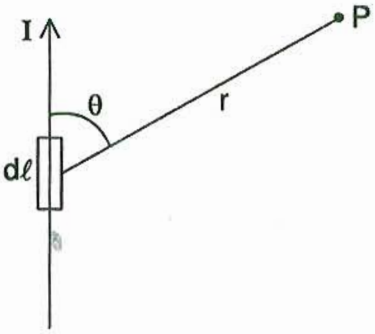
Fleming's left-hand rule states, "when fore finger, middle finger and thumb of left-hand are kept mutually perpendicular and if the fore finger is in the direction of magnetic field and middle finger is in the direction of velocity of the particle, then Lorentz force will act (a) in the direction of thumb, if the particle is positively charged and (b) in the opposite direction of thumb, if the particle is negatively charged.

(iii) Magnetic field induction due to a current element (Biot-Savart law)

$$dB = \frac{\mu_0 I d\ell \sin\theta}{4\pi r^2}$$

If the linear conductor is grasped (held) in the palm of the right–hand with thumb pointing along the direction of current, then the finger tips will point in the direction of lines of force.

(iv) Magnetic field intensity, $H = \frac{B}{\mu}$



$\mu = \mu_r \times \mu_0$

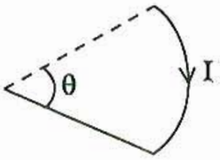
Unit of H is ampere per metre (Am^{-1})

(v) **(a)** Magnetic flux density at the centre of a circular coil of radius R carrying a current I,

$$B = \frac{\mu_0 N I}{2R}$$, where N is the number of turns in the coil.

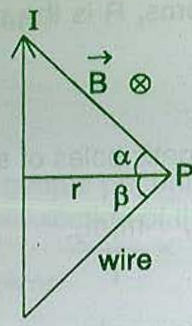
(b) Magnetic induction at the centre due to circular arc conductor carrying current I

$$B = \frac{\mu_0 I \theta}{4\pi R}$$



(vi) **(a)** Magnetic flux density about a long straight conductor at a point P, distant r from its centre,

$$B = \frac{\mu_0 I}{2\pi r}$$



(b) Due to a finite length of conductor,

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta).$$

(vii) The force F on a straight conductor carrying a current placed in a magnetic field of induction B is

$F = BI_1L_1 \sin\theta,$

where θ is the angle between I and B.

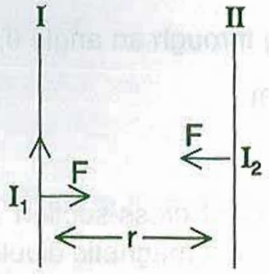
The conductor of length L_1 placed at right angles to B experiences a force,

$F = BI_1L_1$

If the magnetic field is due to a parallel conductor of length L_2 carrying a current I_2 placed at a distance r from the first conductor,

$$B = \frac{\mu_0 I_2}{2\pi r}$$

The force on $L_1 = \frac{\mu_0 I_2}{2\pi r} \cdot I_1 L_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2L_1}{r}$



The force is attractive if the currents are in the same direction, and repulsive if they are in opposite directions.

(viii) Flux density along the axis of a circular coil carrying current I,

$$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$$

where N is the number of turns, R is the radius and x is the distance of point from the center of the coil.

(ix) Permanent Magnets

(a) Force between two magnetic poles of strength m_1 and m_2,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{m_1 m_2}{r^2}$$

(b) Magnetic induction due to a pole,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^2}; \; F = mB$$

(c) Magnetic field intensity due to pole,

$$H = \frac{B}{\mu_0} = \frac{1}{4\pi} \cdot \frac{m}{r^2}$$

(d) Magnetic moment of a bar magnet (dipole) $M = 2m\ell$, where the distance between the poles $= 2\ell$ and m is the pole strength.

(e) Field due to a bar magnet (for tan A, tan B positions of deflection magnitudes),

$$H = \frac{1}{4\pi} \cdot \frac{2Md}{(d^2 - L^2)^2} \text{ (along axis)}$$

$$H = \frac{1}{4\pi} \cdot \frac{M}{(d^2 + L^2)^{\frac{3}{2}}} \text{ (along perpendicular bisector)}$$

(f) Torque on a dipole in a magnetic field,

$\tau = MB_e \sin\theta$, where M is the moment of the dipole, B_e is the external magnetic field and θ is the angle between axis of magnet and B_e.

Work done in rotating through an angle θ,

$$W = MB\,(1 - \cos\theta)$$

(g) Current Loop

A current loop of area of cross-section A, and number of turns N, carrying a current I is equivalent to a magnetic dipole of magnetic moment NIA.

(a) $\mu_m = NIA$

(b) Magnetic potential energy

$$= -\vec{\mu}_m \cdot \vec{B}$$

(c) Magnetic force, $F = \mu_m \dfrac{\partial B}{\partial \ell}$

(h) Moving coil galvanometer

$$I = \frac{c}{BNA}\,\theta$$

where c is the torsional couple per unit twist, N is the number of turns in the coil, and A is the area of cross-section of the coil, and θ is the deflection in radians.

(i) Electromagnetic Induction

e.m.f. induced, $E = -N\frac{d\phi}{dt}$, where N is the number of turns in the coil and ϕ is the magnetic flux linked with the coil.

(j) Mutual Inductance: $M = N\frac{d\phi_B}{dt}$, ϕ_B is in weber, M is in henry.

Self-inductance of a solenoid, $L = \frac{\mu N^2 A}{\ell}$, where N is the number of turns per unit length and ℓ is the length of the coil.

$$\text{e.m.f. induced} = -L\frac{di}{dt}$$

$$\phi_B = Li$$

$$\text{Energy in a solenoid} = \frac{1}{2}Li^2$$

(x) Cyclotron

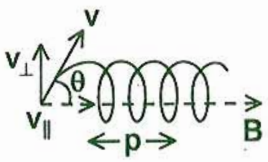
$$\frac{mv^2}{r} = Bqv$$

Period of uniform circular motion,

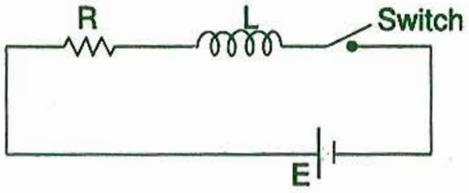
$$T = \frac{1}{f} = \frac{2\pi m}{Bq}$$

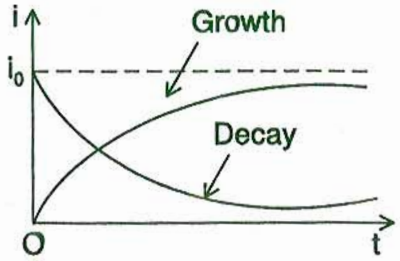
(xi) If particle with charge q and mass m is shot with a speed of v at an angle of θ to a magnetic field B, the path is spiral (or helix) if pitch $p = v_{\parallel}\,T$; $r = \frac{mv_{\perp}}{qB}$; period, $T = \frac{2\pi r}{v_{\perp}}$

pitch, $p = v\cos\theta\left(\frac{2\pi m}{Bq}\right)$



(xii) LR circuit





For growth of current, $i = i_0\left[1 - e^{-\frac{R}{L}t}\right]$

For decay of current, $i = i_0\, e^{-\frac{R}{L}t}$

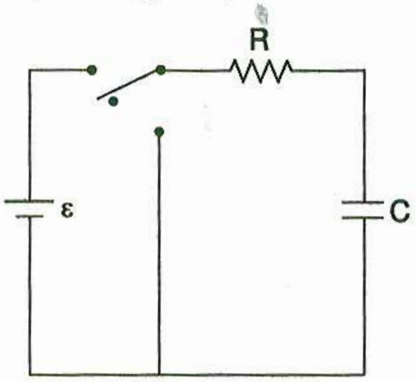
The quantity $\left(\frac{L}{R}\right)$ is called the time constant τ of the circuit.

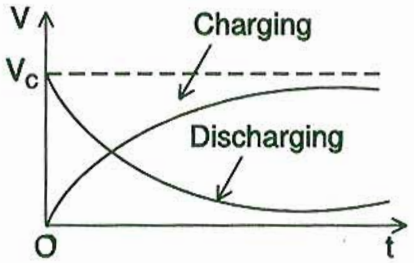
(xiii) CR circuit

During charging: $q = C\,\varepsilon\left(1 - e^{-t/RC}\right)$

During discharging: $q = q_0\, e^{-t/RC}$

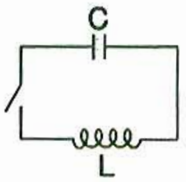
(where $q_0 = C\varepsilon$)





Time constant, $\tau = RC$

(xiv) LC oscillations (analogy with mass and spring)



$$\omega = 2\pi f$$

$$f = \frac{1}{T}$$

LC circuit	Mass-spring
L, C	$m, \frac{1}{k}$
$U_m = \frac{1}{2} Li^2$	$E_k = \frac{1}{2} mv^2$
$U_e = \frac{1}{2} \cdot \frac{q^2}{C}$	$E_p = \frac{1}{2} kx^2$
$q = Q \cos\left(\sqrt{\frac{1}{LC}} \cdot t\right)$	$x = A \cos\left(\sqrt{\frac{k}{m}} \cdot t\right)$
$\omega = \sqrt{\frac{1}{LC}}$	$\omega = \sqrt{\frac{k}{m}}$

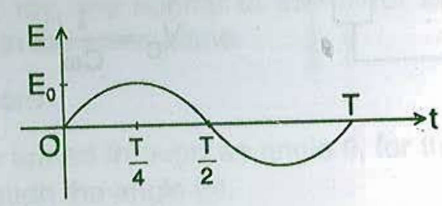
Alternating current (a.c.)

$$E = E_0 \sin \omega t$$

$$E_0 = \omega NBA$$

$$i_0 = \frac{E_0}{R_L}$$

$$i = i_0 \sin \omega t$$



For currents or voltages, the following ratios hold good.

$$\frac{\text{r.m.s. value}}{\text{peak value}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\frac{\text{mean value}}{\text{peak value}} = \frac{2}{\pi} = 0.637$$

$$\frac{\text{r.m.s. value}}{\text{mean value}} = \frac{\pi}{2\sqrt{2}} = 1.11 = \text{form factor}$$

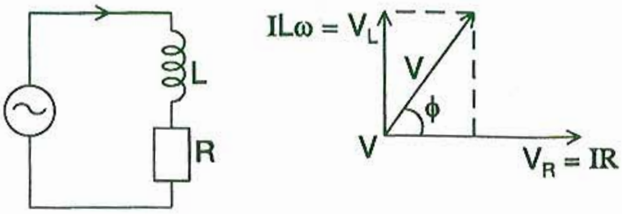
$$\text{Power in a.c. circuit} = \frac{1}{2} V_{peak} \cdot I_{peak} \cos \phi$$

where $\cos \phi$ is called power factor.

Impedance $= \frac{V}{I}$, Reactance = X (which could be capacitive or inductive)

Note: $I_{r.m.s.} \sin \phi$ is wattless.

LR a.c. circuit



$$V = \sqrt{V_R^2 + V_L^2}$$

$$Z = \sqrt{R^2 + X_L^2}$$

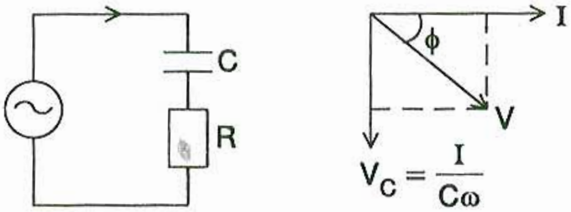
where $X_L = L\omega$

$$\tan\phi = \frac{L\omega}{R}$$

CR a.c. circuit

$$V = I\sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

$$\tan\phi = \frac{1}{C\omega R}$$



LCR – Resonant circuit

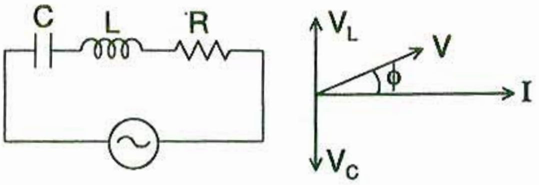
Series

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}, \cos\phi = \frac{R}{Z}$$

$$\text{Resonant frequency } \omega = \frac{1}{\sqrt{LC}}$$

(with R = Z, $\phi = 0$)



6. OPTICS

New Cartesian Convention

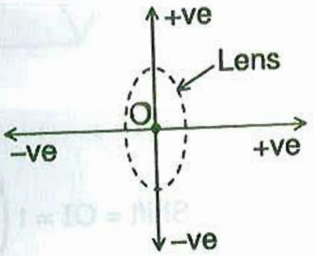
(1) All the distances are measured from pole.

(2) Distances measured in the direction of incident ray are taken to be positive while the distances measured in opposite direction to the incident ray are taken as negative.

(3) Heights above the principle axis are positive while the heights below the principle axis are negative.

(4) Rays are plotted from left to right.

With or without optic centre (it being thin lens) as origin of coordinate system, the distances and heights (sizes) of objects and images are given positive, negative signs as per their location (quadrant) in the rectangular axes system.



6.1 REFLECTION AND REFRACTION

(i) Laws of Reflection

(a) The angle of incidence is equal to the angle of reflection.

(b) The incident ray, the normal to the mirror at the point of incidence, and the reflected ray are in the same plane.

(ii) Rotation of Mirrors

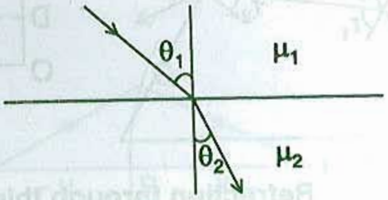
When a mirror is turned through an angle θ, for the same incident ray, the reflected ray is turned through the angle 2θ.

(iii) The image formed by a plane mirror is as far behind the mirror as the object is in front of it. The image is erect, virtual, is of the same size as the object, and is laterally inverted.

(iv) Laws of Refraction

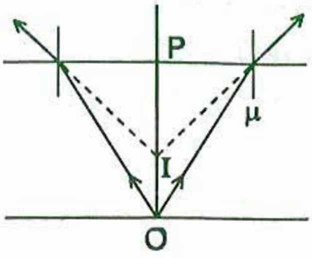
(a) The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray are in the same plane.

(b) The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for the two media.



$$\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$$

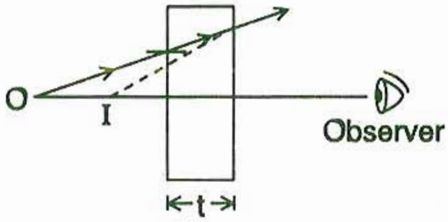
(v) When an object inside a medium of refractive index, μ is viewed normally from air,



$$\mu = \frac{\text{Real Depth}}{\text{Apparent Depth}} = \frac{PO}{PI}$$

(vi) $\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the medium}}$

(vii) When a slab of glass is placed between an object and observer, the object appears shifted through a distance OI.



$$\text{Shift} = OI = t\left(1 - \frac{1}{\mu}\right)$$

(viii) If C is the critical angle of a medium, the refractive index, $\mu = \frac{1}{\sin C}$

(ix) **Refraction through prism**

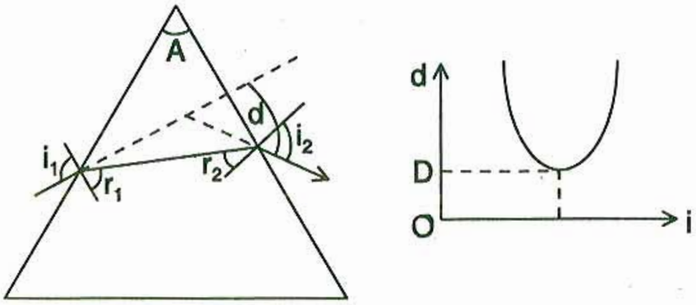
Angle of deviation,

$d = i_1 + i_2 - A$

$A = r_1 + r_2$

In minimum deviation,

$$\mu = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$$



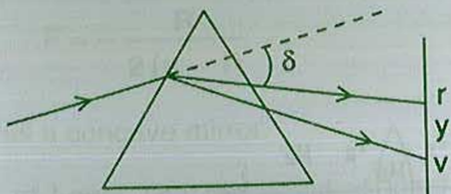
Refraction through thin prism

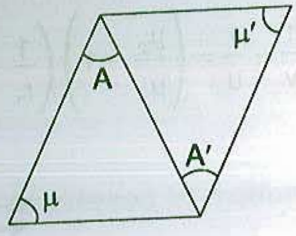
Angle of deviation, $d = (\mu - 1) A$

For a prism

Dispersive power, $\omega = \dfrac{\delta_v - \delta_r}{\delta_y}$

Angular dispersion = $\delta_v - \delta_r$





Deviation without dispersion

$A(\mu_v - \mu_r) = \left(\mu'_v - \mu'_r\right) A'$

For dispersion without deviation

Condition is $(\mu - 1)A = (\mu' - 1)A'$

(i) **Refraction at Spherical Surfaces**

(a) For mirrors,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Magnification, $m = -\dfrac{v}{u}$

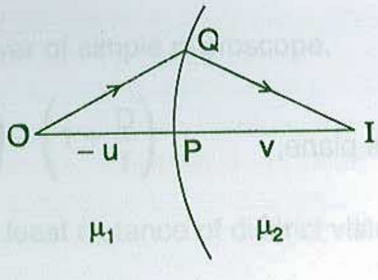
(b) For thin lenses,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Magnification, $m = \dfrac{v}{u}$

(ii) **Refraction at a curved surface**

$$\frac{\mu_1}{u} - \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r}$$

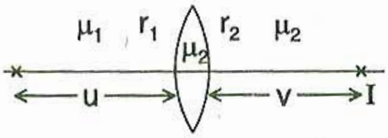


(iii) Lens maker's formula,

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

(iv) Thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

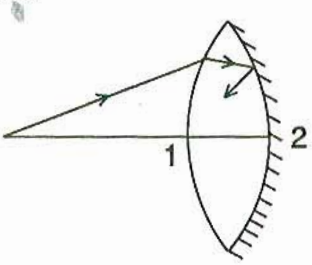


(v) Effect of silvering one of the refracting surface of a lens.

If one of the surfaces of a lens is silvered as shown in **Figure**. The rays are refracted at surface 1, reflected at surface 2 and again refracted at surface 1. The focal length, F of the effective mirror is given by,

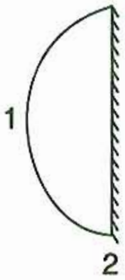
$$\frac{1}{F}=\sum_{n=1,2}\frac{1}{f_n}$$

where f_n = focal length of lens or mirror repeated as many times as there are reflections and refractions.



In the case shown in **Figure**, there are two refractions and one reflection.

$$P_{\text{mirror}}=P_{\text{lens}}+P_{\text{mirror}}+P_{\text{lens}}$$



Hence, $-\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_1}-\frac{1}{f_m}=\frac{2}{f_1}-\frac{1}{f_m}$

where f_1 = focal length of lens,

f_m = focal length of spherical mirror

$$f_m=\left(-\frac{R}{2}\right)$$

If the second surface is plane,

$$-\frac{1}{F}=\frac{2}{f_1}$$

$$F = -\frac{f_1}{2}$$

$$\frac{1}{f_1} = (\mu - 1)\frac{1}{R}$$

$$F = -\frac{R}{2(\mu - 1)}$$

It acts as a concave mirror.

(vi) Power of Lens: The reciprocal of focal length (expressed in metres) of a lens is called its power (unit is 'Dioptre').

Equivalent focal length of two lenses in contact,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power, $P = P_1 + P_2$

If two lenses are separated by a distance d, then the focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$P = P_1 + P_2 - d\,P_1P_2$$

(vii) Focal length f_ℓ of lens immersed in liquid of refractive index μ_ℓ

$$_\ell\mu_g = \frac{\mu_g}{\mu_\ell}$$

$$f_\ell = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_\ell} - 1\right)} \times f_a$$

where f_a is the focal length of the lens in air.

(viii) (a) Magnifying power of simple microscope,

$$M = \left(1 + \frac{v}{f}\right) = \left(1 + \frac{D}{f}\right)$$

where D is the least distance of distinct vision.

(b) Magnifying power of compound microscope,

$$M = \frac{\text{Angle subtended by image at eye } (\beta)}{[\text{Angle subtended by the object at eye when the object is supposed to be at the position of final image } (\alpha)]}$$

$$= -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

If the final image is formed at infinity,

$$M = -\frac{v_o}{u_o}\frac{D}{f_e}$$

(ix) For the Astronomical telescope,

$$M = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

For the normal adjustment, $M = -\frac{f_o}{f_e}$

Maximum length of the telescope,

$$L = f_o + f_e$$

For the Terrestrial telescope,

$$M = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

For normal adjustment, $M = -\frac{f_o}{f_e}$

Maximum length of the telescope, $L = f_o + f_e + 4f$, where f is the focal length of the erecting lens.

(x) **(a)** Limit of resolution of telescope $= \frac{1.22\,\lambda}{d}$

Limit of resolution of microscope $\propto \frac{\lambda}{\text{cone angle}}$.

Resolving limit of eye = 1 minute.

(b) Methods of reducing spherical aberrations in lens is

(i) using stops

(ii) by using lenses of large focal lengths

(iii) by using plano-convex lenses

(iv) by using crossed lenses

(v) by combining convex and concave lenses

(vi) by using two plano-convex lenses separated by a distance $d = f_1 - f_2$

(vii) by using paraboloidal mirror

(c) To eliminate chromatic aberration,

(i) Condition of achromatism for two thin lenses in contact

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

If $\omega_1 = \omega_2$, $f_1 = -f_2$

(ii) Condition for achromatism for two thin lenses separated by the distance d,

$$d = \frac{f_1 + f_2}{2}$$

6.2 INTERFERENCE

(i) The wavelengths of visible light are between 4.3×10^{-7} m and 6.9×10^{-7} m.

(ii) Result of superposition of two wave trains with a phase difference, ϕ

$$y = \left(2y_0 \cos\frac{\phi}{2}\right) \sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

(iii) Phase difference and path difference,

$$\phi = \frac{2\pi\delta}{\lambda}$$

where δ is the path difference and λ is the wavelength.

If two coherent waves with intensity I_1 and I_2 are superimposed with a phase difference of ϕ, the resulting wave intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

The resulting amplitude,

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi,$$

where $A_1^2 = I_1$; $A_2^2 = I_2$.

(iv) Young's Double Slit

(a) Condition for dark band,

$$\delta = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots\ldots$$

(b) Condition for bright band,

$\delta = m\lambda$, where m = 0, 1, 2, ...

(c) Fringe width, $\beta = \frac{D\lambda}{a}$

where D is the distance between the double slits and the screen and a is the distance between the two slits.

(d) Distance of mth dark fringe from centre $= \frac{(2m-1)}{2a} D\lambda$,

where m = 1, 2, 3, etc.

7. MODERN PHYSICS

7.1 ATOMIC PHYSICS

(i) Charge of an electron $= -1.602 \times 10^{-19}$ C

(ii) Photoelectric effect

(a) $h\nu = W + \frac{1}{2} m_e v_m^2$

where W is the photoelectric work function, ν is the frequency of incident radiation, and v_m is the maximum velocity of emitted electron.

(b) If ν_0 is the threshold frequency, $W = h\nu_0$

If V is the stopping potential,

$$eV = \frac{1}{2} m_e v_m^2$$

where v_m is the maximum speed of ejection.

(c) Energy of photon $= \frac{hc}{\lambda}$

(iii) Bohr Atom Model (S.I. units)

(a) Radius of the first Bohr orbit

$$r_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10} \text{ m}$$

(b) Energy of electron in the nth orbit

$$= -\frac{1}{\varepsilon_0^2}\cdot\frac{Z^2me^4}{8n^2h^2} = -\frac{Z^2\,(13.6\text{ eV})}{n^2}$$

(c) Spectral Series:

Wave number, $\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$

where, $R = \frac{me^4}{8\,\varepsilon_0^2 ch^3} = 1.097 \times 10^7\text{ m}^{-1}$

(d) Ionisation energy of hydrogen atom from ground state = 13.6 eV

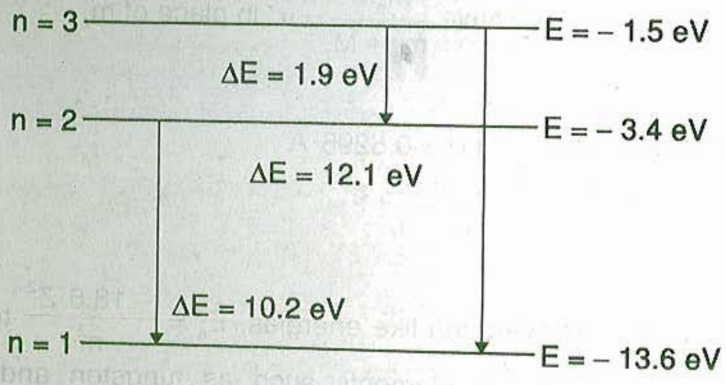
Energy of hydrogen atom in the ground state = – 13.6 eV

Energy of hydrogen atom in the first excited state (n = 2)

$$= -\frac{13.6}{4} = -3.4\text{ eV}$$

Energy of hydrogen atom in the second excited state (n = 3)

$$= -\frac{13.6}{9} = -1.5\text{ eV}$$



(e) The values of following combinations of universal constants are useful sometimes in solving a question.

$$\frac{hc}{e} = 1240\text{ eV-nm}$$

$$\frac{e^2}{\hbar c} = \frac{1}{137} \qquad (\text{where } \hbar = \frac{h}{2\pi})$$

$$\frac{h}{mc} = 0.024\text{ Å (for electron)}$$

Bohr magneton, $\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\text{ J/T}$

r_1 = First Bohr-radius

$$= \frac{\varepsilon_0 h^2}{\pi m_e e^2} = 0.5293\ \text{Å}$$

(f) **For nth Bohr-orbit**

$$E_n = \frac{-13.6}{n^2}\ \text{eV for Hydrogen atom}$$

$$E_n = \frac{-Z^2}{n^2} \times 13.6\ \text{eV (for Hydrogen like atom)}$$

$$r_n = \frac{n^2 r_1}{Z}$$

$$v_n = v\frac{Z}{n},\ \text{where}\ v = \frac{e^2}{2h\varepsilon_0}$$

(g) **Series limits** (of Balmer etc. in hydrogen atomic spectra) are calculated for transitions from $n = \infty$ state.

If mass of nucleus, M is also taken into account, the above formulae (of Bohr's theory) will have $\frac{m_e M}{m_e + M} = \mu'$ in place of m_e.

$$\text{Thus, } r_1' = \left(1 + \frac{m_e}{M}\right) r_1 = 0.5296\ \text{Å}$$

(iv) **X-rays**

(a) One can use hydrogen like energies, $E_n = \frac{-13.6\ Z^2}{n^2}$ to estimate electron energies for heavy elements such as tungsten and then equate the difference in those energies to $\frac{hc}{\lambda}$ to find λ of X-rays.

(b) **Mosley's law**

Frequency (say K_α like X-ray)

$\nu = a(Z - b)^2$ (Here a is constant)

$b = 1$ for K_α line.

(c) **Bragg's Law**

$$2d \sin\theta = n\lambda$$

where n = 1, 2, 3, ...

d is lattice distance,

and θ is glancing angle

7.2 NUCLEAR PHYSICS

(i) Law of radioactive decay

$$N = N_0 e^{-\lambda t}$$

where λ is the decay constant, N_0 the number of atoms at t = 0 and N, the number of atoms remaining after time t.

$$\text{Activity} = \frac{dN}{dt} = -\lambda N \text{ (unit is becquerel)}$$

$$\text{Half-life period, } T_{1/2} = \frac{0.693}{\lambda}$$

(ii) Variation of mass with velocity,

$$m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}$$

where m_0 is the rest mass, u is the velocity of the particle and c is the speed of light in vacuum.

Newton's second law in relativity,

$$F = \frac{dp}{dt} = \frac{d}{dt}\left[\frac{m_0 u}{\sqrt{1 - \frac{u^2}{c^2}}}\right]$$

Energy, $E = mc^2$

1 unified mass unit = 931 MeV (denoted as a.m.u or u)

$$= 931 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= \frac{931 \times 1.6 \times 10^{-13}}{3 \times 3 \times 10^{16}}$$

$$= 165.5 \times 10^{-29} = 1.655 \times 10^{-27} \text{ kg}$$

(iii) Nuclear binding energy $= (Zm_p + Nm_n - M)c^2$

$$= (Zm_H + Nm_n - M')c^2$$

$$= \Delta m \times 931 \text{ MeV}$$

where M is the mass of the nucleus and M′ is the mass of the atom.